

Section 1: Primary Ratios

3. Grave and Acute Tones

The French word *son* (sound), as used here, expresses the pitch of the vibrations. An *acute sound* differs from a *grave sound* by the greater (acute) or smaller (grave) number of vibrations that it executes in a given interval of time.

The word sound, therefore, has three very different meanings: it expresses first, everything that can be heard (in German, *Schall*); second, regular or perceptible vibrations (in German, *Klang*) as opposed to noise; third, the frequency of the vibrations (in German, *Ton*). The word *ton* is not used in French, as it is in other languages, to express the frequency of the vibrations in each sound. This word has several other meanings; for example, it expresses a major second. It also indicates the mode, the intensity with which one executes the music, etc.

4. Explanation of the Words *Interval*, *Melody*, *Chord*, *Harmony*, etc.

An *interval* is the ratio of the frequency of one sound to another. Usually, a very deep or grave sound is regarded as the basis for comparison with a more acute sound.

A *melody* is a sequence of tones.

A *chord* is the coexistence of several tones.

Harmony is a sequence of chords, or the coexistence of several melodies.

Music makes use of materials, for which acoustics furnishes the theory, in order to excite sensations.

5. Absolute Frequency of the Vibrations of Each Tone

(*Note: Description of a tonometer.*) In the deepest (gravest) sounds that are perceptible to the human ear, the vibrating body makes at least 30 vibrations per second; but acute sounds can be sensed in which up to 8000 or 12,000 vibrations occur per second. We do not go far from the truth if, to facilitate comparison of the absolute number of vibrations to relative numbers, we regard each *do*¹ as representing the power of two, taking the fundamental *do* to be unity. We therefore attribute to the lowest *do* on the piano or violoncello the value of 128 vibrations (or simple oscillations per second), which is enough to find the absolute number of vibrations of every other sound, by multiplying the relative numbers (Pars. 19 and 26) for the first lower octave by 128, for the second by 256, for the first octave above by 512, etc.

The value that one has assigned to these instruments has not always been the same in different countries and in different eras. Thus, Euler assigned a value of 118 to *do* (in his *Tentam. nov. theor. mus.*, ch. 1) and 125 in another (in *Nov. Comment. Acad. Petrop.* vol. 16). Marpurg at Berlin found this same latter result while Sarti (*Nov. Act. Acad. Petrop.* 1796) noted that the *la* of the third string of a violin makes 436 double, or 872 single, vibrations per second, in St. Petersburg.² This gives a value of approximately 131 vibrations per second for this same *do*. There has been a tendency to increase these values in tuning instruments since the days of Euler and Marpurg, and, at present, in several orchestras, they have gone somewhat beyond the number of vibrations that result from using powers of two. However, one can still adopt some power of two for the average value of *do*.³

(continued)

¹ Chladni used the symbol *ut* for the first note in the musical scale, but we have replaced it by *do* regularly used in modern texts. We have left the seventh note as *si*, since both *si* and *ti* are used today in different countries.—RTB

² In the early mathematical studies of music, a debate existed between those who used the number of half oscillations (simple vibrations) per second, and those who used the number of full oscillations (double vibrations). The latter school won out.—RTB

³ This “powers-of-two” scale is no longer used in music but is commonly employed in instruction in physics, and is sometimes called the physicists’ scale.—RTB

I have found a very simple method of determining, by the judgment of the eyes and ears, the number of vibrations that corresponds to each sound. It is necessary that a vibrating body be of a sufficient length (which can be shortened if required) for one to see and count the vibrations that take place in a certain interval of time, in order to compare the sounds and the lengths of the parts with the number of vibrations counted, and also compare them to the length of the entire body. I used very long strings for this effect; but I did not succeed, because the vibrations of the different parts were added to vibrations of the entire string, as well as many circular and lateral motions, which made the accuracy of the observations difficult. It is therefore preferable to make use of a thin sheet of iron or brass, of about a half-line of thickness and of a half inch in width and of a length sufficient to vibrate very slowly. It was necessary that the thickness be exactly uniform. A metal wire could have been used, but the width of the sheet served to prevent lateral motions. The frequency of vibrations of such a sheet is inversely proportional to the squares of their lengths, when other conditions remain the same. The end of the sheet was clamped in a fixed vise, making it project out, more or less, to the length at which it makes, in each second of time, a certain number of vibrations visible to the eye. These vibrations can be compared to the oscillations of a second pendulum, which is understandable, as it is done in astronomical observations. When one is a little used to it, it is not difficult to count up to eight vibrations per second. I propose to use 4 vibrations per second to mark, exactly, the length of the projection from the sheet and to divide it into two, four, eight, and other numbers of parts. If one fixes the sheet in the vise in such a way that one half is extended, it will make 16 vibrations per second. These vibrations will be too slow to be heard and too rapid to be counted; but one can hear a distinct sound by making the sheet vibrate in two unequal portions, so as to establish a node of vibrations, at a distance from the free end, slightly less than one third of its length. This sound, which I call the *second sound*,⁴ because it corresponds to the second figure of the plate (Fig. 21), will make 100 vibrations per second, like the stationary sound of Sauveur. It will be *sol*[#], approximately a major third below *do*, the lowest note on the keyboard. If the part sticking out from the vise is shortened, so that it equals one fourth of the thickness of the plate, it will make 64 vibrations, one octave below the first octave on the keyboard. The second sound, which makes 400 vibrations, will give the *sol*, two octaves higher than the one which has 100 vibrations. Whatever may be the manner in which it partakes

(continued)

⁴ What Chladni refers to is the second partial (often the second harmonic).—TDR

of the motion of the plate, the results of these experiments conform very well to the theory. The best manner of producing this sound will be to use the bow of a violin. Before making the experiments, one should read what I have said regarding the transverse vibrations of a straight rod, in Sect. 5 of Part II.

6. Difference of Consonant and Dissonant Intervals

The greater or lesser simplicity of the numerical ratios of the vibrations is the sole basis of harmony. An *interval* is *consonant* when the ratio is very simple; when the ratio is less simple, the interval is *dissonant*. The consonant intervals can be expressed by the numbers 1, 2, 3, 4, 5, 6 or by 1, 3, 5 and the doubling of any of these numbers; dissonance results from different combinations of the same numbers. A consonant interval is pleasing by itself, while dissonance is only pleasing when it is returned and when it passes to another simpler ratio.

The ear, without counting the numbers themselves, perceives the effect of the relationships of the concurrence of simultaneous vibrations when they arrive together. It does for time what the eye does for space, when it is affected in an agreeable manner by the fair relationship of forms, without measuring and without calculating the ratios themselves.

Leibniz expressed himself very well on this subject (*Epistolae ad diversos*, vol. 1, epist. 154):

*Musica est exercitium arithmeticae occultem nescient is se numerare animi; multa enim facit in perceptionibus confuies seu insensibilibus, quae distincta a perceptione notare nequit. Errant enim, qui nihil in animâ fieri putant, ujus ipsa non sit conscia. Anima igitur etsi se nunerare non sentiat, sentit tamen hujus numerationis insensibilis effectum, seu voluptatem in consonantiis, molestiam in dissonantiis inde resultantem. Ex multis enim congruentiis insensibilibus oritur voluptas, etc.*⁵

Descartes also proposed the same principles (*Tract de homine*, p. 3, sect. 36, and *Comp. mus.*).

It does not conform to nature to want to derive, as several authors do, all the harmony of vibrations, and in particular the coexistence of some with the

(continued)

⁵ Unlike Chladni, we do not assume that the modern reader readily understands Latin. The first line of this quote from Leibniz is a famous definition of music: "Music is a hidden arithmetic exercise of a soul that does not know it is counting." Leibniz says that the pleasure we obtain from music comes from this unconscious counting.—CBH

fundamental sound, from those of a string. A string is only one kind of sounding body. In many other bodies, the general laws of vibrations, which one had not known, are modified differently; consequently, one cannot apply the laws of a sounding body to that which must be common to all. A monochord cannot serve to establish the principles of harmony; but only to give an idea of the effect of ratios.

7. Unison and the Octave

The simplest ratio is 1:1, in which two vibratory motions occur at the same time, and is known as *unison*.

The interval 1:2, in which the frequency of one vibration is double that of the other, is known as an *octave*. It is called thus because it is the eighth step in the ordinary scale, as each other interval takes its name from the step of the scale on which it is found. Experience shows that two sounds which are in the ratio of 1:2 have such a resemblance that we can regard one as the repetition of the other, from whence it follows that:

1. The nature of an interval does not change if one takes the sound of which it is composed one or two octaves lower or higher; that which returns to take double or one half of the smallest number; except that in the case in which one of these numbers becomes larger than the other; for one must regard this interval as an *inversion* of the first. Thus, 2:3, 1:3, 1:6 are the same interval; but 3:4 or 4:3 will be the inverse of that interval.
2. One can regard all the intervals as comprised in a single octave, so that one can express all of them by fractions contained between 1 and 2.

The calculations of intervals are the same as those of fractions.

8. Other Consonant Intervals

All the consonant intervals that can be expressed by the numbers 1, 2, 3, 4, 5, 6, or by doubling of these numbers, when one arranges them between 1 and 2 according to their distance from unity, will be

$$\frac{6}{5}, \frac{5}{4}, \frac{4}{3}, \frac{3}{2}, \frac{8}{5}, \frac{5}{3}$$

of which the last three are also the inversions of the first three. Of all the intervals, that of 3:2, of the *fifth*, is the simplest that the ear perceives as the most perfect

consonant after the octave. The *fourth* $\frac{4}{3}$ is an inversion of the fifth. It is consonant by itself, but it is customary in practice to treat it as dissonance because the combinations require a resolution in another interval.⁶ The interval $\frac{5}{4}$ is the *major third*, the interval $\frac{6}{5}$ the *minor third*, the *minor sixth* $\frac{8}{5}$, and the *major sixth* $\frac{5}{3}$ are their inversions. Ordinarily, the unison, the octave, and the fifth are called the *perfect consonants*, and the thirds and the sixths, the *imperfect consonants*.

9. Consonant Chords

According to these six consonant intervals, one can judge very easily how many here will be of *chords* or of combinations of more than two *consonant* sounds.

Let $m = 1$, $n = \frac{6}{5}$, $p = \frac{5}{4}$, $q = \frac{4}{3}$, $r = \frac{3}{2}$, $s = \frac{8}{5}$, $t = \frac{5}{3}$. The possible combinations will be:

mnp, mpq, mqr, mrs, mst
mnq, mpr, mqs, mrt
mnr, mps, mqt
mns, mpt
mnt

In many of these combinations, the last two intervals are not consonant with one another. They are related in *mnp* as $1 : \frac{25}{24}$, in *mnq* as $1 : \frac{10}{9}$, in *mnt* as $1 : \frac{15}{18}$, in *mpq* as $1 : \frac{16}{15}$, in *mps* as $1 : \frac{32}{15}$, in *mqr* as $1 : \frac{9}{8}$, in *mrs* as $1 : \frac{16}{15}$, in *mrt* as $1 : \frac{10}{9}$, and in *mst* as $1 : \frac{25}{24}$. All these combinations do not give a consonant chord. But *mpr* or $1 : \frac{5}{4} : \frac{3}{2}$ makes another, since $\frac{5}{4}$ is to $\frac{3}{2}$ as $1 : \frac{6}{5}$ and *mnt* where $1 : \frac{6}{5} : \frac{3}{2}$ gives another because $\frac{6}{5}$ is as $\frac{3}{2}$ to $1 : \frac{5}{4}$. The combinations *mns*, *mpt*, *mqt*, and *mqs* reduce to these two chords if one multiplies or divides the number by 2, and if one expresses them in the smallest numbers. It will never be possible to add a fourth consonant interval to all the others. There will therefore never be a consonant chord composed of more than three tones, except if one wishes to add the octave of one of the three sounds. Such a chord as $1 : \frac{5}{4} : \frac{3}{2}$ or $1 : \frac{6}{5} : \frac{3}{2}$ is a *perfect chord*; the first is the *major perfect chord*, the other the *minor perfect chord*. The consonant combinations $1 : \frac{5}{4} : \frac{5}{3}$ and $1 : \frac{6}{5} : \frac{8}{5}$ (the *chord of the sixth*) and $1 : \frac{4}{3} : \frac{5}{3}$ and $1 : \frac{4}{3} : \frac{8}{5}$ (the *chord of the sixth fourth*) are the inversions of the major perfect chord and the minor perfect chord.

Experience shows that the two perfect chords have different effects. The major is more suitable for expressing joy. It soothes the ear more than the minor. The cause of this different effect is the greater simplicity of the major chord. In reducing these ratios to their lowest terms, the vibrations of the major perfect chord will be as 4:5:6

⁶ Most musicians now consider the perfect fourth to be consonant.—TDR

and those of the minor as 10:12:15. Both are composed of a major third and a minor, which together make a fifth; the only difference lies on the position of the thirds.

The manner in which I have shown here the formation of the perfect chords is fundamentally the same as that used by Mr. Mercadier de Belestá (*Système de Musique*, Paris 1776), who set forth several subjects pertaining to the numerical theory of sound better than many others.

10. Dissonant Chords

A *dissonant chord* is one that contains one or more dissonant intervals. The principal of these chords is the *seventh chord*, in which one adds a seventh to a perfect chord. It is susceptible to three inversions, in which one must always regard the sound, which by origin is a seventh, as the dissonance in the position it is found. Several other dissonances result from the delay or anticipation of a sound.

11. Ordinary Scale

The major perfect chord, because of its simplicity, could serve better than the other to find the ordinary *scale* of sound; that is, the series of the most agreeable and suitable of sounds, by which one can pass from a fundamental tone to its octave, and from one octave to another, without losing the sensation of the fundamental sound. The perfect chord of a fundamental tone, joined to its octave, excites the most perfect of the sounds, reinforced by the most suitable consonants. When we regard *do* as the fundamental, we will have:

$$\begin{array}{ccccccc} 1 & : & \frac{5}{4} & : & \frac{3}{2} & : & 2 \\ do & & mi & & sol & & do \end{array}$$

But this is still not a scale because the distances are too great and too unequal. It is therefore necessary to add the perfect chords of these tones, which approach the fundamental more than the others, such as the fifth ($\frac{3}{2}$) and the fourth ($\frac{4}{3}$). The fifth $\frac{3}{2}$ produces, by its perfect chord, the tones $\frac{3}{2} \times \frac{5}{4}$ or $\frac{15}{8} = si$ and $\frac{3}{2} \times \frac{3}{2}$ or $\frac{9}{4} = re$.⁷ The fourth $\frac{4}{3} = fa$, which is inserted by itself, produces its major third $\frac{4}{3} \times \frac{5}{4}$ or $\frac{5}{3} = la$.

⁷ Although $\frac{3}{2} \times \frac{3}{2}$ does not appear to equal $\frac{9}{8}$, this is the original text in both French and German versions of this work.—MAB

Its fifth is the same as the octave of the fundamental tone. We will therefore have the scale:

$$\begin{array}{cccccccc} 1, & \frac{9}{8}, & \frac{5}{4}, & \frac{4}{3}, & \frac{3}{2}, & \frac{5}{3}, & \frac{15}{8}, & 2 \\ do, & re, & mi, & fa, & sol, & la, & si, & do \end{array}$$

This scale has seven intervals of different sizes. The interval of the third to the fourth and that of the seventh to the eighth are approximately one half of the others. One calls the greater intervals *tones* and the smaller ones, *semi-tones*. Each interval draws its enumeration from the interval that it represents, so that the distance from *do* to *re* is a second, from *do* to *mi* a third, from *do* to *fa* a fourth, from *do* to *sol* a fifth, from *do* to *la* a sixth, from *do* to *si* a seventh, and from *do* to *do* an *octave*. If one compares all these tones to the octave above, one will have intervals that must be regarded as the inversions of the previous ones, and which do not differ much from it, so far as the effect and the manner of treating them are concerned. Thus, the distance from *re* to *do* will be a seventh, from *mi* to *do* a sixth, from *fa* to *do* a fifth, from *sol* to *do* a fourth, from *la* to *do* a third, and from *si* to *do* a *second*.

12. Intervals

This scale will acquaint us with most of the dissonant intervals. The first interval is to the second as 1 to $\frac{9}{8}$, or as 8:9 and the second to the third, as $\frac{9}{8}$ to $\frac{5}{4}$ or as 9:10. These two intervals, which differ by $\frac{81}{80}$, we call a tone; the one a *major* tone, the other a *minor* tone.

The major third 1 : $\frac{5}{4}$ or 4:5 differs from the minor third $\frac{6}{5}$ by the interval $\frac{25}{24}$, which is the smallest interval one can make practical use of. If an interval is raised or lowered (a *sharp* or a *flat*) to the same degree, the difference is always $\frac{25}{24}$. Each smaller difference is a *comma*. The inversion of a *minor semi-tone* $\frac{25}{24}$ is the *diminished octave* $\frac{24}{25}$.

The difference between the third sound $\frac{5}{4}$ and the fourth $\frac{4}{3}$ is $\frac{16}{15}$; this interval is called a *major semi-tone*; it differs from the minor semi-tone by $\frac{128}{125}$. Sound inversion gives the *major seventh* $\frac{15}{8}$.

The fourth sound differs from the fifth by $\frac{9}{8}$, or a major tone; this one differs from the sixth by $\frac{10}{9}$, or a minor tone. The difference of the major sixth and the minor is that of the third $\frac{5}{4}$. The seventh differs from the octave by $\frac{16}{15}$, or a major semi-tone.

The ratio of these tones gives us still other intervals. The one from *re* to *fa* or $\frac{9}{8} : \frac{4}{3} = \frac{32}{37}$ is a minor third, diminished by a comma $\frac{81}{80}$. That from *fa* to *si* $\frac{4}{3}$ to $\frac{15}{8}$ is an *augmented fourth*, which is also called a *tri-tone* because it results from the combination of three tones; its inversion is the *diminished fifth* $\frac{64}{45}$.

13. Some Other Intervals

There are therefore major and minor seconds, thirds, sixths, and sevenths, but there are no such fifths and fourths. If a fifth or a fourth, as also a major second third, sixth, or seventh, is augmented by a minor semi-tone $\frac{25}{24}$, it is called *augmented*; if a fifth or a fourth as also a minor second, third, sixth, or seventh is lowered by the same interval, it is called *diminished*.

The inversion of a major interval gives a minor and that of a minor gives a major; the inversion of a diminished interval yields an augmented one and the inversion of an augmented one gives a diminished. The augmented and diminished intervals of which we will make use of include:

The *augmented second* $\frac{9}{8} \times \frac{25}{24} = \frac{75}{64}$ or $\frac{10}{9} \times \frac{25}{24} = \frac{125}{108}$, and its inversion, the *diminished seventh* $\frac{16}{9} \times \frac{24}{25} = \frac{128}{75}$ or $\frac{9}{5} \times \frac{24}{25} = \frac{216}{125}$.

The *diminished third* $\frac{6}{5} \times \frac{24}{25} = \frac{144}{125}$ and its inversion, the *augmented sixth* $\frac{5}{3} \times \frac{25}{24} = \frac{125}{72}$.

The *diminished fourth* $\frac{4}{3} \times \frac{24}{25} = \frac{32}{25}$ and its inversion, the *augmented fifth* $\frac{3}{2} \times \frac{25}{24} = \frac{25}{16}$.

The *augmented fourth* $\frac{4}{3} \times \frac{25}{24} = \frac{25}{18}$ and its inversion, the *diminished fifth*, which is also called the *false fifth* $\frac{3}{2} \times \frac{24}{25} = \frac{36}{25}$.

An *augmented third* $\frac{5}{4} \times \frac{25}{24} = \frac{125}{96}$ and its inversion, the *diminished sixth* $\frac{8}{5} \times \frac{24}{25} = \frac{192}{125}$ are not in use.

14. Diatonic, Chromatic, and Enharmonic Progressions

(Note: Names of tones in different languages.) The scale mentioned in Par. 11, as also every other scale composed of major tones and semi-tones, is known as a diatonic scale, and the progression from one of these sounds to another contained in the same scale is known as a *diatonic progression*. Sometimes, the major semi-tone $\frac{16}{15}$ is the smallest degree on such a scale, the diatonic semi-tone. If one augments one of the sounds of the scale *do*, *re*, *mi*, *fa*, *sol*, *la*, *si* by the minor semi-tone $\frac{25}{24}$, this is expressed by the sign #, which is called a *sharp*; but if we lower the sound by the same interval $\frac{25}{24}$, it is expressed by the sign ♭, which is known as a *flat*. The sign of restoration is the *natural* ♮. A progression from a raised or lowered sound to the natural tone of the same denomination, or of the natural sound to the raised or lowered one, for example, from *do* to *do*[#] or from *mi* to *mi*[♭], is called a *chromatic progression*. Sometimes the minor semi-tone $\frac{25}{24}$, by which the progression of a raised sound to its neighboring lowered sound is made, is called the *chromatic semi-tone*. For example, from *do*[#] to *re*[♭] or from *re*[♭] to *do*[#] is called an *enharmonic progression*.

The origin of the names *do, re, mi, fa, sol, la, si* is too well known to be repeated here.⁸ In different countries, the names of these sounds are different. In Italy, the syllables *do, re, mi, fa, sol, la* are used to express the degrees of any scale. In place of *si*, one then sings *mi* because it expresses the advance from the major semi-tone by *i:fa*; one then ordinarily changes the preceding syllable into a *re*. At the present time, some people add the syllable *si* because there are too many useless difficulties in wanting to express seven different objects by six signs. To express the same sounds, one makes use of the letters *C, D, E, F, G, A, B*, to which one adds the syllables that are suitable to the degrees of the ancient hexachord, in which this sound is found. Thus, for example, *do* is called *C sol fa do*, *re* is called *D la sol re*, etc. For elevation of the sound, a sharp is added, and for the lowering, a flat. In Germany, the sounds are called (beginning with *do*): *c, d, e, f, g, a, h* (which is pronounced *ha*). To express the sharp semi-tone, the termination *is* is added, saying *cis, dis, eis, fis, gis, ais, his* and to express the semi-tone flat, the termination *es* is added: *ces, des, es, fes, ges, as*; but to express *si^b*, one makes an exception, calling it *b*. One conforms more to the analogies of the other denominations if, as some have proposed, one wants to express the *si* by *b*, the *si[#]* by *bis*, and *si^b* by *bes*. One sees that the Italian denominations are more wordy, the German more precise.

The English and the Dutch call the sounds *c, d, e, f, g, a, b*. To express the semi-tone sharp, the English use the word *sharp* and the Dutch *kruis*, and to express the semi-tone flat, the English add *flat* and the Dutch *mol*.

15. Scales of Different Tones

All the sharp and flat intervals are necessary because, to avoid monotony, it is necessary that one be able to regard each sound as a fundamental sound, and to assign a fair scale to it. The series of sounds *do, re, mi, fa, sol, la, and si* does not contain all the degrees of these scales. If we regard the sound *sol*, for example as the fundamental, the sixth step to the seventh (*mi* to *fa*) will be only a semi-tone. It is

⁸ It is popularly believed that Guido of Arezzo took the opening syllable letters from each line of a hymn to Saint John the Baptist to form the names of notes in the musical scale:

UTqueant laxis
REsonare fibris
MIRA gestorum
FAMuli tuorum
SOLve polluti
LABii reatum
Sancte Ioanne—RTB

therefore necessary, in order that it be a tone, to use $fa^\sharp$ in place of fa . In the same way, to use re as the fundamental sound, we must change fa into $fa^\sharp$ and do into $do^\sharp$. For other fundamental sounds, we must flatten several sounds. For example, to have the fair scale of fa , it would be necessary to change si into si^b and to have the scale of this si it would also be necessary to change mi into mi^b . Proceeding by fifths, it would always be necessary to sharpen one more sound, and in proceeding by fourths, or inverse fifths, it is also necessary to have a higher sound or a flattened one. One will therefore have the following diatonic scales:

<i>do, re, mi, fa, sol, la, si, do</i>	
<i>sol, la, si, do, re, mi, fa[♯], sol</i>	<i>fa, sol, la, si^b, do, re, mi, fa</i>
<i>re, mi, fa[♯], sol, la, si, do[♯], re</i>	<i>si^b, do, re, mi^b, fa, sol, la, si</i>
<i>la, si, do[♯], re, mi, fa[♯], sol[♯], la</i>	<i>mi^b, fa, sol, la^b, si^b, do, re, mi^b</i>
<i>mi, fa[♯], sol[♯], la, si, do[♯], re[♯], mi</i>	<i>la^b, si^b, do, re^b, mi^b, fa, sol, la^b</i>
<i>si, do[♯], re[♯], mi, fa[♯], sol[♯], la[♯], si</i>	<i>re^b, mi^b, fa, sol^b, la^b, si^b, do, re^b</i>
<i>fa[♯], sol[♯], la[♯], si, do[♯], re[♯], mi[♯], fa[♯]</i>	<i>sol^b, la^b, si^b, do^b, re^b, mi^b, fa, sol^b</i>
<i>do[♯], re[♯], mi[♯], fa[♯], sol[♯], la[♯], si[♯], do[♯]</i>	<i>do^b, re^b, mi^b, fa^b, sol^b, la^b, si^b, do^b</i>

These changes of the sounds in all the possible scales can be expressed by the arithmetic progression.

$$n^\sharp \dots 3^\sharp, 2^\sharp, 1^\sharp, 0, 1^b, 2^b, 3^b \dots n^b$$

If one wants to regard other sounds, for example, $sol^\sharp$ or fa^b as the fundamentals, it is necessary to sharpen or flatten some sounds twice. When this becomes necessary, we express the double sharp by the sign x and the double flat by $b b$.

The fundamental sound series *do, re, mi, fa, sol, la, si* (*c, d, e, f, g, a, b*) is the *range*, and the fundamental sound with the sounds that depend on it is the *mode*. If the fundamental sound has a major third, as in the series mentioned, it forms a *major mode*, if it has a *minor third*, it forms a *minor mode*.

16. Scale of the Minor Mode

To form the scale of the minor mode, we must give the perfect minor chords to the fundamental sound and to the sounds that approach it closer than the others, such as the fifth and the fourth. If we regard *la* as the fundamental sound, the perfect minor chord of this sound is *la, do, mi*; that of the fifth *mi, sol, si*; and that of the fourth *re, fa, la*. This will give the scale:

$$la, si, do, re, mi, fa, sol, la$$

But the ear requires that, in going up the scale, the step from the seventh to the eighth note be only a major semi-tone, which is called the *sensible note* (*subsemitonium modi*), because it determines every major or minor mode. It is therefore necessary, in going up the scale, to give to the fifth *mi* the major third $sol^\sharp$.

But, through this change, the step from the sixth note *fa* to the seventh *sol*[#] would be too large; it is therefore often necessary to use *fa*[#] instead of *fa*, and to regard the scale of the minor mode, in going up, as *la, si, do, re, mi, fa*[#], *sol*[#], *la*. This augmentation of the sixth and seventh notes is regarded as accidental, and they are pointed out every time they are used. In going down the scale, the scale remains unchanged.

Each scale of a minor mode contains the same notes as the major mode of its third minor; thus, for example, the scale of the minor mode of:

<i>la</i> is the same as that of the major mode:	<i>do</i>
<i>mi</i>	<i>sol</i>
<i>si</i>	<i>re</i>
<i>fa</i> [#]	<i>la</i> , etc.

17. Explanation of Several Words

When one mode contains more or less sharp or flat notes than the other, we say that they differ by so many *degrees*. A major and a minor that contain the same notes are *relative modes*. Sometimes we call the fundamental note the *tonic*, its fifth the *dominant*, its fourth the *subdominant*, and its third the *mediant*.

18. Progressions from One Chord to Another

The most natural progressions from one chord to another are that of the fifth or fourth, or to another which differs only by one degree. When one proceeds to more distant modes, one ordinarily makes by an enharmonic substitution of an augmented note to its neighboring diminished note, or from a diminished to its neighboring augmented, or one forces the ear to neglect the comma $\frac{128}{125}$ by which the major semi-tone $\frac{16}{15}$ differs from the minor $\frac{25}{14}$.

I will not develop any further the passages from one note to another or from one chord to another chord, since there are sufficient treatises on harmony that can provide instruction.

19. Relative Frequencies of Sounds Contained in an Octave

To provide a more exact idea of the size of each interval, I have furnished in the following Table the relative numbers of vibrations and the lengths of the corresponding chords, in fractions and in decimals, for each interval, reduced to the fundamental note *do*.

	Number of vibrations		Lengths of strings	
Unison, <i>do:do</i>	1	1	1	1
Minor semitone, <i>do:do</i> [#]	25/24	1.0416 2/3	24/25	0.96
Minor second or the major semi-tone, <i>do:re</i> ^b	16/15	1.066 2/3	15/16	0.9375
Major second, <i>do:re</i>	10/9	1.1111 1/9	9/10	0.9
(minor tone)				
or	9/8	1.125	8/9	0.888 8/9
(major tone)				
Diminished third, <i>do:mi</i> ^{b b}	144/125	1.152	125/144	0.8680 5/9
(or, rather, <i>do</i> [#] : <i>mi</i> ^b)				
Augmented second, <i>do:re</i> [#]	125/108	1.574 3/27	108/125	0.864
or	75/64	1.718 3/4	64/75	0.8533 1/3
Minor third, <i>do:mi</i> ^b	6/5	1.2	5/6	0.8333 1/3
Major third, <i>do:mi</i>	5/4	1.25	4/5	0.8
Diminished fourth, <i>do:fa</i> ^b	32/25	1.28	25/32	0.78125
Fourth, <i>do:fa</i>	4/3	1.3333 1/3	3/4	0.75
Augmented fourth, <i>do:fa</i> [#]	25/18	1.3888 8/9	18/25	0.72
Diminished fifth, <i>do:sol</i> ^b	36/25	1.44	25/36	0.6944 4/9
Fifth, <i>do:sol</i>	3/2	1.5	2/3	0.6666 2/3
Augmented fifth, <i>do:sol</i> [#]	25/16	1.5625	16/25	0.64
Minor sixth, <i>do:la</i> ^b	8/5	1.6	5/8	0.625
Major sixth, <i>do:la</i>	5/3	1.6666 2/3	3/5	0.6
Diminished seventh, <i>do:si</i> ^{b b} (or <i>do</i> [#] : <i>si</i> ^b)	128/75	1.7066 2/3	75/128	0.5959 3/8
or	216/125	1.728	125/216	0.5787 1/27
Augmented sixth, <i>do:la</i> [#]	125/72	1.7361 1/9	72/125	0.576
Minor seventh, <i>do:si</i> ^b	16/9	1.7777 7/9	9/16	0.5625
or	9/5	1.8	8/9	0.5555 5/9
Major seventh, <i>do:si</i>	15/8	1.875	8/15	0.5333 1/3
Diminished octave, <i>do:do</i> ^b	48/25	1.92	24/48	0.5208 1/3
Octave, <i>do:do</i>	2	2	1/2	0.5

Some people who are involved in practice have found fault with the theory in that it yields a minor semi-tone $\frac{25}{24}$, for example, *do* to *do*[#], that is smaller than the major semi-tone $\frac{16}{15}$, *do* to *re*, although the minor sometimes has a better effect if one makes it slightly more acute; however, the theory is fair, and the reason why a minor semi-tone sometimes supports or requires slightly higher value is that ordinarily an augmented note rises to its more acute neighbor, and the ear likes to prepare and anticipate a little the tendency toward the following note.

20. Several Other Intervals Contained in the Natural Series of Numbers

The natural series of numbers still gives intervals that are not received in the ordinary system of sounds, and which, however, are produced by some musical instruments, such as the horn and the trumpet, where one must make use of such sounds for some others which they approach. The sound corresponding to the number 7, of which the effect is intermediate between the consonances and the dissonances, can be produced on these instruments, but they are not used. It would be useless to want to introduce them, because one would have to multiply too much the number of intervals which could scarcely be distinguished from those which already exist. It can, however, be presumed that the reason why the seventh chord (*do, mi, sol, si*) and the augmented sixth (*do, mi, sol, la[#]*) are also not disagreeable to the ear, which one could believe from their complex number, is due to the fact that the ear substitutes for these numbers the ratios 4:5:6:7, in which the interval $\frac{7}{4}$ differs from the seventh $\frac{16}{9}$ by the comma $\frac{64}{63}$, and from the augmented sixth $\frac{125}{72}$ by the still smaller comma $\frac{126}{125}$. In the same instruments, the sound corresponding to the number 11 is replaced by *fa*, but the interval $\frac{11}{8}$ is more acute by $\frac{33}{32}$ than the fourth $\frac{4}{3}$ or the true *fa*. Sometimes one makes it still more acute by employing more force and then we use it in place of *fa[#]*. The sound that corresponds to the number 13 is used for *la*, but the interval $\frac{13}{8}$ is graver by $\frac{40}{39}$ than the major sixth $\frac{5}{3}$. Sounds that surpass the number 16 are not used with horn or trumpet.

If one wishes to continue the natural series of numbers, even to infinity, one could never express certain intervals exactly, counting from the fundamental note; because there does not exist an integer to which some power of two were to be as 3 to 4 or as 5 to 6. However, the interval $\frac{9}{16}$ approaches the minor third $\frac{6}{5}$ closely, being no less than a comma $\frac{96}{95}$ away. Perhaps, when one sometimes uses the chord of the perfect minor chord in place of the major, or the major in place of the minor, the ear is less injured, because it substitutes for the minor third $\frac{6}{5}$ the interval $\frac{19}{16}$, thus hearing a variety of ratios like 16:19:24 and 16:20:24.

I would express these intervals in decimals in order to compare them with those numbers that were found in the previous paragraph.

Number of vibrations		Length of strings	
7/4	1.75	4/7	0.5714 2/7
11/8	1.375	8/11	0.7272 8/11
13/8	1.625	8/13	0.6153 11/13
19/16	1.1875	16/19	0.8157 17/19

Treatise on Acoustics

The First Comprehensive English Translation of E.F.F.

Chladni's *Traité d'Acoustique*

Chladni, E.F.F.

2015, XXXI, 227 p., Hardcover

ISBN: 978-3-319-20360-7