

Chapter 2

A Continuum Damage Model Based on Experiments and Numerical Simulations—A Review

Michael Brüning

Abstract The paper summarizes the author's activities in the field of damage mechanics. In this context, a thermodynamically consistent anisotropic continuum damage model is reviewed. The theory is based on consideration of damaged as well as fictitious undamaged configurations. The modular structure of the continuum model is accomplished by kinematic decomposition of strain rates into elastic, plastic and damage parts. A generalized yield condition is used to adequately describe the plastic flow properties of ductile metals and the plastic strain rate tensor is determined by a non-associated flow rule. Furthermore, a stress-state-dependent damage criterion as well as evolution equations of damage strains are proposed. Different branches of the respective criteria are considered corresponding to various damage and failure mechanisms depending on stress state. Since it is not possible to propose and to validate stress-state-dependent criteria only based on tests with uniaxially loaded specimens for a wide range of stress states, numerical calculations on the micro-level have been performed to be able to study stress-state-dependent mechanisms of micro-defects and their effect on macroscopic behavior. In addition, new experiments with two-dimensionally loaded specimens have been developed. Corresponding numerical simulations of these experiments show that they cover a wide range of stress triaxialities and Lode parameters in the tension, shear and compression domains.

Keywords Ductile damage · Stress triaxiality · Lode parameter · Experiments · Numerical simulations

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2.1 Introduction

The accurate and realistic modeling of inelastic deformation and failure behavior of engineering materials is essential for the solution of numerous boundary-value problems. For example, microscopic defects cause reduction in strength of materials and shorten the life time of engineering structures. Therefore, a main issue in engineering applications is to provide realistic information on the stress distribution within material elements or assessment of safety factors against failure.

Continuum damage mechanics analyzes systematically the effect of damage on mechanical properties of materials. Critical values of damage variables can be seen as major parameters at the onset of fracture. An important issue in such phenomenological constitutive approaches is the appropriate choice of the physical nature of mechanical variables characterizing the damage state of materials and their tensorial representation. Therefore, to be able to develop a realistic, accurate and efficient phenomenological model it is important to analyze and to understand the complex stress-state-dependent processes of damage and fracture as well as its respective mechanisms acting on different scales. In this context, in the last years various damage models have been published based on experimental observations as well as on multi-scale approaches (Brünig 2001, 2003; Gurson 1977; Murakami and Ohno 1981; Lemaitre 1996; Voyiadjis and Kattan 1999). In this context, Brünig et al. (2011b; 2008); Brünig and Gerke (2011) have proposed a generalized and thermodynamically consistent, phenomenological continuum damage model which has been implemented as user-defined material subroutines in commercial finite element programs allowing analyses of static and dynamic problems in differently loaded metal specimens. To be able to detect stress triaxiality dependence of the constitutive equations tension tests with carefully designed specimens have been developed. For example, differently pre-notched specimens and corresponding numerical simulations have been used by Bai and Wierzbicki (2008); Bao and Wierzbicki (2004); Becker et al. (1988); Bonora et al. (2005); Brünig et al. (2011b; 2008); Dunand and Mohr (2011); Gao et al. (2010). However, these experiments with unnotched and differently notched flat specimens showed stress triaxialities only in a small region of positive values. Larger triaxialities appear in tension tests with cylindrical (axisymmetric) specimens but they cannot be manufactured when the behavior of thin sheets is investigated. Thus, specimens with new geometries have been designed to be able to analyze stress states with small hydrostatic parts. Tension tests with these specimens have been performed (Bao and Wierzbicki 2004; Gao et al. 2010) leading to shear mechanisms in their centers. Similar specimens have been developed and tested by Brünig et al. (2008); Driemeier et al. (2010). Furthermore, to be able to take into account other regions of stress triaxialities butterfly specimens have been manufactured (Bai and Wierzbicki 2008; Dunand and Mohr 2011; Mohr and Henn 2007) which can be tested in different directions using special experimental equipment. Alternatively, series of new tests with biaxially loaded flat specimens taken from thin sheets have been developed by Brünig et al. (2014a; 2015) leading to

experimental results on inelastic behavior, damage and fracture of ductile metals for a wide range of stress triaxialities not obtained by the experiments discussed above.

Further information on damage and failure mechanisms can be obtained by performing numerical simulations on the micro-level (Brocks et al. 1995; Brünig et al. 2011a, 2013, 2014b; Chew et al. 2006; Kuna and Sun 1996; Needleman and Kushner 1990; Ohno and Hutchinson 1984; Zhang et al. 2001) considering individual behavior of growth and coalescence of voids and micro-shear-cracks as well as their accumulation to macro-cracks. These numerical calculations have been carried out with different loading conditions covering a wide range of macroscopic stress states. The numerical results elucidated which parameters had remarkable effect on macroscopic stress-strain relations and on evolution equations for the damage variables and which ones only had marginal influence. Thus, it was possible to detect different damage mechanisms which have not been exposed by experiments.

In the present paper main ideas and fundamental governing equations of the phenomenological continuum damage model proposed by the author are reviewed. Some experimental and numerical results are summarized to demonstrate the efficiency and applicability of the approach.

2.2 Continuum Damage Model

Large deformations as well as anisotropic damage and failure of ductile metals are predicted by the continuum model proposed by Brünig (2003) which is based on experimental results and observations as well as on numerical simulations on the micro-level detecting information of microscopic mechanisms due to individual micro-defects and their interactions. Similar to the theories presented by Betten (1982); Grabacki (1991); Murakami and Ohno (1981); Voyiadjis and Park (1999) the phenomenological approach is based on the introduction of damaged and corresponding fictitious undamaged configurations and has been implemented into finite element programs. Extended versions of this model (Brünig et al. 2011b, 2008, 2014a, 2015) propose a stress-state-dependent damage criterion based on different experiments and data from corresponding numerical simulations. Furthermore, it takes into account numerical results of various analyses using unit cell models (Brünig et al. 2011a, 2013, 2014b). Based on these numerical results covering a wide range of stress states it was possible to propose damage equations as functions of the stress triaxiality and the Lode parameter.

2.2.1 Kinematics

The kinematic approach is based on the introduction of initial, current and elastically unloaded configurations each defined as damaged and fictitious undamaged configurations, respectively (Brünig 2001, 2003). This leads to the multiplicative

decomposition of the metric transformation tensor

$$\mathbf{Q} = \mathbf{R}^{o-1} \bar{\mathbf{Q}}^{pl} \mathbf{R}^* \mathbf{Q}^{el} \quad (2.1)$$

where $\mathbf{Q}$ describes the total deformation of the material body due to loading, $\mathbf{R}^o$ represents the initial damage, $\bar{\mathbf{Q}}^{pl}$ denotes the plastic deformation of the fictitious undamaged body, $\mathbf{R}^*$ characterizes the deformation induced by evolution of damage and $\mathbf{Q}^{el}$ represents the elastic deformation of the material body. In addition, the elastic strain tensor

$$\mathbf{A}^{el} = \frac{1}{2} \ln \mathbf{Q}^{el} \quad (2.2)$$

and the damage strain tensor

$$\mathbf{A}^{da} = \frac{1}{2} \ln \mathbf{R}^* \quad (2.3)$$

are introduced.

Furthermore, the strain rate tensor

$$\dot{\mathbf{H}} = \frac{1}{2} \mathbf{Q}^{-1} \dot{\mathbf{Q}} \quad (2.4)$$

is defined, which using Eq. (2.1) can be additively decomposed

$$\dot{\mathbf{H}} = \dot{\mathbf{H}}^{el} + \mathbf{R}^{-1} \dot{\mathbf{H}}^{pl} \mathbf{R} + \mathbf{Q}^{el-1} \dot{\mathbf{H}}^{da} \mathbf{Q}^{el} \quad (2.5)$$

into the elastic

$$\dot{\mathbf{H}}^{el} = \frac{1}{2} \mathbf{Q}^{el-1} \dot{\mathbf{Q}}^{el}, \quad (2.6)$$

the plastic

$$\dot{\mathbf{H}}^{pl} = \frac{1}{2} \mathbf{Q}^{el-1} \bar{\mathbf{Q}}^{pl-1} \dot{\bar{\mathbf{Q}}}^{pl} \mathbf{Q}^{el}, \quad (2.7)$$

and the damage strain rate tensor

$$\dot{\mathbf{H}}^{da} = \frac{1}{2} \mathbf{R}^{*-1} \dot{\mathbf{R}}^*. \quad (2.8)$$

With the identity (see Brünig (2003) for further details)

$$\mathbf{R}^* \mathbf{Q}^{el} = \mathbf{Q}^{el} \mathbf{R} \quad (2.9)$$

the damage metric transformation tensor with respect to the current loaded configurations, $\mathbf{R}$ (see Eq. (2.5)), has been introduced.

2.2.2 Constitutive Equations

Dislocations and growth of micro-defects are the most common modes of irreversible micro-structural deformations at each stage of the loading process. In particular, pure plastic flow is caused by dislocation motion and sliding phenomena along crystallographic planes whereas damage-related irreversible deformations are due to residual opening of micro-defects after unloading. These dissipative processes are distinctly different in their nature as well as in the manner how they affect the compliance of the material and are active on different scales. Consequently, in damage-coupled elastic-plastic theories two sets of internal variables corresponding to formation of dislocations (plastic internal variables) as well as to nucleation and growth of micro-defects (damage internal variables) are separately used to adequately describe the irreversible constitutive behavior and to compute corresponding strain rate tensors.

The effective undamaged configurations are considered to characterize the behavior of the undamaged matrix material. In particular, the elastic behavior of the undamaged matrix material is described by an isotropic hyperelastic law leading to the effective Kirchhoff stress tensor

$$\bar{\mathbf{T}} = 2G \mathbf{A}^{el} + \left(K - \frac{2}{3}G \right) \text{tr} \mathbf{A}^{el} \mathbf{1} \quad (2.10)$$

where G and K represent the constant shear and bulk modulus of the undamaged matrix material, respectively, and $\mathbf{1}$ denotes the second order unit tensor.

In addition, onset and continuation of plastic flow of ductile metals is determined a yield criterion. Experiments on the influence of hydrostatic stress on the behavior of metals carried out by Spitzig et al. (1975; 1976) have shown that the flow stress depends approximately linearly on hydrostatic stress. Additional numerical studies performed by Brünig (1999) elucidated that hydrostatic stress terms in the yield condition affect onset of localization and associated deformation modes leading to notable decrease in ductility. Thus, plastic yielding of the matrix material is governed by the Drucker-Prager-type yield condition

$$f^{pl}(\bar{I}_1, \bar{J}_2, c) = \sqrt{\bar{J}_2} - c \left(1 - \frac{a}{c} \bar{I}_1 \right) = 0, \quad (2.11)$$

where $\bar{I}_1 = \text{tr} \bar{\mathbf{T}}$ and $\bar{J}_2 = \frac{1}{2} \text{dev} \bar{\mathbf{T}} \cdot \text{dev} \bar{\mathbf{T}}$ denote invariants of the effective stress tensor $\bar{\mathbf{T}}$, c is the strength coefficient of the matrix material and a represents the hydrostatic stress coefficient.

The experiments (Spitzig et al. 1975, 1976) also showed that plastic deformations are nearly isochoric. Thus, the plastic potential function

$$g^{pl}(\bar{\mathbf{T}}) = \sqrt{\bar{J}_2} \quad (2.12)$$

is assumed to depend only on the second invariant of the effective stress deviator leading to the non-associated isochoric effective plastic strain rate

$$\dot{\mathbf{H}}^{pl} = \dot{\lambda} \frac{\partial g^{pl}}{\partial \bar{\mathbf{T}}} = \dot{\lambda} \frac{1}{2\sqrt{J_2}} \text{dev} \bar{\mathbf{T}} = \dot{\gamma} \bar{\mathbf{N}}. \quad (2.13)$$

In Eq. (2.13) $\dot{\lambda}$ is a non-negative scalar-valued factor, $\bar{\mathbf{N}} = \frac{1}{\sqrt{2J_2}} \text{dev} \bar{\mathbf{T}}$ denotes the normalized deviatoric stress tensor and $\dot{\gamma} = \bar{\mathbf{N}} \cdot \dot{\mathbf{H}}^{pl} = \frac{1}{\sqrt{2}} \dot{\lambda}$ represents the equivalent plastic strain rate measure used in the present continuum model.

Furthermore, the damaged configurations are considered to characterize the behavior of anisotropically damaged material samples. In particular, since damage remarkably affects the elastic behavior and leads to material softening, the elastic law of the damaged material sample is expressed in terms of both the elastic and the damage strain tensor, (2.2) and (2.3). The Kirchhoff stress tensor of the ductile damaged metal is given by

$$\begin{aligned} \mathbf{T} = & 2 \left(G + \eta_2 \text{tr} \mathbf{A}^{da} \right) \mathbf{A}^{el} \\ & + \left[\left(K - \frac{2}{3} G + 2\eta_1 \text{tr} \mathbf{A}^{da} \right) \text{tr} \mathbf{A}^{el} + \eta_3 \left(\mathbf{A}^{da} \cdot \mathbf{A}^{el} \right) \right] \mathbf{1} \\ & + \eta_3 \text{tr} \mathbf{A}^{el} \mathbf{A}^{da} + \eta_4 \left(\mathbf{A}^{el} \mathbf{A}^{da} + \mathbf{A}^{da} \mathbf{A}^{el} \right) \end{aligned} \quad (2.14)$$

where the parameters $\eta_1 \dots \eta_4$ describe the deteriorating effect of damage on the elastic material properties. The parameters η_1 and η_2 are related to the isotropic character of damage whereas η_3 and η_4 correspond to anisotropic evolution of damage. With these additional parameters it is possible to simulate the decreases in Young's modulus, Poisson's ratio, shear and bulk moduli measured in experiments (Brünig 2003).

Based on many experiments it is well known that the stress state remarkably affects damage mechanisms occurring in ductile metals which is illustrated in Fig. 2.1 (Brünig et al. 2011a,b). For example, under tension dominated loading conditions (high positive stress triaxialities, $\eta \geq \eta_t$) damage in ductile metals is mainly caused

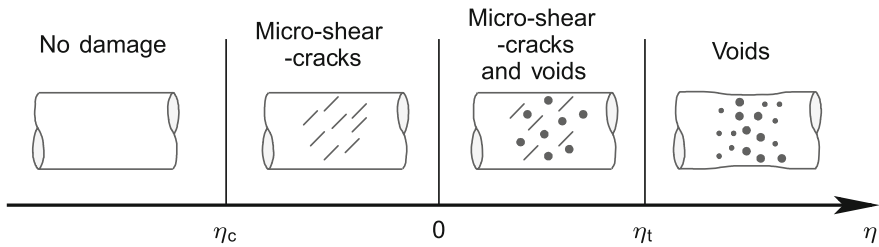


Fig. 2.1 Damage mechanisms depending on stress triaxialities

by nucleation and isotropic growth of voids whereas under shear and compression dominated conditions (negative stress triaxialities, $\eta_c \leq \eta \leq 0$) evolution of micro-shear-cracks is the predominant damage mechanism. Between these regions, $0 < \eta < \eta_t$ damage is a combination of both basic mechanisms with simultaneous growth of voids and formation of micro-shear-cracks. In addition, no damage occurs in ductile metals for high negative stress triaxialities $\eta < \eta_c$. Therefore, to be able to develop a realistic, accurate and efficient phenomenological model it is important to analyze and to understand the complex stress-state-dependent processes of damage as well as its respective mechanisms acting on different scales.

To model onset and continuation of damage the concept of damage surface is adopted. Following the ideas given in Brünig (2003); Chow and Wang (1987) the damage surface is formulated in stress space at the macroscopic damaged continuum level

$$f^{da} = \alpha I_1 + \beta \sqrt{J_2} - \sigma = 0 \quad (2.15)$$

with the stress invariants $I_1 = \text{tr} \mathbf{T}$ and $J_2 = \frac{1}{2} \text{dev} \mathbf{T} \cdot \text{dev} \mathbf{T}$ and the damage threshold σ . In Eq. (2.15) the variables α and β represent damage mode parameters depending on the stress intensity (von Mises equivalent stress) $\sigma_{eq} = \sqrt{3J_2}$, the stress triaxiality

$$\eta = \frac{\sigma_m}{\sigma_{eq}} = \frac{I_1}{3\sqrt{3J_2}} \quad (2.16)$$

with the mean stress $\sigma_m = 1/3 I_1$ as well as on the Lode parameter

$$\omega = \frac{2\tilde{T}_2 - \tilde{T}_1 - \tilde{T}_3}{\tilde{T}_1 - \tilde{T}_3} \quad \text{with } \tilde{T}_1 \geq \tilde{T}_2 \geq \tilde{T}_3 \quad (2.17)$$

expressed in terms of the principal stress components $\tilde{T}_1$, $\tilde{T}_2$ and $\tilde{T}_3$.

Furthermore, evolution of macroscopic irreversible strains caused by the simultaneous nucleation, growth and coalescence of different micro-defects is modeled by a stress-state-dependent damage rule. Thus, the damage potential function

$$g^{da}(\tilde{\mathbf{T}}) = g^{da}(I_1, J_2, J_3) \quad (2.18)$$

is introduced where $\tilde{\mathbf{T}}$ represents the stress tensor formulated in the damaged configuration which is work-conjugate to the damage strain rate tensor $\dot{\mathbf{H}}^{da}$ (see Brünig 2003, for further details) and I_1 , J_2 and J_3 are corresponding invariants. This leads to the damage strain rate tensor

$$\dot{\mathbf{H}}^{da} = \dot{\mu} \frac{\partial g^{da}}{\partial \tilde{\mathbf{T}}} = \dot{\mu} \left(\frac{\partial g^{da}}{\partial I_1} \mathbf{1} + \frac{\partial g^{da}}{\partial J_2} \text{dev} \tilde{\mathbf{T}} + \frac{\partial g^{da}}{\partial J_3} \text{dev} \tilde{\mathbf{S}} \right) \quad (2.19)$$

where $\dot{\mu}$ is a non-negative scalar-valued factor and

$$\text{dev}\tilde{\mathbf{S}} = \text{dev}\tilde{\mathbf{T}} \text{dev}\tilde{\mathbf{T}} - \frac{2}{3} J_2 \mathbf{1} \quad (2.20)$$

represents the second order deviatoric stress tensor (Brünig et al. 2013, 2014b). Alternatively, the damage strain rate tensor (2.19) can be written in the form

$$\dot{\mathbf{H}}^{da} = \dot{\mu} \left(\bar{\alpha} \frac{1}{\sqrt{3}} \mathbf{1} + \bar{\beta} \mathbf{N} + \bar{\delta} \mathbf{M} \right) \quad (2.21)$$

where the normalized deviatoric tensors $\mathbf{N} = \frac{1}{2\sqrt{J_2}} \text{dev}\tilde{\mathbf{T}}$ and $\mathbf{M} = \frac{1}{\|\text{dev}\tilde{\mathbf{S}}\|} \text{dev}\tilde{\mathbf{S}}$ have been used. In Eq. (2.21) the parameters $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\delta}$ are kinematic variables describing the portion of volumetric and isochoric damage-based deformations. The damage rule (2.21) takes into account isotropic and anisotropic parts corresponding to isotropic growth of voids and anisotropic evolution of micro-shear-cracks, respectively. Therefore, both basic damage mechanisms (growth of isotropic voids and evolution of micro-shear-cracks) acting on the micro-level are involved in the damage rule (2.21).

2.3 Uniaxial Tension Tests

Uniaxial tension tests with unnotched specimens taken from thin sheets are used to identify basic elastic-plastic material parameters appearing in the constitutive equations discussed above. From these experiments equivalent stress–equivalent plastic strain curves are easily obtained from load-displacement curves as long as the uniaxial stress field remains homogeneous between the clip gauges fixed on the specimens during the tests (Brünig et al. 2011b, 2008). For example, fitting of numerical curves and experimental data for an aluminum alloy leads to Young’s modulus $E = 65000 \text{ MPa}$ and Poisson’s ratio is taken to be $\nu = 0.3$. In addition, the power law function for the equivalent stress–equivalent plastic strain function appearing in the yield criterion (2.11)

$$c = c_0 \left(\frac{H}{n} \frac{\gamma}{c_0} + 1 \right)^n \quad (2.22)$$

is used to model the work-hardening behavior. Good agreement of experimental data and numerical results is achieved for the initial yield strength $c_0 = 175 \text{ MPa}$, the hardening modulus $H = 2100 \text{ MPa}$ and the hardening exponent $n = 0.22$ (Brünig et al. 2015).

Since uniaxial tension tests with differently pre-notched specimens are characterized by different hydrostatic stress states in the small region between the notches (Brünig et al. 2011b, 2008; Driemeier et al. 2010), these experiments can be used to determine the hydrostatic-stress-dependent parameter a/c appearing in the yield

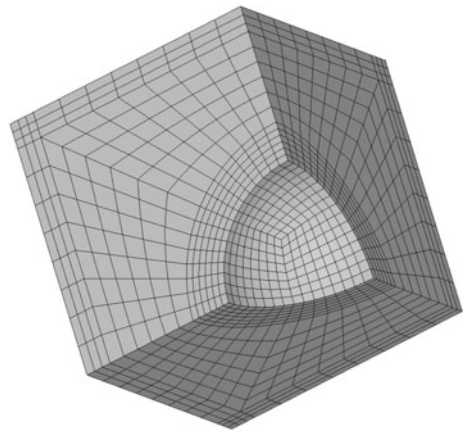
criterion (2.11). However, in these tests with pre-notched specimens stress and strain fields are not homogeneous and only quantities in an average sense can be evaluated from these experiments. Therefore, corresponding finite element simulations of these uniaxial tests have been performed. Comparison of the numerically predicted load-displacement curves and the experimental ones leads with an inverse identification procedure to the parameter $a/c = 0.000055 \text{ MPa}^{-1}$.

Moreover, various parameters corresponding to the damage part of the constitutive model have to be identified. For example, onset of damage is determined by comparison of experimental load-displacement curves of the uniaxial tension tests with those predicted by numerical analyses based on the elastic-plastic model only (Brünig et al. 2011b, 2015). First deviation of these curves is an indicator of onset of deformation-induced damage. Using this procedure, the damage threshold appearing in the damage condition (2.15) is $\sigma = 300 \text{ MPa}$ for the aluminum alloy under investigation.

2.4 Numerical Analyses on the Micro-Scale

Identification of further damage parameters is more complicated because most criteria and evolution equations are motivated by observations on the micro-level. Therefore, stress state dependence of the parameters in the damage condition (2.21) and in the damage rule (2.21) has been studied in detail by Brünig et al. (2013; 2014b) performing numerical simulations with micro-void-containing representative volume elements under various loading conditions. In particular, the numerical analyses of differently loaded micro-defect containing unit cells have been performed using the finite element program ANSYS enhanced by an user-defined material subroutine. One eighth of the symmetric cell model as well as the finite element mesh with 3123 3D-elements of type Solid185 are shown in Fig. 2.2. Various loading conditions with

Fig. 2.2 Finite element mesh of one eighth of the unit cell



principal macroscopic stresses acting on the outer bounds of the representative volume elements are taken into account to be able to cover the wide range of stress triaxiality coefficients (16) $\eta = -1, -2/3, -1/3, 0, 1/3, 2/3, 1, 5/3, 2, 7/3$ and 3 as well as of the Lode parameters (2.23) $\omega = -1, -1/2, 0, 1/2$, and 1.

In all micro-mechanical numerical calculations the stress triaxiality coefficient and the Lode parameter are kept constant during the entire loading history of the unit cell to be able to accurately analyze their effect on damage and failure behavior of ductile metals. Based on the numerical results of these unit-cell calculations (Brünig et al. 2013) different damage mode parameters have been identified for the aluminum alloy: In particular, the damage mode parameter α (see Eq. (2.15)) is given by

$$\alpha(\eta) = \begin{cases} 0 & \text{for } -\frac{1}{3} \leq \eta \leq 0 \\ \frac{1}{3} & \text{for } \eta > 0 \end{cases} \quad (2.23)$$

whereas β is taken to be the non-negative function

$$\beta(\eta, \omega) = \beta_0(\eta, \omega = 0) + \beta_\omega(\omega) \geq 0, \quad (2.24)$$

with

$$\beta_0(\eta) = \begin{cases} -0.45\eta + 0.85 & \text{for } -\frac{1}{3} \leq \eta \leq 0 \\ -1.28\eta + 0.85 & \text{for } \eta > 0 \end{cases} \quad (2.25)$$

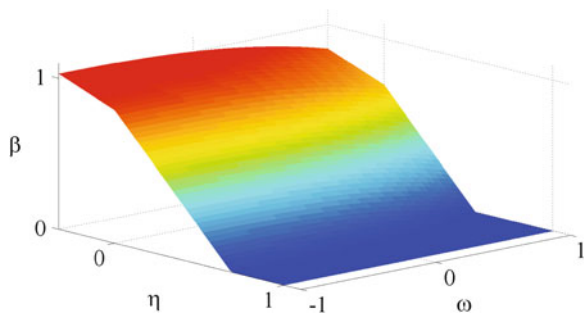
and

$$\beta_\omega(\omega) = -0.017\omega^3 - 0.065\omega^2 - 0.078\omega. \quad (2.26)$$

This function $\beta(\eta, \omega)$ is visualized in Fig. 2.3.

In addition, based on the results of the unit cell analyses the stress-state-dependent parameters $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\delta}$ in the damage rule (2.27) have been identified by Brünig et al. (2013). The non-negative parameter $\bar{\alpha} \geq 0$ characterizing the amount of volumetric damage strain rates caused by isotropic growth of micro-defects is given by the relation

Fig. 2.3 Damage mode parameter β versus stress triaxiality η and Lode parameter ω



$$\bar{\alpha}(\eta) = \begin{cases} 0 & \text{for } \eta < 0.09864 \\ -0.07903 + 0.80117 \eta & \text{for } 0.09864 \leq \eta \leq 1 \\ 0.49428 + 0.22786 \eta & \text{for } 1 < \eta \leq 2 \\ 0.87500 + 0.03750 \eta & \text{for } 2 < \eta \leq 3.33333 \\ 1 & \text{for } \eta > 3.33333 \end{cases} . \quad (2.27)$$

The parameter $\bar{\alpha}$ is high for high stress triaxialities, smaller for moderate ones and zero for negative triaxialities. Dependence on the Lode parameter ω has not been revealed by the numerical simulations on the micro-scale. In addition, the parameter $\bar{\beta}$ characterizing the amount of anisotropic isochoric damage strain rates caused by evolution of micro-shear-cracks is given by the relation

$$\bar{\beta}(\eta, \omega) = \bar{\beta}_0(\eta) + f_{\beta}(\eta) \bar{\beta}_{\omega}(\omega) \quad (2.28)$$

with

$$\bar{\beta}(\eta, \omega) = \begin{cases} 0.94840 + 0.11965 \eta + f_{\beta}(\eta) \bar{\beta}_{\omega}(\omega) & \text{for } -\frac{1}{3} \leq \eta \leq \frac{1}{3} \\ 1.14432 - 0.46810 \eta + f_{\beta}(\eta) \bar{\beta}_{\omega}(\omega) & \text{for } \frac{1}{3} < \eta \leq \frac{2}{3} \\ 1.14432 - 0.46810 \eta & \text{for } \frac{2}{3} < \eta \leq 2 \\ 0.52030 - 0.15609 \eta & \text{for } 2 < \eta \leq \frac{10}{3} \\ 0 & \text{for } \eta > \frac{10}{3} \end{cases} \quad (2.29)$$

with

$$f_{\beta}(\eta) = -0.0252 + 0.0378 \eta \quad (2.30)$$

and

$$\bar{\beta}_{\omega}(\omega) = 1 - \omega^2 . \quad (2.31)$$

The parameter $\bar{\delta}$ also corresponding to the anisotropic damage strain rates caused by the formation of micro-shear-cracks is given by the relation

$$\bar{\delta}(\eta, \omega) = f_{\delta}(\eta) \bar{\delta}_{\omega}(\omega) \quad (2.32)$$

with

$$\bar{\delta}(\eta, \omega) = \begin{cases} (-0.12936 + 0.19404 \eta)(1 - \omega^2) & \text{for } -\frac{1}{3} \leq \eta \leq \frac{2}{3} \\ 0 & \text{for } \eta > \frac{2}{3} \end{cases} \quad (2.33)$$

It is worthy to note that this parameter $\bar{\delta}$ only exists for small stress triaxialities and mainly depends on the Lode parameter ω . Although its magnitude is small in comparison to the parameters $\bar{\alpha}$ and $\bar{\beta}$ its effect on the evolution of macroscopic damage strain rates $\dot{\mathbf{H}}^{da}$ is not marginal (Brünig et al. 2013).

2.5 Experiments and Numerical Simulations with Biaxially Loaded Specimens

The damage related functions discussed above are only based on numerical calculations on the micro-level and their validation has to be realized by experiments also covering a wide range of stress states. This is not possible by the uniaxial tests mentioned above and, therefore, new experiments with biaxially loaded specimens have been developed by Brünig et al. (2014a; 2015). The experiments are performed using the biaxial test machine (Type LFM-BIAX 20 kN from Walter · Bai, Switzerland) shown in Fig. 2.4. It is composed of four electro-mechanically, individually driven cylinders with load maxima and minima of ± 20 kN (tension and compression loading is possible). The specimens are fixed in the four heads of the cylinders where clamped or hinged boundary conditions are possible. The geometry of the flat specimens and the loading conditions are shown in Fig. 2.5. The load F_1 will lead to shear mechanisms in the center of the specimen whereas simultaneous loading with F_2 leads to superimposed tension or compression modes leading to shear-tension or shear-compression deformation and failure modes. Therefore, these tests cover the full range of stress states corresponding to the damage and failure mechanisms discussed above with focus on high positive as well as low positive, nearly zero and negative stress triaxialities.

Figure 2.6 clearly shows that different load ratios $F_1 : F_2$ have remarkable influence on the damage and final fracture modes. For example, for the tension test without

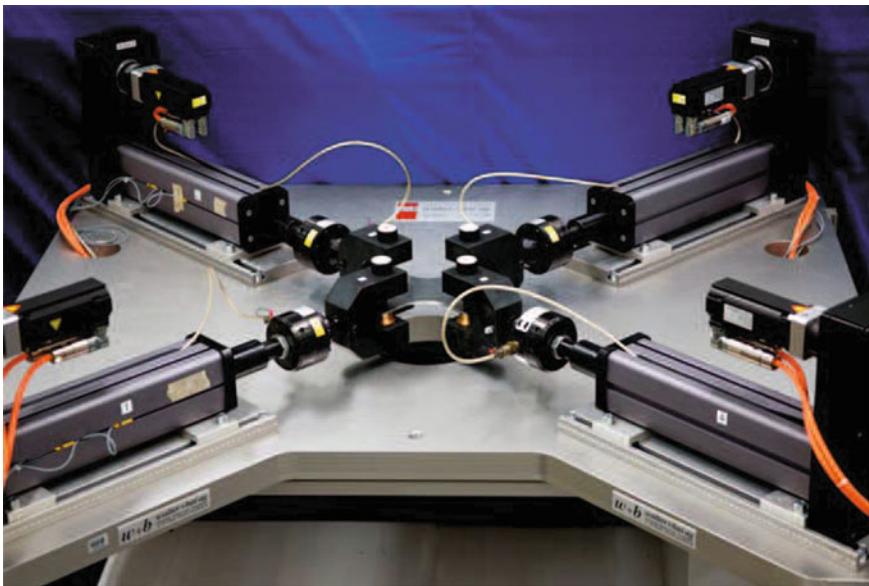
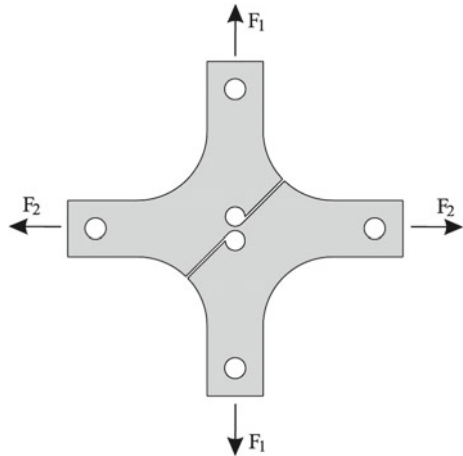


Fig. 2.4 Biaxial test machine

Fig. 2.5 Specimen and loading conditions



shear loading, $F_1 : F_2 = 0 : 1$, a nearly vertical fracture line is obtained. Under this loading condition, damage is mainly caused by growth and coalescence of voids with small influence of micro-shear-cracks leading to the macroscopic tensile fracture mode with small cup-cone fracture effect (Fig. 2.6a). On the other hand, for pure shear loading, $F_1 : F_2 = 1 : 0$, shear fracture is observed where the fracture line has an angle of about 25° with respect to the vertical line. Under this loading condition, damage is mainly caused by formation of micro-shear-cracks leading to the macro-shear-crack shown in Fig. 2.6b. For shear-tension loading, $F_1 : F_2 = 1 : 0.5$, shear-tension fracture is obtained and the fracture line has an angle of only about 10° with respect to the vertical line. Under this loading condition, damage is caused by the simultaneous growth of voids and formation of micro-shear-cracks leading to the macroscopic fracture mode shown in Fig. 2.6c which is between tensile fracture (Fig. 2.6a) and shear fracture (Fig. 2.6b). However, for shear-compression loading, $F_1 : F_2 = 1 : -0.5$, again shear fracture is observed and the fracture line again has an angle of about 25° with respect to the vertical line. Under this loading condition damage seems to be caused only by formation of micro-shear-cracks leading to the macro-shear-crack shown in Fig. 2.6d. It is worthy to note that the damage and failure mechanisms for $F_1 : F_2 = 1 : 0$ (Fig. 2.6b) and $F_1 : F_2 = 1 : -0.5$ (Fig. 2.6d) seem to be very similar and damage modes characteristic for shear loading will not be remarkably affected by superimposed small compression loads.

Furthermore, corresponding numerical simulations have been performed to be able to get detailed information on amounts and distributions of different stress and strain measures as well as further parameters of interest especially in critical regions (Brüning et al. 2015). For example, numerically predicted distributions of the stress triaxiality η at the onset of damage are shown in Fig. 2.7 for different load ratios. In particular, for uniaxial tension loading $F_1 : F_2 = 0 : 1$ remarkable high stress triaxialities up to $\eta = 0.84$ are numerically predicted in the center of the specimen. These high values are caused by the notches in horizontal and thickness direction

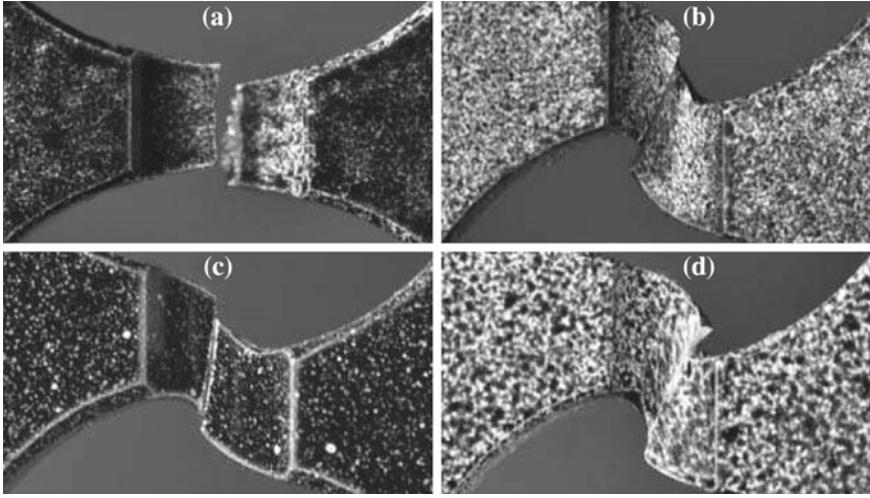


Fig. 2.6 Fracture modes of differently loaded specimens: **a** $F_1 : F_2 = 0 : 1$, **b** $F_1 : F_2 = 1 : 0$, **c** $F_1 : F_2 = 1 : 0.5$, **d** $F_1 : F_2 = 1 : -0.5$

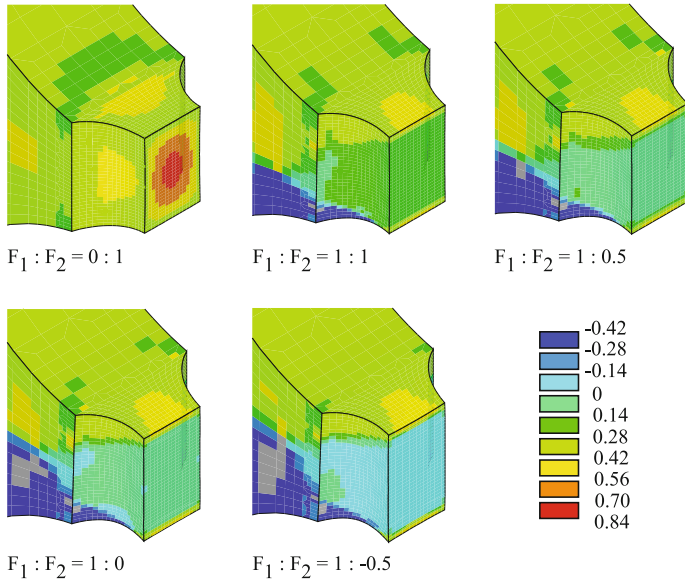


Fig. 2.7 Distribution of the stress triaxiality η for different load ratios

leading to high hydrostatic stress during elongation of the specimen. This will lead to damage and failure mainly due to growth of voids. In addition, when the specimen is only loaded by $F_1 : F_2 = 1 : 0$ (shear loading condition) the stress triaxiality η is numerically predicted to be nearly zero in the whole vertical section shown in Fig. 2.7.

Thus, damage will start in the specimen's center and will be caused by formation and growth of micro-shear-cracks. Furthermore, combined loading in vertical and horizontal direction, F_1 and F_2 , will lead to combination of these basic damage modes. For example, for the load ratio $F_1 : F_2 = 1 : 1$ the stress triaxiality is again nearly constant in the vertical section with $\eta = 0.25$, and very similar distribution is numerically predicted for $F_1 : F_2 = 1 : 0.5$ with $\eta = 0.14$. On the other hand, the load ratio $F_1 : F_2 = 1 : -0.5$ represents combined shear-compression loading with the stress triaxiality $\eta = -0.14$ which is nearly constant in the vertical section shown in Fig. 2.7. For this negative stress triaxiality η damage and failure will be caused by formation and growth of micro-shear-cracks only. These numerical results nicely correspond to the fracture modes discussed above and shown in Fig. 2.6.

The stress triaxialities η covered by experiments with different flat specimens manufactured from thin sheets are shown in Fig. 2.8. In particular, for unnotched dog-bone-shape specimens (green) nearly homogeneous stress states occur in the small part during uniaxial tension tests with stress triaxiality $\eta = 1/3$. Higher stress triaxialities can be obtained in uniaxial tension tests when notches with different radii are added in the middle part of the specimens (red). Decrease in notch radius will lead to an increase in stress triaxiality in the specimen's center up to $\eta = 1/\sqrt{3}$. In addition, shear specimens (blue) elongated in uniaxial tension test will lead to stress triaxialities of about $\eta = 0.1$ when notches in thickness direction are added in the central part (Brünig et al. 2011b, 2008; Driemeier et al. 2010) whereas without additional notch they will also lead to onset of damage at nearly $\eta = 1/3$. However, with these flat specimens taken from thin sheets elongated in uniaxial tension tests only the stress triaxialities shown in Fig. 2.8 (green, red and blue points) can be taken into account whereas no information is obtained for high positive ($\eta > 1/\sqrt{3}$), low positive (between $0.1 < \eta < 1/3$) or negative stress triaxialities ($\eta < 0$). However, further experiments with new specimens (grey) tested under biaxial loading conditions will lead to stress triaxialities in the requested regimes. The grey points shown in Fig. 2.8 correspond to the loading conditions discussed above but variation

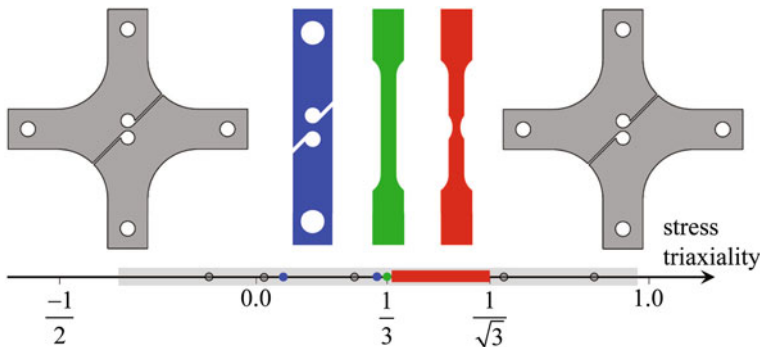


Fig. 2.8 Stress triaxialities covered by different specimens

of the load ratios $F_1 : F_2$ may lead to stress triaxialities marked by the grey zone shown in Fig. 2.8. Therefore, biaxial tests with 2D specimens discussed above cover a wide range of stress triaxialities and Lode parameters.

2.6 Conclusions

The author's activities in the field of damage mechanics have been summarized. The continuum model taking into account stress-state-dependent damage criteria and damage evolution laws has been reviewed. Elastic-plastic material parameters as well as onset of damage have been identified by uniaxial tension tests with smooth and pre-notched flat specimens and corresponding numerical simulations. Further damage parameters depending on stress triaxiality and Lode parameter have been determined using numerical results from unit-cell calculations on the micro-level. Since the proposed functions are only based on numerical analyses validation of the stress-state-dependent model was required. In this context, a series of experiments with biaxially loaded specimens has been presented. Different load ratios led to shear-tension and shear-compression mechanisms with different fracture modes. Corresponding finite element simulations of the experiments revealed a wide range of stress states covered by the tests depending on biaxial loading conditions. They could be used to validate the proposed stress-state-dependent functions of the continuum model.

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Altenbach, H.; Matsuda, T.; Okumura, D. (Eds.)

2015, XVIII, 601 p. 350 illus., 207 illus. in color.,

Hardcover

ISBN: 978-3-319-19439-4