

## Chapter 2

# Physics Beyond the Standard Model: Supersymmetry

In this chapter, one model for physics beyond the SM is introduced: supersymmetry (SUSY). As it will be shown, this model will “solve” several of the problems mentioned in Sect. 1.4.

The concept behind SUSY is quite simple: There exists a symmetry relating fermions and bosons, or to be more precise: There exists an operator changing the spin of a particle by  $1/2$ . As this operator changes only the spin, all superpartners of the SM particles should already have been observed, as they would have the same masses. This is clearly not the case. However, an escape to this is SUSY breaking, which can lead to larger masses for the superpartners.

The transformation between fermions and bosons can be simply expressed as

$$Q|b\rangle = |f\rangle, \quad Q|f\rangle = |b\rangle, \quad (2.1)$$

where  $Q$  is the symmetry operator,  $|f\rangle$  a fermionic state, and  $|b\rangle$  a bosonic state.

The operator  $Q$  itself must carry spin  $1/2$  and is fermionic. This symmetry operator<sup>1</sup> must satisfy the following algebra [2, 3]:

$$\{Q_\alpha, Q_\beta^\dagger\} = 2\sigma_{\alpha\beta}^\mu P_\mu, \quad (2.2)$$

$$\{Q_\alpha, Q_\beta\} = \{Q_\alpha^\dagger, Q_\beta^\dagger\} = 0, \quad (2.3)$$

$$[P^\mu, Q_\alpha] = [Q_\beta^\dagger, P^\mu] = 0, \quad (2.4)$$

$$[M_{\mu\nu}, Q_\alpha] = \frac{1}{2}(\sigma_{\mu\nu})_\alpha^\beta Q_\beta, \quad (2.5)$$

with  $\sigma^{1,2,3}$  being the Pauli matrices,  $\sigma^0$  the identity matrix, and  $\alpha, \beta$  being spinor indices. The generator of the translation and Lorentz groups are  $P_\mu$  and  $M_{\mu\nu}$ , respectively, and  $(\sigma_{\mu\nu})_\alpha^\beta = \frac{1}{4}((\sigma_\mu)_{\alpha\gamma}(\sigma_\nu)^{\gamma\beta} - (\sigma_\nu)_{\alpha\gamma}(\sigma_\mu)^{\gamma\beta})$ .

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<sup>1</sup>The representation here is the so-called  $N = 1$  supersymmetry, see for example [1].

Equation (2.5) simply states that  $Q_1$  ( $Q_2$ ) changes the  $z$  component of the spin by  $+\frac{1}{2}$  ( $-\frac{1}{2}$ ). Similarly,  $Q_1^\dagger$  ( $Q_2^\dagger$ ) changes the  $z$  component of the spin by  $-\frac{1}{2}$  ( $+\frac{1}{2}$ ). As  $Q$  has to commute with the Hamiltonian (it is a symmetry operator), so has its anticommutator: the Eq. (2.2) follows, as the anticommutator has to be a conserved spin-1 operator, and therefore  $\sim P_\mu$ . In principle, states with spins between  $-2$  up to  $2$  are allowed, thus up to eight SUSY operators  $Q$  are possible, since an operator cannot be applied twice (follows from Eq. (2.3)). The discussion in this thesis is restricted to SUSY with one such operator. The symmetry operator  $Q$  is the only possible extension to the Poincaré group (follows from the commutator relations of Eqs. (2.4) and (2.5) and [2, 3]).

In theory, the fields of the SM particles and their supersymmetric partners can be arranged into supermultiplets. As the symmetry operators commute with the generators of the gauge group, all particles within the same supermultiplet have the same electric, weak, and color charges. An elegant formulation of SUSY can be obtained using the superspace formulation. One displays all fields within a supermultiplet in one object, the superfield [4]. While we do not engage in the discussion of superfields here (for more details, see for instance [5]), we want to mention that the properties of SUSY can be derived directly in this formulation, such as the fact that the numbers of degrees of freedom for a boson field and its fermion partner field have to be identical. This argument will also lead to the introduction of the non-propagating auxiliary fields  $F$  and  $D$ , as discussed later.

The naming convention of the partner fields, that will be used hereafter, is simply constructed out of the names of the SM partner fields: for fermions, the partners will be called *sfermions*, like squarks for quark partners.<sup>2</sup> For gauge and Higgs bosons, the partners are denoted with an appended *ino*, like higgsino for the Higgs boson partner. The symbol for the partner particle has the same letter with a tilde, for example the gluino  $\tilde{g}$  is the partner particle of the gluon  $g$ .

In the following, the basic principles of the supersymmetric theory will be introduced in Sect. 2.1. The consequences on the Higgs sector are reviewed in Sect. 2.2, and on the masses of the supersymmetric particles in Sect. 2.3. Also, the SM problems from Sect. 1.4 are revisited in Sect. 2.4. Possible experimental signatures of SUSY will be discussed in Chap. 3.

This and the following chapters are mainly based on [5–7].

## 2.1 The Minimal Supersymmetric Standard Model

In its minimal form of adding one supersymmetric operator on top of the SM theory, the extended theory is called the Minimal Supersymmetric Standard Model (MSSM).

The Lagrangian of the unbroken MSSM can be written as

$$\mathcal{L}_{\text{MSSM}}^{\text{unbroken}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{chiral}} - V. \quad (2.6)$$

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<sup>2</sup>For the name of flavored squarks, we omit the word *quark* (that is *stop* instead of *stop quark* or *top squark*). The same holds for the word *boson*.

### The Kinematic Part of the Lagrangian

The kinematic term of Eq. (2.6) can be written as

$$\begin{aligned} \mathcal{L}_{\text{kin}} = & -D^\mu \phi^{*i} D_\mu \phi_i - i\psi^{\dagger i} \bar{\sigma}^\mu D_\mu \psi_i - \frac{1}{4} F_{\mu\nu,j}^a F_j^{\mu\nu,a} - i\lambda_j^{\dagger a} \bar{\sigma}^\mu \nabla_\mu \lambda_j^a \\ & - \sqrt{2} g_j \left[ (\phi^{*i} T_j^a \psi_i) \lambda_j^a + \lambda_j^{\dagger a} (\psi^{\dagger i} T_j^a \phi_i) \right]. \end{aligned} \quad (2.7)$$

Here,  $\psi$  denotes the fermion fields and  $\phi$  its partner fields; the index  $i$  runs over all flavors and chiralities. The gauge boson and its partner fields are  $A_\mu^a$  and  $\lambda^a$ , respectively, with couplings  $g_j$  and generators  $T_j^a$ . The index  $j$  denotes the gauge group. In this and following equations, we apply the Einstein notation for summing over indices appearing twice. The field strength tensors  $F_{\mu\nu}^a$  are known from Eq. (1.9) as well as the covariant derivatives  $D_\mu$  from Eq. (1.8). The covariant derivative for the gaugino fields,  $\nabla_\mu$ , is defined as

$$\nabla_\mu \lambda^a = \partial_\mu \lambda^a - g f^{abc} A_\mu^b \lambda^c. \quad (2.8)$$

The gauge couplings are the same for the SM particles and their supersymmetric partner particles.

### The Chiral Part of the Lagrangian

In order to obtain the chiral part of Eq. (2.6), the superpotential  $W$ , a function leading to the MSSM Lagrangian that is invariant under supersymmetric transformations, is defined:

$$W = \epsilon_{ij} \left[ G_{ab}^l H_d^i (\tilde{L}_L^j)_a (\tilde{e}_R)_b + G_{ab}^d H_d^i (\tilde{Q}_L^j)_a (\tilde{d}_R)_b - G_{ab}^u H_u^i (\tilde{Q}_L^j)_a (\tilde{u}_R)_b + \mu H_u^i H_d^j \right] \quad (2.9)$$

with  $G^{l,u,d}$  being Yukawa couplings,  $\mu$  the Higgs mixing parameter,<sup>3</sup>  $H_{u,d}$  the Higgs doublets, and  $\tilde{Q}_L, \tilde{L}_L, \tilde{u}_R, \tilde{d}_R, \tilde{e}_R$  the left- and right-handed partner multiplets for all three fermion generations. The antisymmetric tensor  $\epsilon_{ij} = -\epsilon_{ji}$  is defined with  $\epsilon_{12} = 1$ . The superpotential has to be a holomorphic function of the scalar fields. Therefore, terms like  $HH^*$  are forbidden, and two Higgs doublets are needed to provide masses for up- and down-type quarks and leptons.

The chiral Lagrangian is then given as

$$\mathcal{L}_{\text{chiral}} = -\frac{1}{2} \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \psi_i \psi_j + h.c. \quad (2.10)$$

Here,  $\phi$  are any of the scalar fields of Eq. (2.9) and  $\psi$  their partner fields.

This Lagrangian resembles the well known SM Yukawa coupling of Eq. (1.16), but also describes similar vertices, where the Higgs boson and one fermion are exchanged

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<sup>3</sup>This parameter should not be confused with the Higgs mass parameter from the SM, Eq. (1.11).

by their superpartners. Also vertices with charged Higgs bosons or higgsinos are described by it. The  $\mu$ -term of Eq. (2.9) leads to masses for the higgsinos.

### The Potential of the Lagrangian

The final term in Eq. (2.6), the potential, can be written as

$$V(\phi) = F_i^* F^i + \frac{1}{2} D^a D^a \quad (2.11)$$

with

$$F_i = \frac{\partial W}{\partial \phi^i}, \quad D^a = -g \phi_i^* T_{ij}^a \phi_j.$$

These  $F$  and  $D$  fields are auxiliary fields. They are needed to match the numbers of degrees of freedom for fermion supermultiplets ( $F$ -term) and the gauge supermultiplets ( $D$ -term) in the case of off-shell fields.

The  $F$ -term can be separated into three types of interactions. First, the Yukawa part of Eq. (2.9), leads to quartic interactions between two Higgs bosons and two sfermions of the same flavor generation. Secondly, there exists a part with interactions between one Higgs boson and two sfermions. The final part is the terms coming from the  $\mu$ -term of Eq. (2.9):

$$V_H = |\mu|^2 \left( |H_u^0|^2 + |H_d^0|^2 + |H_u^+|^2 + |H_d^-|^2 \right).$$

Note, that this potential is similar to the  $\mu^2$  term of Eq. (1.11). However, it is positive definite, and thus electroweak symmetry breaking does not occur, if SUSY is unbroken.

The contribution to the Lagrangian, introduced by the  $D$ -term of Eq. (2.11) is quadratic in gauge couplings and quartic in scalar fields. Quartic Higgs boson couplings have already been observed in Eq. (1.11). SUSY also adds these couplings to the other type of scalar fields, the sfermion fields. One should note, that in SUSY these quartic couplings are determined by gauge couplings through the definition of  $D^a$ .

Summarizing our current findings: So far, we have reproduced the interaction vertices from the SM. The theory doubles the particle spectrum, and the chiral and gauge eigenstates are summarized in Table 2.1. Further interactions are introduced by exchanging two SM particles by their supersymmetric partners at the SM vertices. These interactions happen at the same strength as the ones in the SM, because the coupling constants are equal. It is important to note, that SUSY does not introduce new couplings. Additional interaction vertices emerge like the quartic coupling between sfermion fields, where no analogon in the SM exists.

This extended theory has some problems: First, we have no electroweak symmetry breaking. Also, we introduced a lot of new particles, which only differ in spin from their SM partner particles, but not in mass. However, these particles have not

**Table 2.1** The particle content of the MSSM in terms of the chiral and gauge eigenstates and the representation in the SM gauge groups

| Name                                                 |       | Spin-0                         | Spin-1/2                         | Spin-1 | $SU(3)_C \times SU(2)_L \times U(1)_Y$ |
|------------------------------------------------------|-------|--------------------------------|----------------------------------|--------|----------------------------------------|
| Squarks,<br>quarks<br>( $\times 3$<br>generations)   | $Q_L$ | $(\tilde{u}_L, \tilde{d}_L)$   | $(u_L, d_L)$                     | —      | $(3, 2, +\frac{1}{6})$                 |
|                                                      | $u_R$ | $\tilde{u}_R$                  | $u_R$                            | —      | $(3, 1, +\frac{2}{3})$                 |
|                                                      | $d_R$ | $\tilde{d}_R$                  | $d_R$                            | —      | $(3, 1, -\frac{1}{3})$                 |
| Sleptons,<br>leptons<br>( $\times 3$<br>generations) | $L_L$ | $(\tilde{\nu}_L, \tilde{e}_L)$ | $(\nu_L, e_L)$                   | —      | $(1, 2, -\frac{1}{2})$                 |
|                                                      | $e_R$ | $\tilde{e}_R$                  | $e_R$                            | —      | $(1, 1, -1)$                           |
| Gauginos,<br>gauge bosons                            | $g$   | —                              | $\tilde{g}$                      | $g$    | $(8, 1, 0)$                            |
|                                                      | $W$   | —                              | $\tilde{W}$                      | $W$    | $(1, 3, 0)$                            |
|                                                      | $B$   | —                              | $\tilde{B}$                      | $B$    | $(1, 1, 0)$                            |
| Higgs boson,                                         | $H_u$ | $(H_u^+, H_u^0)$               | $(\tilde{H}_u^+, \tilde{H}_u^0)$ | —      | $(1, 2, +\frac{1}{2})$                 |
| higgsino                                             | $H_d$ | $(H_d^0, H_d^-)$               | $(\tilde{H}_d^0, \tilde{H}_d^-)$ | —      | $(1, 2, -\frac{1}{2})$                 |

been observed. In order to obtain a SUSY model with realistic phenomenological observations, SUSY needs to be broken.

### 2.1.1 Breaking of Supersymmetry

SUSY should be broken in such a way that the symmetry itself is preserved, but the vacuum state is not symmetric under supersymmetric transformations.

Unlike the Higgs mechanism, the underlying mechanism to break SUSY is not yet fully understood, and only a parameterization of the breaking terms can be written down. The way SUSY is broken should not introduce new “problems” like the ones mentioned in Sect. 1.4. For example, dimensionless couplings lead to quadratic divergencies as observed in the hierarchy problem [5]. Thus, only *soft* SUSY breaking terms are allowed. A possible parameterization can be

$$\begin{aligned}
\mathcal{L}_{\text{MSSM}}^{\text{soft}} = & -\frac{1}{2} (M_1 \tilde{B} \tilde{B} + M_2 \tilde{W} \tilde{W} + M_3 \tilde{g} \tilde{g} + c.c.) \\
& - \left( \tilde{u}_R A_u \tilde{Q}_L H_u - \tilde{d}_R A_d \tilde{Q}_L H_d - \tilde{e}_R A_e \tilde{L}_L H_u + c.c. \right) \\
& - \tilde{Q}_L^\dagger m_Q^2 \tilde{Q}_L - \tilde{L}_L^\dagger m_L^2 \tilde{L}_L - \tilde{u}_R^\dagger m_u^2 \tilde{u}_R - \tilde{d}_R^\dagger m_d^2 \tilde{d}_R - \tilde{e}_R^\dagger m_e^2 \tilde{e}_R \\
& - M_{H_u}^2 |H_u|^2 - M_{H_d}^2 |H_d|^2 + (B\mu H_u H_d + c.c.). \tag{2.12}
\end{aligned}$$

In total, 105 new parameters are introduced: the gaugino mass terms  $M_1, M_2, M_3$ , the sfermion  $3 \times 3$  mass matrices  $m_Q^2, m_L^2, m_u^2, m_d^2, m_e^2$ , the Higgs mass terms  $M_{H_u}^2, M_{H_d}^2$ , trilinear sfermion-sfermion-Higgs coupling  $3 \times 3$  matrices  $A_u, A_d, A_e$ , and off-diagonal Higgs mass term  $B\mu$ . These terms should be roughly of the SUSY breaking scale  $M_{\text{soft}}$ .

This new Lagrangian introduces flavor-changing neutral currents and CP-violating effects that have been excluded experimentally. One potential way out is diagonalizing the newly introduced  $3 \times 3$  mass matrices,  $m_X^2 = m_{X0}^2 \mathbb{I}$ , making the trilinear couplings proportional to the Yukawa couplings,  $A_x = A_{x0} G^x$ , and removing new complex phases,  $\text{Im}(M_{1,2,3}) = \text{Im}(A_{u0,d0,e0}) = 0$ .

### 2.1.2 The Hidden Sector—The Origin of Supersymmetry Breaking

If global SUSY is broken, the potential of Eq. (2.11) is required to have a non-zero vacuum expectation value, and thus  $\langle F \rangle \neq 0$  or  $\langle D \rangle \neq 0$ . As none of the MSSM fields can obtain a non-zero vacuum expectation value without spoiling gauge invariance, this breaking must occur in the “hidden sector”. It is assumed that this hidden sector either couples extremely weakly or only indirectly via messenger particles to the “visible sector”. One consequence of SUSY breaking is the existence of a neutral and massless fermion, the goldstino. Two popular models of spontaneous SUSY breaking are discussed here. Other SUSY breaking scenarios have been developed, for instance anomaly mediated breaking [8, 9], but are not discussed.

In supergravity mediated models (SUGRA), the hidden sector is around the Planck scale,  $M_P$ , and interacts with the visible sector via gravitation. When gravitation is taken into account, SUSY becomes a local symmetry, supergravity, with the addition of the spin-2 graviton and its spin-3/2 partner, the gravitino. The gravitino absorbs the goldstino, acquiring the mass  $M_{3/2} \sim \langle F \rangle / M_P$ . The scale requested for the soft breaking terms is at the TeV scale,

$$M_{\text{soft}} \sim \frac{\langle F \rangle}{M_P} \sim 1 \text{ TeV},$$

hence one estimates  $\sqrt{\langle F \rangle} \sim 10^{11} \text{ GeV}$ . In this scenario the gravitino is heavy. For collider physics, typically it does not play a role due to the weakness of the gravitational interaction.

Another popular model of SUSY breaking is the gauge-mediated SUSY breaking (GMSB). Instead of coupling directly to the hidden sector, GMSB introduces messenger fields with SM gauge quantum numbers: gauge interactions with these messenger fields mediate the breaking by quantum fluctuations. Hence, the soft breaking scale can be estimated as

$$M_{\text{soft}} \sim \frac{\alpha}{2\pi} \frac{\langle F \rangle}{M_{\text{mess}}} \sim 1 \text{ TeV}$$

with  $\alpha$  being the gauge couplings involved in the loop interaction. Assuming that  $M_{\text{mess}}^2$  is of the same scale as  $\langle F \rangle$ , one concludes that  $\sqrt{\langle F \rangle} \sim 10^4 - 10^5$  GeV. A direct consequence of this is that the gravitino is extremely light:  $M_{3/2} \ll v \approx 246$  GeV.

### 2.1.3 *R-parity*

The Lagrangian introduced in Eqs. (2.6) and (2.12) conserves the discrete quantum number

$$R = (-1)^{3B+L+2S}, \quad (2.13)$$

with  $B$  being the baryon number,  $L$  the lepton number, and  $S$  the spin. This quantum number has values  $R = +1$  for all the SM particles and  $R = -1$  for their supersymmetric partners.

Requiring  $R$ -parity conservation has several phenomenological consequences:

- both the baryon and lepton numbers are conserved,
- sparticles can only be produced in pairs,
- sparticles can only decay into an odd number of other sparticles, thus requiring that the lightest supersymmetric particle (LSP) is stable,
- due to cosmological constraints, the LSP must be electrically neutral and interact only very weakly.

Previously,  $R$ -parity conservation was assumed without mentioning. In fact, more terms in Eq. (2.9) are allowed:

$$\begin{aligned} W_{RPV} = & \lambda_{ijk} (\tilde{L}_L)_i (\tilde{L}_L)_j (\tilde{e}_R)_k + \lambda'_{ijk} (\tilde{L}_L)_i (\tilde{Q}_L)_j (\tilde{d}_R)_k \\ & + \lambda''_{ijk} (\tilde{u}_R)_i (\tilde{d}_R)_j (\tilde{d}_R)_k + \mu'_i (\tilde{L}_L)_i H_d \end{aligned} \quad (2.14)$$

The first, second and fourth term violate lepton number conservation, while the third term violates baryon number conservation. The coupling  $\lambda_{ijk}$  permits slepton-lepton-lepton interactions, the second coupling,  $\lambda'_{ijk}$ , slepton-quark-quark or lepton-squark-quark interactions, while the third coupling,  $\lambda''_{ijk}$ , corresponds to a squark-quark-quark vertex.

Allowing for  $R$ -parity violation (RPV) has severe consequences: sparticles can be produced as single particle and also decay into an even number (including zero) of other sparticles: the LSP does neither have to be stable nor to be neutral. Besides direct SUSY phenomenology, RPV allows for several interactions forbidden by the SM. For example, protons can decay at strength  $\lambda' \lambda''$  via squark exchange, like  $p \rightarrow e^+ \pi^0$ . A very concise review of RPV can be found in [10]. Indirect measurements put strong limits on the couplings of Eq. (2.14). Taking the example of the proton decay  $p \rightarrow e^+ \pi^0$ , experiments have limited the proton lifetime  $\gtrsim 10^{33}$  years. This yields a stringent limit of  $\lambda'_{11k} \lambda''_{11k} \lesssim \frac{1}{2} \cdot 10^{-27} (\tilde{M}_k / 100 \text{ GeV})^2$ .

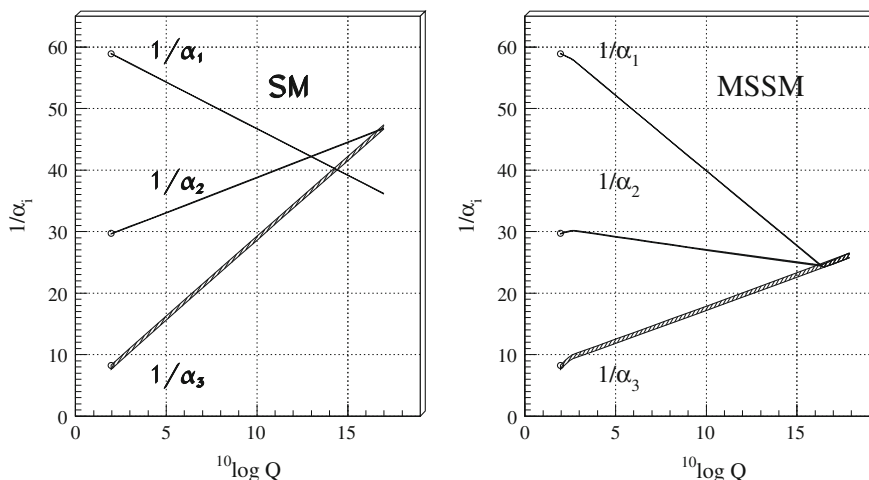
For the rest of this writing,  $R$ -parity conservation is assumed, unless stated otherwise.

### 2.1.4 Unification of Gauge Couplings

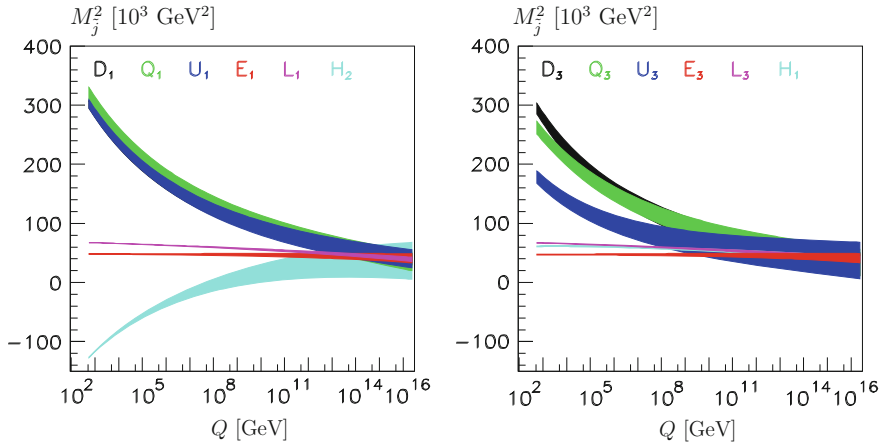
In Sect. 1.2.1, the evolution of the strong coupling constant  $\alpha_s$  as a function of  $Q^2$  was shown. The  $\beta_i$  coefficients of Eq. (1.21) depend on the particles entering the loops. If this evolution is considered for all three coupling constants, no scale  $Q^2$  is found, for which the three coupling constants intersect within the SM, see Fig. 2.1, left.

The framework of MSSM introduces new particles at a scale of the order of 1 TeV. These particles alter the coefficients  $\beta_i$  and all three gauge couplings meet at a scale  $M_{\text{GUT}} \sim 10^{15} - 10^{16}$  GeV below the Planck scale  $M_P$ , allowing for a unification, see Fig. 2.1, right. The unified coupling  $\alpha_{\text{GUT}} \simeq 1/25$  is well within the perturbative regime. Such a theory is known as Grand Unification Theory (GUT).

If unification occurs, the three SM gauge groups unify into one group. Consequences depend on the unification group. Some of them are phenomenologically interesting: A  $SU(5)$  unification can predict the electroweak mixing angle compatible with measurements, or explain the quantization of charge  $q(e) = -3q(\bar{d})$ . On the other hand, also new phenomena open up, like the introduction of leptoquarks (couple to  $lq$ ) and diquarks (couple to  $qq$ ). These can be derived from the representations for quarks and leptons, which are, for  $SU(5)$ ,  $\bar{\mathbf{5}} = \{d_R, L_L\}$  and  $\mathbf{10} = \{Q_L, u_R, e_R\}$ . The interactions with leptoquarks or diquarks lead to tree-level contributions to the



**Fig. 2.1** Evolution of the inverse of the gauge coupling constants with energy  $Q$  [GeV]. Left for SM, right for MSSM with particle masses of  $\sim 1$  TeV. Based on [11] and taken from [12]



**Fig. 2.2** Example of a mass spectrum: *left* for sfermions of the first and second generations and the Higgs boson mass parameter  $M_{H_u}$  (here  $M_{H_2}$ ), *right* for sfermions of the third generation and  $M_{H_1}$ . The parameter  $M_{H_u}^2$  becomes negative for  $Q \ll M_{\text{GUT}}$ . Taken from [15]

proton decay. However, the contributions are suppressed compared to GUT scenarios within the SM alone. Another popular group is SO(10). In this model, all SM leptons of a given generation, including  $\nu_R$ , are unified in one supermultiplet within the representation  $\mathbf{16} = \{Q_L, u_R, d_R, L_L, e_R, \nu_R\}$ . A review on the popular SU(5), SO(10), and other unification groups can be found in [13].

One possible result of gauge unification is discussed here: The positive Higgs boson mass parameter  $M_{H_u}^2$  (at  $M_{\text{GUT}}$ ) becomes negative at the electroweak scale,  $v$ , due to the influence of the strong top quark Yukawa coupling in the RGE. This natural explanation of electroweak symmetry breaking via quantum corrections is called *radiative electroweak symmetry breaking* [14]. An example of a mass spectrum is shown in Fig. 2.2.

## 2.2 The Higgs Sector in the Minimal Supersymmetric Standard Model

In the MSSM, two Higgs doublets  $H_u = (H_u^+, H_u^0)$  and  $H_d = (H_d^0, H_d^-)$  are required, as anomaly cancellation involving chiral fermions requires the existence of two fermionic partners in the Higgs sector with  $Y = \pm 1/2$ . Minimizing the Higgs potential for  $H_u^0$  and  $H_d^0$ , obtained from all relevant terms of Eqs. (2.6) and (2.12), one arrives at the conditions

$$2B\mu < 2|\mu|^2 + M_{H_u}^2 + M_{H_d}^2 \quad \text{and} \quad (B\mu)^2 > (|\mu|^2 + M_{H_u}^2)(|\mu|^2 + M_{H_d}^2).$$

Due to the running of  $M_{H_u}^2$  described in Sect. 2.1.4, these conditions can be met easily. Defining the vacuum expectation values  $v_{u/d} = \langle H_{u/d}^0 \rangle / \sqrt{2}$ , one finds the relation

$$v^2 = v_u^2 + v_d^2 = \frac{4M_Z^2}{g^2 + g'^2}. \quad (2.15)$$

The two Higgs doublets have eight degrees of freedom. As described in Sect. 1.1.1, three of them are absorbed by the massive gauge bosons ( $W^\pm$  and  $Z$ ), leaving five degrees of freedom, forming five Higgs bosons:  $h$ ,  $H$ ,  $A$ , and  $H^\pm$ . Two of them ( $h$ ,  $H$ ) are scalar bosons with convention  $M_h \leq M_H$ ,  $A$  is a pseudoscalar boson, while  $H^\pm$  are two charged bosons.

Calculating the pseudoscalar boson mass

$$M_A^2 = B\mu \frac{v_u^2 + v_d^2}{v_u v_d} \quad (2.16)$$

and defining the ratios ( $-\pi/2 < \alpha < 0$ )

$$\tan \beta = \frac{v_u}{v_d}, \quad \tan(2\alpha) = \tan(2\beta) \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}, \quad (2.17)$$

one can obtain the Higgs boson masses at leading order

$$M_{H^\pm}^2 = M_W^2 + M_A^2, \quad M_{H,h}^2 = \frac{1}{2} \left( M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2(2\beta)} \right). \quad (2.18)$$

It is interesting to note, that at tree-level  $M_h \leq M_Z |\cos(2\beta)|$ . However, including radiative corrections from stop- and top-quark loops,

$$\Delta(M_h) \approx \frac{3}{4\pi} \cos^2 \alpha (G')^2 M_t^2 \ln \left( \frac{M_{\tilde{t}_1} M_{\tilde{t}_2}}{M_t^2} \right) \quad (2.19)$$

an upper limit of  $M_h \lesssim 135$  GeV is obtained. The newly found Higgs-like boson at a mass of  $M_h \sim 125$  GeV fulfills this requirement.

The relative coupling strengths of the neutral MSSM Higgs bosons compared to the ones of the SM Higgs boson are expressed in Table 2.2.

The case of  $M_A^2 \gg M_Z^2$ , that is for  $M_A \gtrsim 300$  GeV, is known as the decoupling limit. In this limit,  $A$ ,  $H$ , and  $H^\pm$  have the same mass scale ( $M_A \sim M_H \sim M_{H^\pm}$ ) and decouple from  $h$ . As  $\beta - \alpha \approx \pi/2$ , the couplings of the light MSSM Higgs boson  $h$  become identical to the ones of the SM Higgs boson and the discovery potential for the heavier MSSM Higgs bosons might be challenging [16].

**Table 2.2** The MSSM Higgs couplings relative to the SM Higgs couplings expressed in  $\alpha$  and  $\beta$ 

| Higgs state | $h_V$                  | $h_A$                   | $h_u$                      | $h_d$                       |
|-------------|------------------------|-------------------------|----------------------------|-----------------------------|
| SM $h$      | 1                      | 1                       | 1                          | 1                           |
| MSSM $h$    | $\sin(\beta - \alpha)$ | $\cos(\beta - \alpha)$  | $\cos \alpha / \sin \beta$ | $-\sin \alpha / \cos \beta$ |
| MSSM $H$    | $\cos(\beta - \alpha)$ | $-\sin(\beta - \alpha)$ | $\sin \alpha / \sin \beta$ | $\cos \alpha / \cos \beta$  |
| MSSM $A$    | 0                      | 0                       | $\cot \beta$               | $\tan \beta$                |

The relative coupling strength  $h_V$  denotes couplings to vector bosons,  $h_A$  couplings to  $AZ$ , and  $h_{u/d}$  couplings to up- or down-type quarks and leptons

## 2.3 Sparticle Masses in the Minimal Supersymmetric Standard Model

It is a consequence of the electroweak symmetry breaking and the soft SUSY breaking terms that particle states with equal gauge quantum numbers mix among each other. The mass eigenstates are obtained by diagonalizing the mass matrices in the Lagrangian.

Gluinos do not share quantum numbers with any other sparticle and therefore do not mix. The other neutral gauginos and higgsinos form four mass eigenstates, called neutralinos,  $\tilde{\chi}^0$ , while their charged counterparts form two mass eigenstates, called charginos,  $\tilde{\chi}^\pm$ .

Mixing also appears in the sfermion sector. However, the diagonalization of the mass matrices and trilinear couplings, as mentioned in Sect. 2.1.1, leads to negligible mixing among the flavors. Because of the Yukawa coupling strengths, mixing is usually assumed only for third generation sfermions. The amount of the mixing also depends on  $\tan \beta$ : for small  $\tan \beta$ , the sbottom and staus mixing is small, and the stop mixing is large, while it is the opposite for large  $\tan \beta$ .

In Table 2.3, the gauge and mass eigenstates of the sparticles are summarized. It is conventional that  $M(\tilde{X}_j) \geq M(\tilde{X}_i)$  if  $j > i$  for the mixed mass eigenstates  $X = \tilde{\chi}^0$ ,

**Table 2.3** The gauge and mass eigenstates of the MSSM sparticles

| Type        | Gauge eigenstate                                       | Mass eigenstate                                                          |
|-------------|--------------------------------------------------------|--------------------------------------------------------------------------|
| Neutralinos | $\tilde{B}, \tilde{W}^0, \tilde{H}_u^0, \tilde{H}_d^0$ | $\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$ |
| Charginos   | $\tilde{W}^\pm, \tilde{H}_u^\pm, \tilde{H}_d^\pm$      | $\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$                                 |
| Gluinos     | $\tilde{g}$                                            | $\tilde{g}$                                                              |
| Squarks     | $\tilde{u}_L, \tilde{u}_R, \tilde{d}_L, \tilde{d}_R$   | $\tilde{u}_L, \tilde{u}_R, \tilde{d}_L, \tilde{d}_R$                     |
|             | $\tilde{c}_L, \tilde{c}_R, \tilde{s}_L, \tilde{s}_R$   | $\tilde{c}_L, \tilde{c}_R, \tilde{s}_L, \tilde{s}_R$                     |
|             | $\tilde{t}_L, \tilde{t}_R, \tilde{b}_L, \tilde{b}_R$   | $\tilde{t}_1, \tilde{t}_2, \tilde{b}_1, \tilde{b}_2$                     |
| Sleptons    | $\tilde{e}_L, \tilde{e}_R, \tilde{\nu}_e$              | $\tilde{e}_L, \tilde{e}_R, \tilde{\nu}_e$                                |
|             | $\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\nu}_\mu$        | $\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\nu}_\mu$                          |
|             | $\tilde{\tau}_L, \tilde{\tau}_R, \tilde{\nu}_\tau$     | $\tilde{\tau}_2, \tilde{\tau}_1, \tilde{\nu}_\tau$                       |

$\tilde{\chi}^\pm, \tilde{t}, \tilde{b}$ , or  $\tilde{\tau}$ . For displaying purposes, hereafter, all anti-particles of supersymmetric particle states will be denoted like  $\tilde{q}^* \equiv \tilde{\bar{q}}$ . We also will denote the mass of particle  $X$  as  $M(X)$  instead of  $M_X$  from here on.

The mass spectrum depends strongly on the SUSY parameters of the soft-breaking Lagrangian, Eq. (2.12), as well as the breaking scenario. Some limiting scenarios are worth mentioning.

One limit considers the electroweak symmetry breaking effects to be small compared to the MSSM parameters:  $M(Z) \ll |\mu \pm M_1|, |\mu \pm M_2|$ . Three hierarchies can be distinguished:

- $|\mu| \gg M_2 > M_1$ : The  $\tilde{\chi}_1^0$  is bino-like,  $\tilde{\chi}_2^0$  and  $\tilde{\chi}_1^\pm$  are wino-like, and  $\tilde{\chi}_{3,4}^0$  and  $\tilde{\chi}_2^\pm$  are higgsino-like. Furthermore,  $M(\tilde{\chi}_1^\pm) \simeq M(\tilde{\chi}_2^0)$ .
- $|\mu| \gg M_1 > M_2$ : The  $\tilde{\chi}_1^0$  is wino-like,  $\tilde{\chi}_2^0$  is bino-like, while the other states behave as in the previous case. Furthermore,  $M(\tilde{\chi}_1^\pm) \simeq M(\tilde{\chi}_1^0)$ .
- $|\mu| \ll M_1, M_2$ : Here, both the  $\tilde{\chi}_1^\pm$  and  $\tilde{\chi}_{1,2}^0$  are higgsino-like with  $M(\tilde{\chi}_1^\pm) \simeq M(\tilde{\chi}_{1,2}^0)$ , while  $\tilde{\chi}_2^\pm$  and  $\tilde{\chi}_{3,4}^0$  are gaugino-like.

As for the gauginos and higgsinos, the sfermion masses are obtained from the running of the corresponding soft-term parameters at the SUSY breaking scale down to the electroweak scale. From the D-term contribution of the potential, Eq. (2.11), general sum rules can be stated:

$$M^2(\tilde{e}_L) - M^2(\tilde{\nu}_{eL}) = M^2(\tilde{u}_L) - M^2(\tilde{d}_L) = g^2 \frac{v_u^2 - v_d^2}{2} = -M^2(W) \cos(2\beta). \quad (2.20)$$

Since  $\cos(2\beta) < 0$ , it follows that  $M(\tilde{e}_L) > M(\tilde{\nu}_{eL})$ , and similar for the other cases. If it is further assumed that the sfermion masses unify at the GUT scale, another sum rule holds:

$$2(M^2(\tilde{u}_R) - M^2(\tilde{d}_R)) + M^2(\tilde{d}_R) - M^2(\tilde{d}_L) + M^2(\tilde{e}_L) - M^2(\tilde{e}_R) = \frac{10}{3} M^2(Z) \sin^2 \theta_w \cos(2\beta). \quad (2.21)$$

These rules also apply to the second generation.

For third generation sfermions, the strong Yukawa couplings introduce a mixing of the left- and right-handed scalars and reduce the effect of the RGE evolution. Therefore, in many scenarios,  $\tilde{t}_1$  or  $\tilde{b}_1$  are the lightest squarks, and  $\tilde{\tau}_1$  the lightest slepton.

The explicit mass formula for the mentioned cases can be found in [5], for example.

In R-parity conserving models, the LSP is stable, and thus its content is important to study.

From cosmological considerations, charged sparticles as LSP are excluded, natural candidates are the neutralino  $\tilde{\chi}_1^0$ , the lightest sneutrino  $\tilde{\nu}$ , or the gravitino  $\tilde{G}$ .

In GMSB models, the LSP is  $\tilde{G}$ . The next-to-LSP (NLSP) can be charged, most likely being the  $\tilde{\tau}_1$ , or neutral, usually being the  $\tilde{\chi}_1^0$ . In SUGRA models, all three candidates can have masses of few hundreds GeV. The most likely candidate for the LSP is the lightest neutralino  $\tilde{\chi}_1^0$ , which is assumed to be the LSP from now on.

## 2.4 Revisiting the Limitations of the Standard Model

The theory of SUSY has been introduced in its minimal version, the MSSM. For each SM particle, there exists one partner particle. They are connected via an operator  $Q$  that changes the spin quantum number by  $1/2$ . The doubled particle spectrum requires the presence of two Higgs doublets, leading to the existence of five Higgs bosons.

Having introduced the most important aspects of the theory, the SM limitations of Sect. 1.4 are revisited: How does the MSSM change the picture?

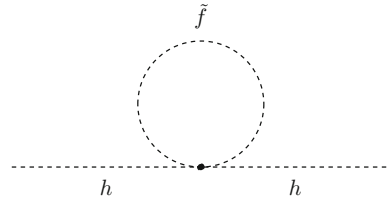
- Solving the hierarchy problem is one of the key motivations for SUSY. Each fermion has a scalar supersymmetric partner particle, coupling to the Higgs boson as sketched in Fig. 2.3. The correction to the Higgs boson mass is:

$$\delta M^2(h)|_{\tilde{f}} = + \frac{(G^f)^2}{8\pi^2} \left[ \Lambda^2 - \mathcal{O} \left( M^2(\tilde{f}) \ln \frac{\Lambda}{M^2(\tilde{f})} \right) \right]. \quad (2.22)$$

The coupling strength is equal to Eq. (1.29), but the mass correction has opposite sign due to the scalar nature of the sfermion: the quadratic divergencies cancel exactly with those of Eq. (1.29). The remaining correction is only logarithmic in  $\Lambda$ . If the differences between the squared masses of the fermions and sfermions are not too large, small Higgs boson masses can be achieved naturally.<sup>4</sup>

- The electroweak symmetry breaking is not constructed adhoc as in the SM (by choosing  $\mu^2 < 0$  and  $\lambda > 0$ ), but can be naturally explained via quantum corrections (radiative electroweak symmetry breaking). The observed mass of the Higgs-like boson,  $M(h) \simeq 125$  GeV fits well within the bound set by the radiative corrections  $M(h) \lesssim 135$  GeV.
- Grand unification is not a limitation of the SM. However, SUSY allowing for unification of the gauge couplings is more than just a coincidence: some free parameters in the SM are determined at the unification scale. For example, SUSY can explain that the proton charge is equal to the positron charge. Also, the electroweak mixing angle is fixed, and agrees with measurements.
- The MSSM is still a theory of the strong, weak, and electromagnetic interactions. Gravitation is not included in SUSY, with the exception of being a possible source

**Fig. 2.3** Sfermion loop contribution to the Higgs boson mass



<sup>4</sup>The observation of a Higgs-like boson with  $M(h) \simeq 125$  GeV raises the so-called *little hierarchy problem*, as large stop masses are required for the Higgs boson acquiring this mass. A fine-tuning of 1–10 % is needed [17, 18].

of SUSY breaking in the hidden sector. However, local SUSY requires the existence of a spin-3/2 particle, the gravitino, and its partner, the graviton. This theory is called supergravity. It might be a step to a more inclusive theory like superstring theory, describing all fundamental interactions.

- There exists no particle candidate within the SM, that could explain the amount of dark matter observed in the universe. In  $R$ -parity conserving SUSY models, the  $\tilde{\chi}_1^0$  is a good particle candidate for dark matter.

The principles of SUSY, a symmetry relating fermions and bosons, provide elegant solutions to several of the limitations in the SM. It is therefore a very well motivated and appealing theory. As of this writing, no direct evidence, especially none of the SM superpartners, has been observed.

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