

## Chapter 2

# Comparing the Fully Stressed Design and the Minimum Constrained Weight Solutions in Truss Structures

David Greiner, José M. Emperador, Blas Galván and Gabriel Winter

**Abstract** The optimization structural design problems of Fully Stressed Design (FSD) and Minimum Constrained Weight (MCW) are compared in this work in a simple truss test case with discrete cross-section type bar sizing, where both optimum designs are coincident. An analysis of the whole search space is included, and the optimization behaviour of evolutionary algorithms are compared with multiple population sizing and mutation rates in both problems. Results of average, best and standard deviation metrics indicate the success and the robustness of the methodology, as well as the fastest and easiest behaviour when considering the FSD case.

**Keywords** Structural design · Truss optimization · Evolutionary algorithms · Fully stressed design · Minimum constrained weight

## 2.1 Introduction

The use of evolutionary algorithms/metaheuristics has allowed the resolution of the global optimum design of many engineering problems (see e.g. [2, 10]), and particularly, in the case of discrete cross-section bar structures since the first nineties of the twentieth century [1, 7, 8]. In this book chapter, it is handled a comparative and relational study of the search algorithm performance in two structural problems: first, the minimization of the constrained weight and, second, the obtainment of the fully stressed design. Results using the above mentioned global search methods in a simple truss structure considering some statistical metrics are

---

D. Greiner (✉) · J.M. Emperador · B. Galván · G. Winter  
Institute of Intelligent Systems and Numerical Applications in Engineering SIANI,  
Universidad de Las Palmas de Gran Canaria ULPGC, 35017 Las Palmas, Spain  
e-mail: david.greiner@ulpgc.es

obtained. First, the structural handled problems are described in Sect. 2.2, then the test case is shown in Sect. 2.3. Section 2.4 presents results and discussion, and finally, the conclusions Sect. 2.5 ends this book chapter.

## 2.2 Structural Problems

Two optimization problems of bar structures with discrete section-types are fronted in this book chapter.

In first place, the problem of minimization of the constrained structural weight (MCW), which is related with the minimization of raw cost of the structure, is considered (Eq. 2.1) (e.g. see [4, 12]). It is the most common structural optimum design problem.

$$MCW = \sum_{i=1}^{Nbars} A_i \cdot l_i \cdot \rho_i \quad (2.1)$$

where  $A_i$  is cross-sectional area,  $l_i$  is length and  $\rho_i$  is specific weight, all corresponding to bar  $i$ ; subjected under constraints of stresses, displacements and/or buckling. In this chapter only stresses constraints are taken into account, and treated as in [6].

In second place, the problem of achieving the fully stressed design (FSD) structure is considered (e.g., see [14]), which has been handled since the beginning of the 20th century. The FSD of a structure is defined as the design in which some location of every bar member in the structure is at its maximum allowable stress for at least one loading condition.

$$FSD = \sqrt{\sum_{i=1}^{Nbars} (\sigma_{MAX-i} - \sigma_{MAX-Ri})^2} \quad (2.2)$$

where  $\sigma_{MAX-i}$  and  $\sigma_{MAX-Ri}$  are the maximum stress and the maximum allowable stress, respectively, both corresponding to bar  $i$ .

Some relation between both previous problems, MCW and FSD, has been established, mainly in trusses structures where the material is allowed to work at its full potential due to the only existence of normal efforts, associated with the cross-sectional area [11, 13]. In this work, we show through the use of metaheuristic global optimization methods in discrete cross section-type trusses, that even in the possible case that both problems (MCW and FSD) share the same optimum solution, the search still has different characteristics and topology, which makes easier or harder to solve for the global search evolutionary algorithm.

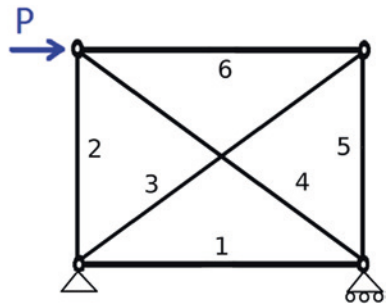
2.3 Test Case

The test case used is a simple test case with truss bar structures based on one in [15] and [16], solving it with discrete cross-section types variables (as in [9]). The computational domain, loading and boundary conditions are shown in Fig. 2.1, with Load  $P = 4450\text{ N}$ . This test case has been solved also for simultaneous (that is, multiobjective optimization) minimization of weight and maximization of the reliability index in Greiner and Hajela [3].

Each bar corresponds to an independent variable. Table 2.1 shows the set of cross section types and their geometric properties (area, radius of gyration). Table 2.2 represents the search space of variables, including the lower and upper limit of each variable.

An own implemented truss bar structure stiffness matrix calculation has been used to evaluate the structural variables, where articulated nodes (that is

**Fig. 2.1** Test case.  
Computational domain,  
loading and boundary  
conditions



**Table 2.1** Cross-section  
types

Order	Cross-section	Area (cm <sup>2</sup> )	Radius of gyration (cm)
1	C1	0.85	0.653
2	C2	0.93	0.652
3	C3	1.01	0.651
4	C4	1.09	0.650
5	C5	1.17	0.649
6	C6	1.25	0.648
7	C7	1.33	0.647
8	C8	1.41	0.646
9	C9	1.49	0.645
10	C10	1.57	0.644
11	C11	1.65	0.643
12	C12	1.73	0.642
13	C13	1.81	0.641
14	C14	1.89	0.640
15	C15	1.97	0.639
16	C16	2.05	0.638

**Table 2.2** Search space of variables

Bar number	Bar variable	Cross-section type set
1	v1	From C1 to C16
2	v2	From C1 to C16
3	v3	From C1 to C16
4	v4	From C1 to C16
5	v5	From C1 to C16
6	v6	From C1 to C16

**Table 2.3** Geometric parameters

	Value (m.)
Height (H)	0.9144
Width (W)	1.2190

**Table 2.4** Material properties (Steel)

Parameter	Value
Density	7850 kg/m3
Young modulus	$2.06 \times 10^5$ MPa
Maximum stress	276 MPa

non-resisting moment capabilities) are considered, elastic behaviour of steel is assumed, and no buckling effect is taken into account in these results. Table 2.3 shows the geometric parameters (height and width) of the structure. Table 2.4 exposes its material properties, -those of standard construction steel-.

In order to define the cross-section type sizing of each bar (that is the structural design), the quantities of interest are the values of the fitness function/s (minimum constrained weight and /or fully stress design) and the maximum stress of each bar.

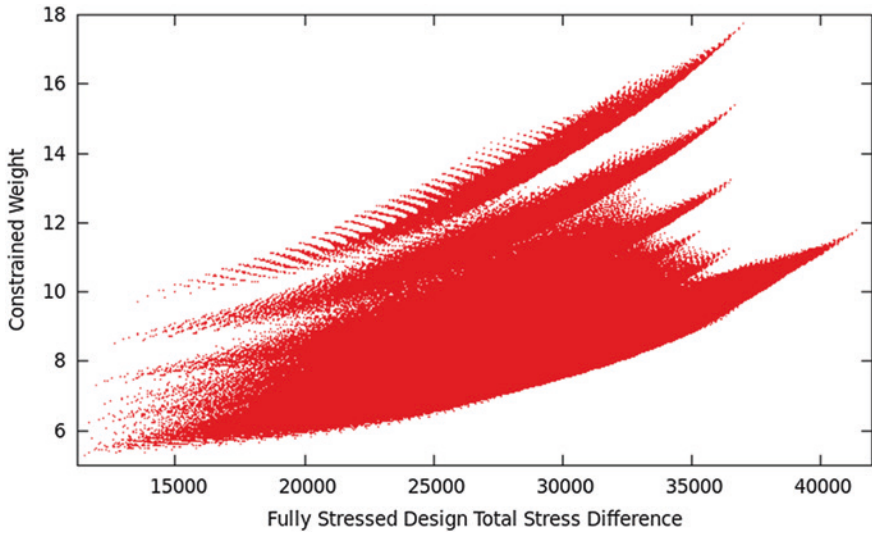
2.4 Results and Discussion

2.4.1 Test Case Analysis

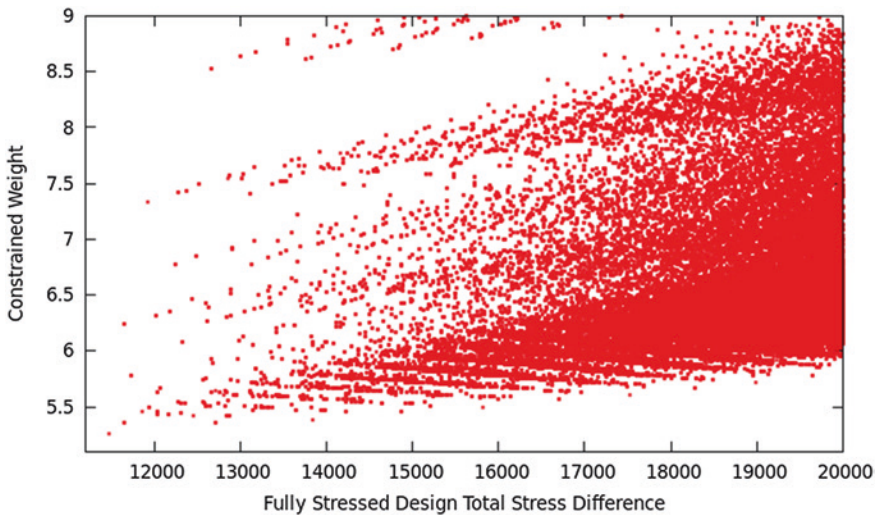
This section studies the relationship between the MCW problem and the FSD problem in our test case (as described in Sect. 2.3).

Therefore, the whole search space of the previous test case has been explored, evaluating both objectives: (a) Minimum Constrained Weight (MCW, in kg.) as shown in Eq. 2.1, and (b) Square root of the sum of squared stress differences of each bar with Fully Stressed Design (FSD) as shown in Eq. 2.2.

Values of the  $2^{24} = 16,777,216$  designs (corresponding to a 6 bar  $\times$  4 bits/bar chromosome = 24 bits) are obtained and shown in Fig. 2.2 (whole search space).



**Fig. 2.2** Whole search space designs of test case



**Fig. 2.3** Zommed vision over Fig. 2.2 (search space designs of test case)

In addition, a zoomed picture of the best solution designs are shown in Fig. 2.3. In this test case, the minimum of both fitness functions is a coincident design, the most left and bottom point in this Fig. 2.3. Moreover, calculating the population Pearson's correlation coefficient  $r$  between both objectives (MCW and FSD) gives a value  $r = 0.71089$  (where 1.0 means a perfect linear relationship).

The detailed best fifty designs of each objective are shown in Table 2.5 (MCW optimum designs) and Table 2.6 (FSD optimum designs). In addition to the optimum design, which is shared by the problem of MCW and the problem of FSD, fifteen designs out of this list of fifty are included in both sets (all shared designs are highlighted in bold type in the tables).

**Table 2.5** Minimum MCW designs (cross-section types as in Table 2.1)

Design order	FSD value	MCW value	Unconstrained weight	Bar 1	Bar 2	Bar 3	Bar 4	Bar 5	Bar 6
<b>1st</b>	<b>11480.7</b>	<b>5.26338</b>	<b>5.26338</b>	<b>C1</b>	<b>C1</b>	<b>C3</b>	<b>C3</b>	<b>C1</b>	<b>C1</b>
2nd	12709.5	5.3581	5.3208	C1	C1	C3	C3	C2	C1
3rd	12709.5	5.3581	5.3208	C1	C2	C3	C3	C1	C1
<b>4th</b>	<b>11645.8</b>	<b>5.35907</b>	<b>5.35907</b>	<b>C1</b>	<b>C1</b>	<b>C4</b>	<b>C3</b>	<b>C1</b>	<b>C1</b>
<b>5th</b>	<b>11645.8</b>	<b>5.35907</b>	<b>5.35907</b>	<b>C1</b>	<b>C1</b>	<b>C3</b>	<b>C4</b>	<b>C1</b>	<b>C1</b>
6th	13834.7	5.37822	5.37822	C1	C2	C3	C3	C2	C1
<b>7th</b>	<b>12449.8</b>	<b>5.41648</b>	<b>5.41648</b>	<b>C2</b>	<b>C1</b>	<b>C3</b>	<b>C3</b>	<b>C1</b>	<b>C2</b>
8th	12840.5	5.4165	5.4165	C1	C2	C3	C4	C1	C1
9th	12875.2	5.4165	5.4165	C1	C1	C3	C4	C2	C1
10th	12840.5	5.4165	5.4165	C1	C1	C4	C3	C2	C1
11st	12875.2	5.4165	5.4165	C1	C2	C4	C3	C1	C1
<b>12nd</b>	<b>12163.8</b>	<b>5.43563</b>	<b>5.43563</b>	<b>C2</b>	<b>C1</b>	<b>C4</b>	<b>C3</b>	<b>C1</b>	<b>C1</b>
<b>13rd</b>	<b>12038.5</b>	<b>5.43563</b>	<b>5.43563</b>	<b>C2</b>	<b>C1</b>	<b>C3</b>	<b>C4</b>	<b>C1</b>	<b>C1</b>
<b>14th</b>	<b>12038.5</b>	<b>5.43563</b>	<b>5.43563</b>	<b>C1</b>	<b>C1</b>	<b>C4</b>	<b>C3</b>	<b>C1</b>	<b>C2</b>
<b>15th</b>	<b>12163.8</b>	<b>5.43563</b>	<b>5.43563</b>	<b>C1</b>	<b>C1</b>	<b>C3</b>	<b>C4</b>	<b>C1</b>	<b>C2</b>
<b>16th</b>	<b>12312.0</b>	<b>5.43998</b>	<b>5.43563</b>	<b>C1</b>	<b>C1</b>	<b>C5</b>	<b>C2</b>	<b>C1</b>	<b>C2</b>
<b>17th</b>	<b>12312.0</b>	<b>5.43998</b>	<b>5.43563</b>	<b>C2</b>	<b>C1</b>	<b>C2</b>	<b>C5</b>	<b>C1</b>	<b>C1</b>
<b>18th</b>	<b>12036.9</b>	<b>5.45477</b>	<b>5.45477</b>	<b>C1</b>	<b>C1</b>	<b>C5</b>	<b>C3</b>	<b>C1</b>	<b>C1</b>
<b>19th</b>	<b>11861.9</b>	<b>5.45477</b>	<b>5.45477</b>	<b>C1</b>	<b>C1</b>	<b>C4</b>	<b>C4</b>	<b>C1</b>	<b>C1</b>
<b>20th</b>	<b>12036.9</b>	<b>5.45477</b>	<b>5.45477</b>	<b>C1</b>	<b>C1</b>	<b>C3</b>	<b>C5</b>	<b>C1</b>	<b>C1</b>
21st	14232.6	5.45478	5.45478	C2	C1	C3	C3	C3	C1
22nd	14232.6	5.45478	5.45478	C1	C3	C3	C3	C1	C2
23rd	14861.9	5.46076	5.43565	C1	C2	C3	C3	C3	C1
24th	14861.9	5.46076	5.43565	C1	C3	C3	C3	C2	C1
25th	13148.3	5.46882	5.39735	C2	C1	C3	C3	C2	C1
26th	13148.3	5.46882	5.39735	C1	C2	C3	C3	C1	C2
27th	13645.3	5.47391	5.47391	C3	C2	C2	C4	C1	C1
28th	13645.3	5.47391	5.47391	C1	C1	C4	C2	C2	C3
29th	13977.5	5.47392	5.47392	C1	C1	C3	C4	C3	C1
30th	13977.5	5.47392	5.47392	C1	C3	C4	C3	C1	C1
31st	13930.1	5.47392	5.47392	C1	C1	C4	C3	C3	C1
32nd	13930.1	5.47392	5.47392	C1	C3	C3	C4	C1	C1
33rd	13970.5	5.47392	5.47392	C1	C2	C4	C3	C2	C1

(continued)

**Table 2.5** (continued)

Design order	FSD value	MCW value	Unconstrained weight	Bar 1	Bar 2	Bar 3	Bar 4	Bar 5	Bar 6
34th	13970.5	5.47392	5.47392	C1	C2	C3	C4	C2	C1
35th	13818.0	5.48927	5.37822	C1	C1	C3	C3	C3	C1
36th	13818.0	5.48927	5.37822	C1	C3	C3	C3	C1	C1
37th	13278.3	5.49303	5.49303	C4	C1	C2	C4	C1	C1
38th	13278.3	5.49303	5.49303	C1	C1	C4	C2	C1	C4
39th	13330.9	5.49305	5.49305	C2	C1	C4	C3	C2	C1
40th	13249.4	5.49305	5.49305	C2	C1	C3	C4	C2	C1
41st	13175.6	5.49305	5.49305	C1	C1	C4	C3	C2	C2
42nd	13175.6	5.49305	5.49305	C2	C2	C3	C4	C1	C1
43rd	13249.4	5.49305	5.49305	C1	C2	C4	C3	C1	C2
44th	13381.0	5.49305	5.49305	C2	C2	C2	C5	C1	C1
45th	13381.0	5.49305	5.49305	C1	C1	C5	C2	C2	C2
46th	13330.9	5.49305	5.49305	C1	C2	C3	C4	C1	C2
47th	13328.1	5.49305	5.49305	C2	C2	C4	C3	C1	C1
48th	13328.1	5.49305	5.49305	C1	C1	C3	C4	C2	C2
49th	15823.2	5.49307	5.49307	C1	C3	C3	C3	C3	C1
<b>50th</b>	<b>11944.9</b>	<b>5.49621</b>	<b>5.33993</b>	<b>C2</b>	<b>C1</b>	<b>C3</b>	<b>C3</b>	<b>C1</b>	<b>C1</b>
<b>51th</b>	<b>11944.9</b>	<b>5.49621</b>	<b>5.33993</b>	<b>C1</b>	<b>C1</b>	<b>C3</b>	<b>C3</b>	<b>C1</b>	<b>C2</b>

**Table 2.6** Minimum FSD designs (cross-section types as in Table 2.1)

Design order	FSD value	MCW value	Unconstrained weight	Bar 1	Bar 2	Bar 3	Bar 4	Bar 5	Bar 6
<b>1st</b>	<b>11480.7</b>	<b>5.26338</b>	<b>5.26338</b>	<b>C1</b>	<b>C1</b>	<b>C3</b>	<b>C3</b>	<b>C1</b>	<b>C1</b>
<b>2nd</b>	<b>11645.8</b>	<b>5.35907</b>	<b>5.35907</b>	<b>C1</b>	<b>C1</b>	<b>C3</b>	<b>C4</b>	<b>C1</b>	<b>C1</b>
<b>3rd</b>	<b>11645.8</b>	<b>5.35907</b>	<b>5.35907</b>	<b>C1</b>	<b>C1</b>	<b>C4</b>	<b>C3</b>	<b>C1</b>	<b>C1</b>
4th	11649.7	6.24373	5.16768	C1	C1	C3	C2	C1	C1
5th	11649.7	6.24373	5.16768	C1	C1	C2	C3	C1	C1
6th	11735.5	5.77910	5.26338	C1	C1	C4	C2	C1	C1
7th	11735.5	5.77910	5.26338	C1	C1	C2	C4	C1	C1
<b>8th</b>	<b>11861.9</b>	<b>5.45477</b>	<b>5.45477</b>	<b>C1</b>	<b>C1</b>	<b>C4</b>	<b>C4</b>	<b>C1</b>	<b>C1</b>
9th	11922.0	7.32865	5.07198	C1	C1	C2	C2	C1	C1
<b>10th</b>	<b>11944.9</b>	<b>5.49621</b>	<b>5.33993</b>	<b>C1</b>	<b>C1</b>	<b>C3</b>	<b>C3</b>	<b>C1</b>	<b>C2</b>
<b>11st</b>	<b>11944.9</b>	<b>5.49621</b>	<b>5.33993</b>	<b>C2</b>	<b>C1</b>	<b>C3</b>	<b>C3</b>	<b>C1</b>	<b>C1</b>
12nd	12026.5	6.31897	5.24423	C2	C1	C2	C3	C1	C1
13rd	12026.5	6.31897	5.24423	C1	C1	C3	C2	C1	C2
<b>14th</b>	<b>12036.9</b>	<b>5.45477</b>	<b>5.45477</b>	<b>C1</b>	<b>C1</b>	<b>C5</b>	<b>C3</b>	<b>C1</b>	<b>C1</b>
<b>15th</b>	<b>12036.9</b>	<b>5.45477</b>	<b>5.45477</b>	<b>C1</b>	<b>C1</b>	<b>C3</b>	<b>C5</b>	<b>C1</b>	<b>C1</b>
<b>16th</b>	<b>12038.5</b>	<b>5.43563</b>	<b>5.43563</b>	<b>C2</b>	<b>C1</b>	<b>C3</b>	<b>C4</b>	<b>C1</b>	<b>C1</b>
<b>17th</b>	<b>12038.5</b>	<b>5.43563</b>	<b>5.43563</b>	<b>C1</b>	<b>C1</b>	<b>C4</b>	<b>C3</b>	<b>C1</b>	<b>C2</b>

(continued)

**Table 2.6** (continued)

Design order	FSD value	MCW value	Unconstrained weight	Bar 1	Bar 2	Bar 3	Bar 4	Bar 5	Bar 6
18th	12041.2	5.63246	5.33993	C2	C1	C2	C4	C1	C1
19th	12041.2	5.63246	5.33993	C1	C1	C4	C2	C1	C2
20th	12069.7	5.67125	5.35907	C1	C1	C5	C2	C1	C1
21st	12069.7	5.67125	5.35907	C1	C1	C2	C5	C1	C1
<b>22nd</b>	<b>12163.8</b>	<b>5.43563</b>	<b>5.43563</b>	<b>C2</b>	<b>C1</b>	<b>C4</b>	<b>C3</b>	<b>C1</b>	<b>C1</b>
<b>23rd</b>	<b>12163.8</b>	<b>5.43563</b>	<b>5.43563</b>	<b>C1</b>	<b>C1</b>	<b>C3</b>	<b>C4</b>	<b>C1</b>	<b>C2</b>
24th	12186.1	6.35272	5.24423	C2	C1	C3	C2	C1	C1
25th	12186.1	6.35272	5.24423	C1	C1	C2	C3	C1	C2
26th	12248.0	6.77926	5.16768	C1	C1	C1	C4	C1	C1
27th	12248.0	6.77926	5.16768	C1	C1	C4	C1	C1	C1
28th	12271.6	7.42623	5.07198	C1	C1	C1	C3	C1	C1
29th	12271.6	7.42623	5.07198	C1	C1	C3	C1	C1	C1
30th	12281.2	5.55047	5.55047	C1	C1	C5	C4	C1	C1
31st	12281.2	5.55047	5.55047	C1	C1	C4	C5	C1	C1
32nd	12309.8	5.53132	5.53132	C2	C1	C4	C4	C1	C1
33rd	12309.8	5.53132	5.53132	C1	C1	C4	C4	C1	C2
<b>34th</b>	<b>12312.0</b>	<b>5.43998</b>	<b>5.43563</b>	<b>C2</b>	<b>C1</b>	<b>C2</b>	<b>C5</b>	<b>C1</b>	<b>C1</b>
<b>35th</b>	<b>12312.0</b>	<b>5.43998</b>	<b>5.43563</b>	<b>C1</b>	<b>C1</b>	<b>C5</b>	<b>C2</b>	<b>C1</b>	<b>C2</b>
36th	12324.0	6.07500	5.33993	C2	C1	C4	C2	C1	C1
37th	12324.0	6.07500	5.33993	C1	C1	C2	C4	C1	C2
38th	12364.8	5.53132	5.53132	C1	C1	C5	C3	C1	C2
39th	12364.8	5.53132	5.53132	C2	C1	C3	C5	C1	C1
40th	12371.7	7.43926	5.14853	C2	C1	C2	C2	C1	C1
41st	12371.7	7.43926	5.14853	C1	C1	C2	C2	C1	C2
42nd	12432.3	6.46520	5.24423	C2	C1	C1	C4	C1	C1
43rd	12432.3	6.46520	5.24423	C1	C1	C4	C1	C1	C2
<b>44th</b>	<b>12449.8</b>	<b>5.41648</b>	<b>5.41648</b>	<b>C2</b>	<b>C1</b>	<b>C3</b>	<b>C3</b>	<b>C1</b>	<b>C2</b>
45th	12491.7	6.84455	5.26338	C1	C1	C5	C1	C1	C1
46th	12491.7	6.84455	5.26338	C1	C1	C1	C5	C1	C1
47th	12525.0	7.50132	5.14853	C2	C1	C1	C3	C1	C1
48th	12525.0	7.50132	5.14853	C1	C1	C3	C1	C1	C2
49th	12562.8	5.74984	5.45477	C1	C1	C6	C2	C1	C1
50th	12562.8	5.74984	5.45477	C1	C1	C2	C6	C1	C1

## 2.4.2 Optimization

In this section, both the MCW and the FSD problems are optimized.

An evolutionary algorithm with two population sizes (80 and 160 individuals), two mutation rates (1.5 and 3 %), uniform crossover and gray codification



(as in [5]) has been executed in 100 independent runs with 30,000 evaluations as stopping criterion. Results are shown in terms of average, best value and standard deviation of the fitness functions.

(a) Solving the problem of minimum constrained weight MCW optimum design: Figure 2.4 shows the average fitness value, Fig. 2.5 shows the best fitness value and Fig. 2.6 shows the standard deviation fitness value; each containing the population size of 80 in black lines and the population size of 160 in pink (gray) lines;

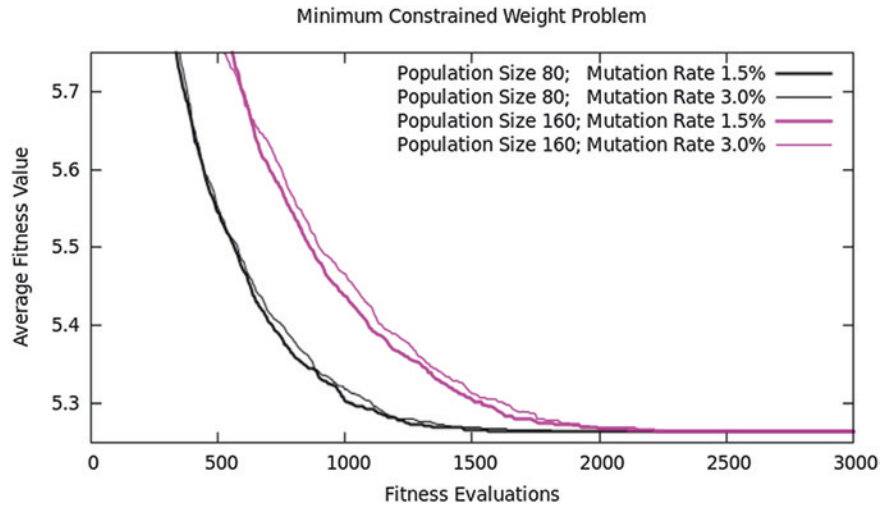


Fig. 2.4 Constrained minimum weight evolution over 100 independent runs in cR00 test case

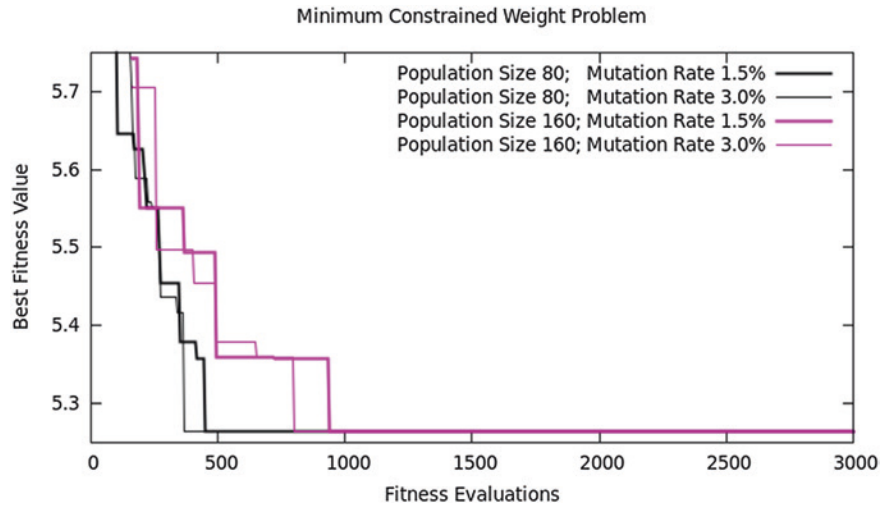


Fig. 2.5 Constrained minimum weight evolution over 100 independent runs in cR00 test case

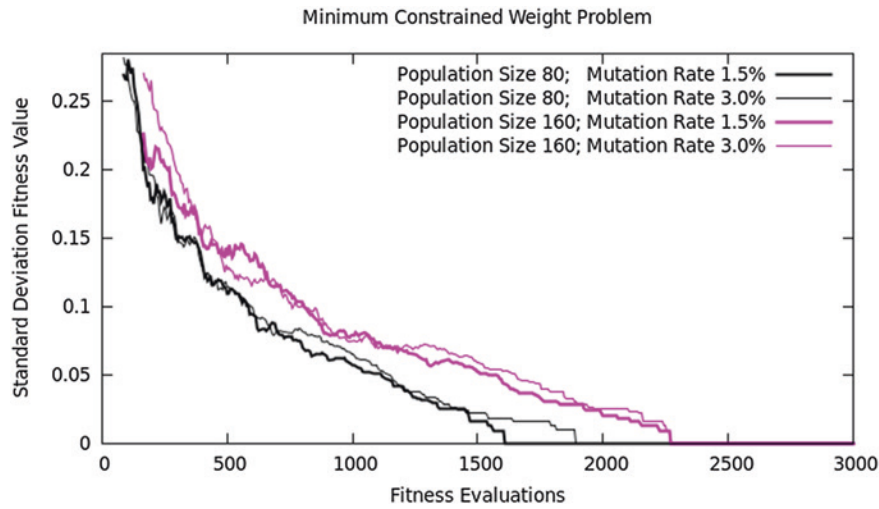


Fig. 2.6 Constrained minimum weight evolution over 100 independent runs in cR00 test case

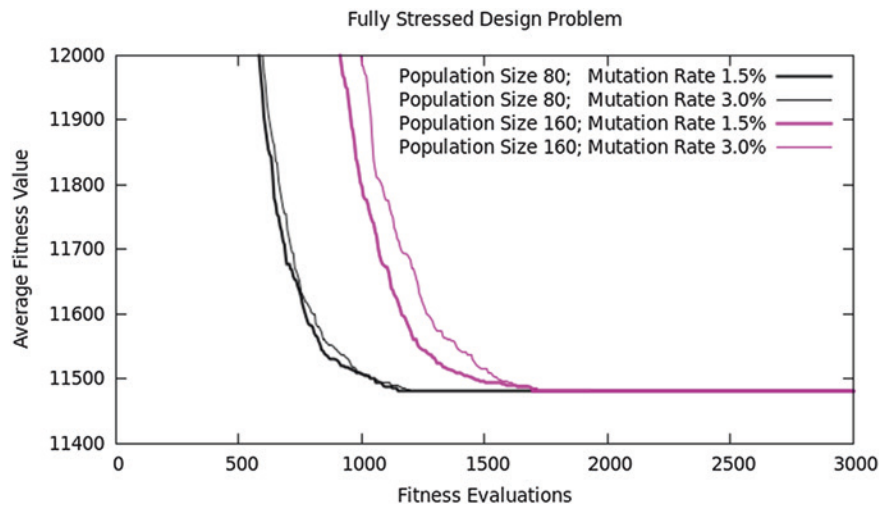


Fig. 2.7 FSD evolution over 100 independent runs in cR00 test case

the thicker line belongs to the 1.5 % mutation rate and the thinner line belongs to the 3.0 % mutation rate.

(b) Solving the problem of fully stressed design FSD optimum design:  
Figure 2.7 shows the average fitness value, Fig. 2.8 shows the best fitness value and Fig. 2.9 shows the standard deviation fitness value; each containing the population size of 80 in black lines and the population size of 160 in pink (gray) lines; the thicker line belongs to the 1.5 % mutation rate and the thinner line belongs to the 3.0 % mutation rate.

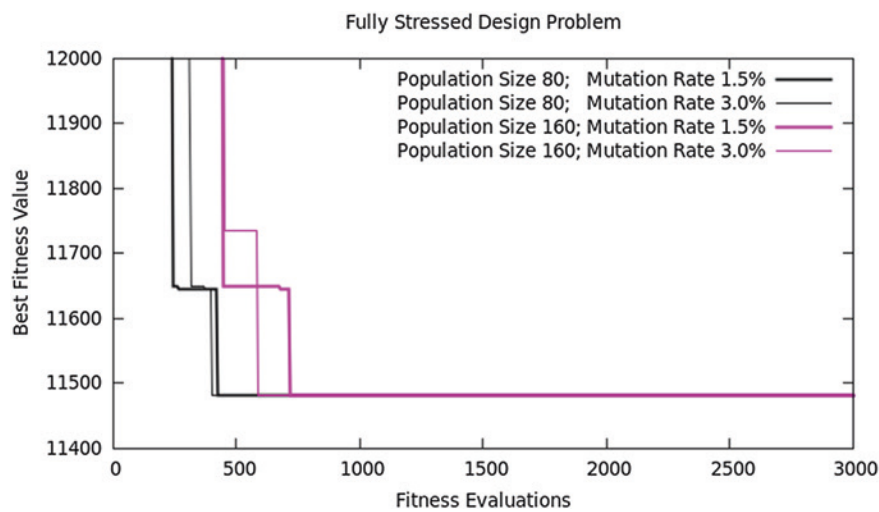


Fig. 2.8 FSD evolution over 100 independent runs in cR00 test case

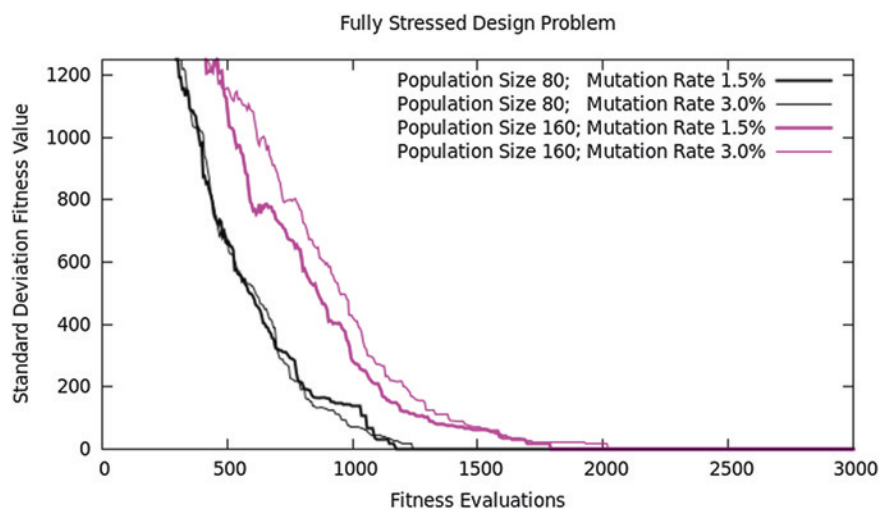


Fig. 2.9 FSD evolution over 100 independent runs in cR00 test case

(c) Comparing FSD and MCW problems with population size 80:  
In this case, fitness values have been scaled between 0 and 1, in order to easily compare visually the behaviour of both problems (FSD and MCW). Figure 2.10 shows the scaled average fitness value, Fig. 2.11 shows the scaled best fitness value and Fig. 2.12 shows the scaled standard deviation fitness value; each containing the FSD optimization problem in black lines and the MCW optimization problem in pink (gray) lines; the thicker line belongs to the 1.5 % mutation rate and the thinner line belongs to the 3.0 % mutation rate.

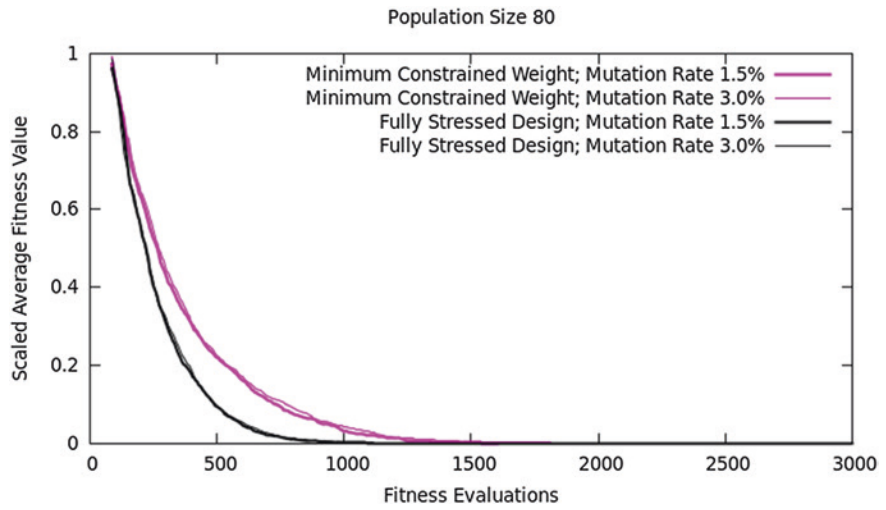


Fig. 2.10 Constrained minimum weight evolution over 100 independent runs in cR00 test case

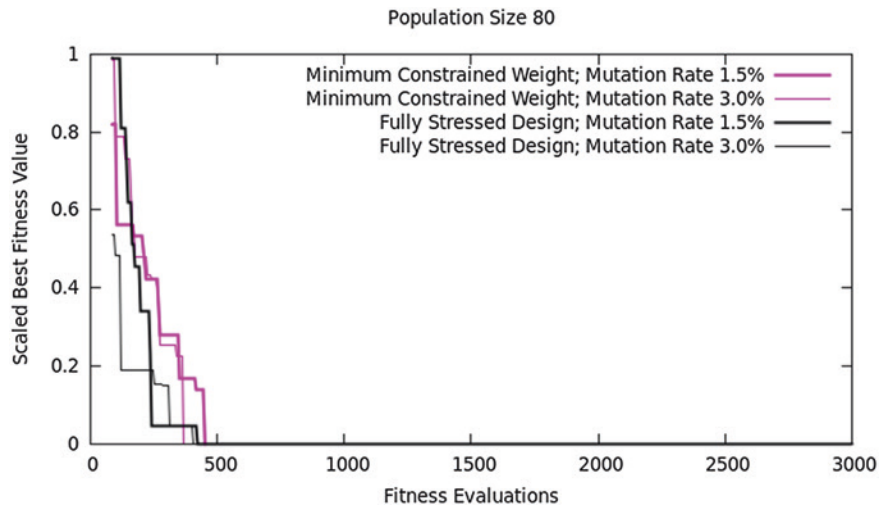


Fig. 2.11 Constrained minimum weight evolution over 100 independent runs in cR00 test case

(d) Comparing FSD and MCW problems with population size 160:  
In this case, fitness values have been scaled between 0 and 1, in order to easily compare visually the behaviour of both problems (FSD and MCW). Figure 2.13 shows the scaled average fitness value, Fig. 2.14 shows the scaled best

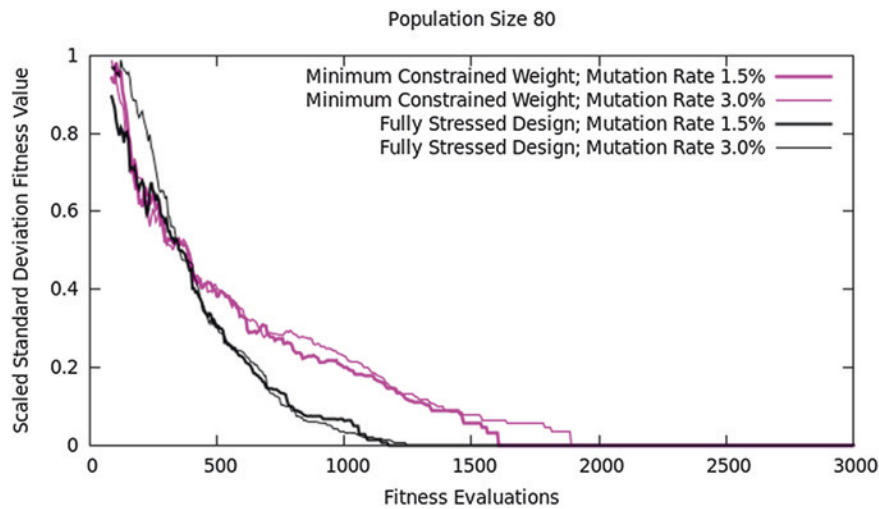


Fig. 2.12 Constrained minimum weight evolution over 100 independent runs in cR00 test case

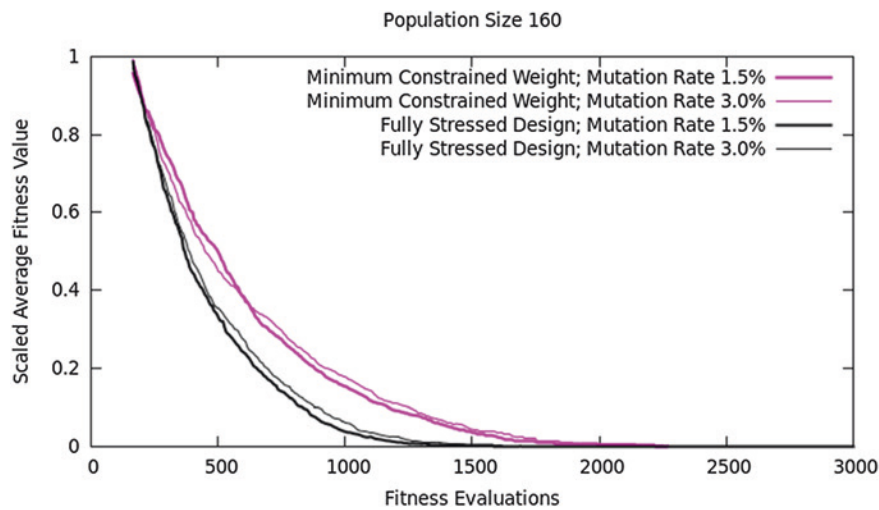


Fig. 2.13 FSD evolution over 100 independent runs in cR00 test case

fitness value and Fig. 2.15 shows the scaled standard deviation fitness value; each containing the FSD optimization problem in black lines and the MCW optimization problem in pink (gray) lines; the thicker line belongs to the 1.5 % mutation rate and the thinner line belongs to the 3.0 % mutation rate.

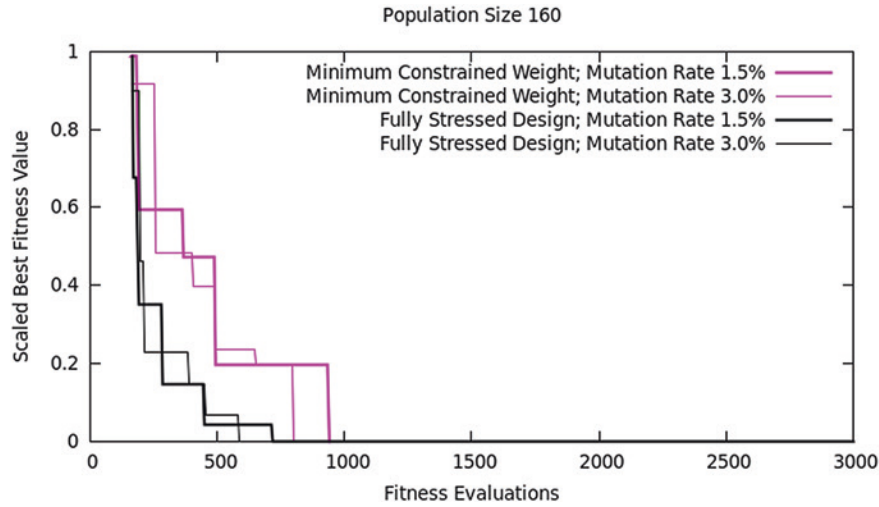


Fig. 2.14 FSD evolution over 100 independent runs in cR00 test case

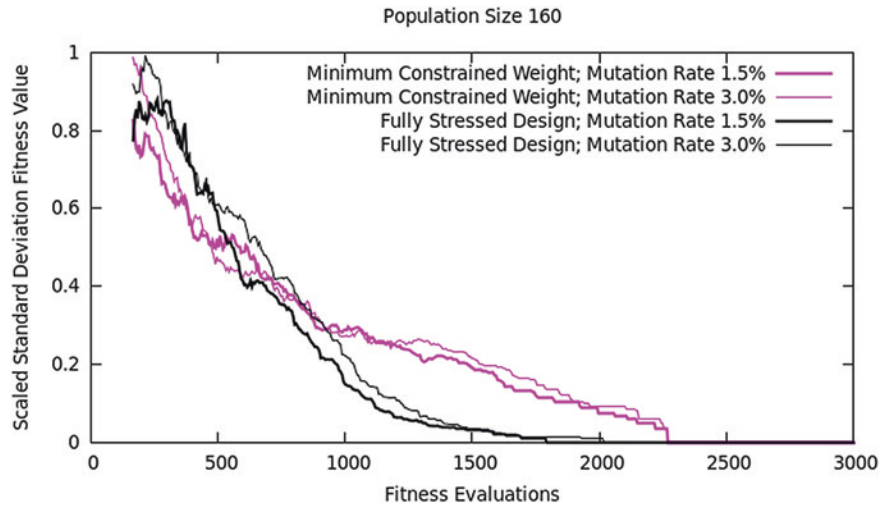


Fig. 2.15 FSD evolution over 100 independent runs in cR00 test case

2.4.3 Discussion

(a) Solving the problem of minimum constrained weight MCW optimum design (Figs. 2.4, 2.5 and 2.6):  
The lower the population size (80 vs. 160), the faster the convergence of the algorithm without worsening the quality of the obtained solution in this test case as shown in Figs. 2.4, 2.5 and 2.6 (the chromosome length of this problem is easily

handled by the evolutionary algorithm). The behaviour of average and standard deviation values are slightly better in the lower mutation rate (1.5 % vs. 3 %). It is remarkable that the range of evaluations required to obtain the best design in all the 100 independent executions in this MCW problem vary from 1610 evaluations (population size 80, mutation rate 1.5 %) to 2272 evaluations (population size 160, mutation rates 1.5 and 3 %).

(b) Solving the problem of fully stressed design FSD (Figs. 2.7, 2.8 and 2.9):

Here also, the lower the population size (80 vs. 160), the faster the convergence of the algorithm without worsening the quality of the obtained solution in this test case as shown in Figs. 2.7, 2.8 and 2.9 (the chromosome length of this problem is easily handled by the evolutionary algorithm). As well, the behaviour of average and standard deviation values are slightly better in the lower mutation rate (1.5 % vs. 3 %). It is remarkable that the range of evaluations required to obtain the best design in all the 100 independent executions in this FSD problem vary from 1172 evaluations (population size 80, mutation rate 1.5 %) to 2020 evaluations (population size 160, mutation rate 3 %).

(c) Comparing FSD and MCW problems with population size 80 and 160 (Figs. 2.10, 2.11, 2.12, 2.13, 2.14, and 2.15):

Figures 2.10, 2.11, 2.12, 2.13, 2.14, and 2.15 show clearly through the scaled fitness, that when comparing the fitness behaviour, in all cases of average, best and standard deviation, the FSD problem has a faster convergence than the MCW problem in this test case (except in the best fitness of MCW problem with population size 80 and 3 % mutation rate), requiring lower number of fitness evaluations to achieve the same optimum design.

## 2.5 Conclusions

The relation of the problems MCW and FSD has been shown through a simple truss test case in discrete cross-section types sizing optimization. This relation has been evidenced by having a coincidental optimum design, a high number of coincidental best designs among the best ones and showing a high correlation coefficient when considering the whole search space of each problem.

The search process of the optimum design of both problems has been possible by using evolutionary algorithms, showing a high robust behaviour where in 100 out of 100 independent runs, this metaheuristic optimization was able to find the best solution in the range of 1000–2000 evaluations -versus a search space of tens of millions-. When comparing them, the fully stressed design (FSD) problem has shown an easier topology for the evolutionary algorithm optimization versus the MCW problem, that is, requiring less number of fitness function evaluations to achieve the same shared best design.

Application of this analysis to an increased number of test cases and generalization in other types of bar structures, -e.g. in the case of frame bar structures-, should be



developed in the future to provide more light about the comparison and relationship of the fully stressed design and the minimum constrained weight problems.

## References

1. Coelho RF (2013) Co-evolutionary optimization for multi-objective design under uncertainty. *J Mech Design* 135–2:1–8
2. Deb K, Bandaru S, Greiner D, Gaspar-Cunha A, Tutum CC (2014) An integrated approach to automated innovation for discovering useful design principles: case studies from engineering. *Appl Soft Comp* 15:42–56
3. Greiner D, Hajela P (2012) Truss topology optimization for mass and reliability considerations—co-evolutionary multiobjective formulations. *Struct Multidiscip O* 45–4:589–613
4. Greiner D, Emperador JM, Winter G (2000) Multiobjective optimisation of bar structures by Pareto GA. In: European congress on computational methods in applied sciences and engineering -ECCOMAS, Barcelona, Spain
5. Greiner D, Emperador JM, Winter G (2005) Gray coding in evolutionary multicriteria optimization: application in frame structural optimum design. *Evol Multi-Criterion Optim Lect Notes Comput Sci* 3410:576–591
6. Greiner D, Emperador JM, Winter G, Galván B (2007) Improving computational mechanics optimum design using helper objectives: an application in frame bar structures. *Evol Multi-Criterion Optim Lect Notes Comput Sci* 4403:575–589
7. Greiner D, Periaux P, Emperador JM, Galvan B, Winter G (2015) A study of Nash evolutionary algorithms for reconstruction inverse problems in structural engineering. In: Greiner D, et al. (eds) *Advances in Evolutionary and Deterministic Methods for Design, Optimization and Control in Engineering and Sciences*, Springer, pp 321–333
8. Greiner D, Emperador JM, Galvan B, Winter G, Periaux J (2014) Optimum structural design using bio-inspired search methods: a survey and applications. In: Becerra V, Vasile M (eds) *Computational intelligence in the aerospace sciences*, American Institute of Aeronautics and Astronautics -AIAA
9. Greiner D, Emperador JM, Galvan B, Winter G (2014) A Comparison of minimum constrained weight and Fully Stressed Design Problems in discrete cross-section type bar structures. In: Oñate E, Oliver X, Huerta A (eds) *Proceedings of the 11th world congress on computational mechanics WCCM-XI, 5th european congress on computational mechanics ECCM-V, the 6th european congress on computational fluid dynamics ECFD-VI*. CIMNE, pp 2064–2072
10. Greiner D, Galván B, Périaux J, Gauger N, Giannakoglou K, Winter G (2015) Advances in evolutionary and deterministic methods for design, optimization and control in engineering and sciences. *Computational Methods in Applied Sciences*. vol 36, Springer, Berlin
11. Gunnlaugsson G, Martin J (1973) Optimality conditions for fully stressed designs. *SIAM J Appl Math* 25–3:474–482
12. Lagaros N, Plevris V, Papadrakakis M (2005) Multi-objective design optimization using cascade evolutionary computations. *Comput Method Appl M* 194(30–33):3496–3515
13. Maxwell JC (1872) On reciprocal figures, frames and diagrams of forces. *Trans Roy Soc Edinburgh* 26(Plates 1–3):1–40
14. Mueller KM, Liu M, Burns SA (2002) Fully stressed design of frame structures and multiple load paths. *J Struct Eng ASCE* 128–6:806–814
15. Murotsu Y, Okada H, Taguchi K, Grimmelt M, Yonezawa M (1984) Automatic generation of stochastically dominant failure modes of frame structures. *Struct Saf* 2:17–25
16. Park S, Choi S, Sikorsky C, Stubbs N (2004) Efficient method for calculation of system reliability of a complex structure. *Int J Solids Struct* 41:5035–5050



Evolutionary Algorithms and Metaheuristics in Civil  
Engineering and Construction Management

Magalhães-Mendes, J.; Greiner, D. (Eds.)

2015, X, 127 p. 66 illus., Hardcover

ISBN: 978-3-319-20405-5