Preface to the English Edition

This is a revised and augmented English edition of my book “Dimension topologique et systèmes dynamiques” which was published in 2005 by the *Société Mathématique de France*. As explained in the preface to the French edition, the goal of the book is to provide a self-contained introduction to mean topological dimension, an invariant of dynamical systems introduced in 1999 by Misha Gromov, and explain how this invariant was successfully used by Elon Lindenstrauss and Benjamin Weiss to answer a long-standing open question about embeddings of minimal dynamical systems into shifts. A large number of revisions and additions have been made to the original text. Chapter 5 contains an entirely new section devoted to the Sorgenfrey line. Two chapters have also been added: Chap. 9 on amenable groups and Chap. 10 on mean topological dimension for continuous actions of countable amenable groups. These new chapters contain material that has never before appeared in textbook form. The chapter on amenable groups is based on Følner’s characterization of amenability and may be read independently from the rest of the book. There are a total of 160 exercises. The hardest ones are accompanied with hints. Although the contents of this book lead directly to several active areas of current research in mathematics and mathematical physics, the prerequisites needed for reading it remain modest, essentially some familiarities with undergraduate point-set topology and, in order to access the final two chapters, some acquaintance with basic notions in group theory.

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Michel Coornaert

Preface to the French Edition

This book grew out from a DEA course I gave at the University of Strasbourg in Spring 2002. The first part of the book presents some fundamental results from dimension theory. The second part is devoted to topological mean dimension and its applications to embeddings problems for dynamical systems.

Dimension theory is the branch of general topology that studies the notion of dimension for topological spaces. It has its root at the origins of geometry and the difficulties encountered by mathematicians when trying to give rigorous definitions of the concepts of curves and surfaces. The theory flourished at the end of the nineteenth century and at the beginning of the twentieth century. Its developments had a deep impact on many other branches of mathematics such as algebraic topology, dynamical systems, and probability theory. Actually, several non-equivalent definitions of dimension for topological spaces may be found in the literature. The most commonly used are the small inductive dimension ind, the large inductive dimension Ind, and the covering dimension dim. Small inductive dimension was introduced by P. Urysohn in 1922 and independently by K. Menger in 1923. Large inductive dimension and covering dimension were introduced by E. Čech in 1931. These three dimensions coincide for separable metrizable spaces.

Mean topological dimension is a conjugacy invariant of topological dynamical systems which was recently introduced by Gromov [44]. This invariant enables one to distinguish systems with infinite topological entropy. It was used by Lindenstrauss and Weiss [74] to answer a long-standing open question about the existence of embeddings of minimal dynamical systems into shifts.

Chapter 1 begins with the definition of the covering dimension of a topological space and the proof of its main properties. We establish in particular the countable union theorem in normal spaces and the monotonicity theorem in metric spaces.

The second chapter is devoted to 0-dimensional topological spaces. Examples of such spaces are given and we investigate the relationship between the class of 0-dimensional spaces and other classes of highly disconnected topological spaces.

The notion of a polyhedron is introduced in Chap. 3. A polyhedron is a topological space that can be triangulated, i.e., is homeomorphic to the geometric realization of some finite simplicial complex. We prove Lebesgue's lemma on open covers of Euclidean cubes. It is used to show that the covering dimension of a polyhedron is equal to the combinatorial dimension of any of its triangulations. We also deduce from Lebesgue's lemma that the covering dimension of $\mathbb{R}^n$ is equal to n as expected.

In Chap. 4, we prove Aleksandrov theorem about topological dimension of compact metrizable spaces and ε-injective maps. We then establish the Menger-Nöbeling embedding theorem that states that any n-dimensional compact metrizable space can be embedded in $\mathbb{R}^{2n+1}$.

Chapter 5 is devoted to the study of counterexamples which played an important role in the history of dimension theory: Erdös and Bing spaces, Knaster-Kuratowski fan, Tychonoff plank. These counterexamples enlighten the validity domains of some of the theorems established in the previous chapters.

In Chap. 6, the mean topological dimension $\mathrm{mdim}(X,T)$ of a discrete dynamical system (X,T), where X is a normal space and $T : X \to X$ a continuous map, is defined and its first properties are established. When X is a compact metric space, an equivalent definition of $\mathrm{mdim}(X,T)$ involving the metric is given.

In Chap. 7, we consider the dynamical system $(K^{\mathbb{Z}}, \sigma)$, where $K^{\mathbb{Z}}$ is the space of bi-infinite sequences of points in a topological space K and σ is the shift map $(x_i) \mapsto (x_{i+1})$. We show that $\mathrm{mdim}(K^{\mathbb{Z}}, \sigma) \leq \dim(K)$ for any compact metrizable space K and that equality holds when K is a polyhedron. By considering appropriate subshifts, we show that mean topological dimension can take any value in $[0, \infty]$.

Chapter 8 discusses embeddings problems of dynamical systems into shifts. We prove the theorem of Jaworski that asserts that any dynamical system (X,T), where T is a homeomorphism without periodic points of a finite-dimensional compact metrizable space X, can be embedded into the shift $(\mathbb{R}^{\mathbb{Z}}, \sigma)$. Finally, we describe the Lindenstrauss-Weiss counterexamples which show that Jaworski's theorem becomes false if the hypothesis on the finiteness of the topological dimension is removed.

There are historical notes and a list of exercises at the end of each chapter. All along the text, I tried to give detailed proofs in order to make them accessible to students who attended a basic course on general topology. The terminology used is that of Bourbaki with the exception that compact (resp. locally compact, resp. normal, resp. scattered) spaces are not required to be Hausdorff.

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Coornaert, M.
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