

Object – Oriented Technology

Chapter 4: Type Systems

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$$\mathcal{T} \vdash x : A \text{ var}, \mathcal{T} \vdash e : A$$

$$\mathcal{T} \vdash x = e : \text{Statement}$$
$$\mathcal{T} \vdash x : \text{int var}, \mathcal{T} \vdash x + 1 : \text{int}$$

$$\mathcal{T} \vdash x = x + 1 : \text{Statement}$$

CONDITIONAL STATEMENT

$$\mathcal{T} \vdash e : \text{boolean}, \mathcal{T} \vdash S1 : \text{Statement}, \mathcal{T} \vdash S2 : \text{Statement}$$

$$\mathcal{T} \vdash \text{if } (e) S1 \text{ else } S2 : \text{Statement}$$
$$\mathcal{T} \vdash x > 0 : \text{boolean}, \mathcal{T} \vdash y = x : \text{Statement},$$
$$\mathcal{T} \vdash y = -x : \text{Statement}$$

$$\mathcal{T} \vdash \text{if } (x > 0) y = x \text{ else } y = -x : \text{Statement}$$

WHILE LOOP

$$\mathcal{T} \vdash e : \text{boolean}, \mathcal{T} \vdash S : \text{Statement}$$

$$\mathcal{T} \vdash \mathbf{while} (e) S : \text{Statement}$$
$$\mathcal{T} \vdash x < \text{highValue} : \text{boolean}, \mathcal{T} \vdash x = x + 1 : \text{Statement}$$

$$\mathcal{T} \vdash \mathbf{while} (x < \text{highValue}) x = x + 1 : \text{Statement}$$

$$\mathcal{T} \vdash e : \text{boolean}, \mathcal{T} \vdash S : \text{Statement}$$

$$\mathcal{T} \vdash \mathbf{do\ S\ while\ (e)} : \text{Statement}$$

$$\mathcal{T} \vdash e1 : int, \mathcal{T} \vdash e2 : int,$$
$$\mathcal{T} \vdash \mathbf{op} \in \{+, -, *, /, \%\}$$

$$\mathcal{T} \vdash e1 \mathbf{op} e2 : int$$
$$\mathcal{T} \vdash x : int, \mathcal{T} \vdash y : int$$

$$\mathcal{T} \vdash x + y : int$$

$$\begin{array}{c} \mathcal{T} \vdash e1 : \textit{boolean}, \mathcal{T} \vdash e2 : \textit{boolean}, \\ \mathcal{T} \vdash \mathbf{op} \in \{\&, |, \&\&, ||\} \end{array}$$

$$\mathcal{T} \vdash e1 \mathbf{op} e2 : \textit{boolean}$$

SAMPLE CLASS

```
class Collection {  
    private int size;  
    private Object[] elements;  
    public Collection() {...}  
    public boolean belongs(Object x){...}  
    public int cardinality(){...}  
    public void add(Object x){...}  
    public void remove(Object x) {...}  
}
```


CLASS TYPE SIGNATURE

fields(C) = < int size; Object[] elements >

constructors(C) = < Collection() >

*methods(C) = < boolean belongs(Object); int cardinality();
void add(Object); void remove(Object) >*

Type signature:

Sig(C) = { fields(C); constructors(C); methods(C) }

TYPE SIGNATURES

- Field signatures:

$fields(A) = \langle F_i \ f_i \rangle_{i=1}^{k^A}$ (a sequence of fields) with F_i the type of field f_i , and $f_i \neq f_j$ for $i \neq j$, where $j = 1, 2, \dots, k^A$.

- Constructor signatures:

$constructors(A) = \{A(C_{c_{i_1}}, C_{c_{i_2}}, \dots, C_{c_{i_n}})\}_{i=1}^{c^A}$
(a collection of constructors) where $C_{m_{i_j}}$ are types for $j = 1, 2, \dots, n$.

- Method signatures:

$methods(A) = \{C_{m_i} \ m_i(C_{m_{i_1}}, C_{m_{i_2}}, \dots, C_{m_{i_n}})\}_{i=1}^{m^A}$
(a collection of methods) where m_i is the method name, and C_{m_i} , $C_{m_{i_j}}$ are types where $j = 1, 2, \dots, n$.

$$\mathcal{T} \cup \{\text{class } D \text{ extends } B\} \Rightarrow \mathcal{T}'$$

$$\mathcal{T}' \vdash D <: B$$

$$\mathcal{T} \vdash T1 <: T2, \mathcal{T} \vdash T2 <: T3$$

$$\mathcal{T} \vdash T1 <: T3$$

$$\begin{array}{c} \mathcal{T} \vdash x : A \text{ var}, \mathcal{T} \vdash e : B, \\ \mathcal{T} \vdash B <: A \end{array}$$

$$\mathcal{T} \vdash x = e : \textit{Statement}$$

$$\begin{array}{c} \mathcal{T} \vdash x : \textit{Person var}, \mathcal{T} \vdash y : \textit{Employee}, \\ \mathcal{T} \vdash \textit{Employee} <: \textit{Person} \end{array}$$

$$\mathcal{T} \vdash x = y : \textit{Statement}$$

$$\begin{array}{l} \mathcal{T} \vdash e : B, \\ \mathcal{T} \vdash B <: A, \end{array}$$

$$\mathcal{T} \vdash e : A$$

$$\begin{aligned}
 &\mathcal{T} \vdash D <: B, \\
 &\mathcal{T} \vdash \text{fields}(B) = \langle F_i^B \ f_i^B \rangle_{i=1}^{k^B}, \\
 &\mathcal{T} \vdash \text{fields}(D) = \langle F_i^D \ f_i^D \rangle_{i=1}^{k^D}, \\
 &\quad k^B \leq k^D, \\
 &\quad j \in \{1, 2, \dots, k^B\}
 \end{aligned}$$

$$\mathcal{T} \vdash \langle F_j^B \ f_j^B \rangle = \langle F_j^D \ f_j^D \rangle$$

$$\begin{aligned} \mathcal{T} \vdash \text{OrderedCollection} <: \text{Collection}, \\ \mathcal{T} \vdash \text{fields}(\text{Collection}) = < \text{int size}; \text{Object[]} \text{elements} >, \\ \mathcal{T} \vdash \text{fields}(\text{OrderedCollection}) = < \\ \text{int size}; \text{Object[]} \text{elements}; \text{other fields} > \end{aligned}$$

$$\frac{\mathcal{T} \vdash D <: B, \quad \mathcal{T} \vdash C_m \ m(C_{m_1}, C_{m_2}, \dots, C_{m_n}) \in \text{methods}(B)}{\mathcal{T} \vdash C_m \ m(C_{m_1}, C_{m_2}, \dots, C_{m_n}) \in \text{methods}(D)}$$

$$\frac{\mathcal{T} \vdash \textit{Collection} <: \textit{Object}, \quad \mathcal{T} \vdash \textit{boolean equals}(\textit{Object}) \in \textit{methods}(\textit{Object})}{\mathcal{T} \vdash \textit{boolean equals}(\textit{Object}) \in \textit{methods}(\textit{Collection})}$$

$$\mathcal{T} \vdash \textit{Collection} <: \textit{Object},$$
$$\mathcal{T} \vdash \textit{Object} \textit{ clone}() \in \textit{methods}(\textit{Object})$$
$$\mathcal{T} \vdash \textit{Collection} \textit{ clone}() \in \textit{methods}(\textit{Collection})$$

$$\begin{array}{l} \mathcal{T} \vdash C_m \ m(C_{m_1}, C_{m_2}, \dots, C_{m_n}) \in \text{methods}(A), \\ \mathcal{T} \vdash C'_m \ m(C'_{m_1}, C'_{m_2}, \dots, C'_{m_n}) \in \text{methods}(A) \end{array}$$

$$\mathcal{T} \vdash (\exists i)((i \in \{1, 2, \dots, n\}), (C_{m_i} \neq C'_{m_i}))$$

$$\begin{array}{c}
 \mathcal{T} \vdash x : A, \\
 \mathcal{T} \vdash C_m \ m(C_{m_1}, C_{m_2}, \dots, C_{m_n}) \in \text{methods}(A), \\
 \mathcal{T} \vdash a_i : A_i, i \in \{1, 2, \dots, n\}, \\
 \mathcal{T} \vdash (\forall i)(i \in \{1, 2, \dots, n\} \Rightarrow A_i <: C_{m_i})
 \end{array}$$

$$\mathcal{T} \vdash x.m(a_1, a_2, \dots, a_n) : C_m$$

$$\begin{array}{l} \mathcal{T} \vdash \text{employees} : \text{Collection var}, \\ \mathcal{T} \vdash \text{boolean add(Object)} \in \text{methods(Collection)}, \\ \mathcal{T} \vdash x : \text{Employee}, \\ \mathcal{T} \vdash \text{Employee} <: \text{Object} \end{array}$$

$$\mathcal{T} \vdash \text{employees.add}(x) : \text{boolean}$$

$$\mathcal{T} \vdash e : A, \mathcal{T} \vdash D <: A$$

$$\mathcal{T} \vdash (D)e : D$$

$$\mathcal{T} \vdash e : \textit{Person}, \mathcal{T} \vdash \textit{Employee} <: \textit{Person}$$

$$\mathcal{T} \vdash (\textit{Employee})e : \textit{Employee}$$

$$\mathcal{T} \vdash e : A, \mathcal{T} \vdash A <: D$$

$$\mathcal{T} \vdash (D)e : D$$

STATIC and DYNAMIC TYPE

$$\mathcal{T} \vdash e : A$$

$$\mathcal{T} \vdash \text{eval}(e.\text{getClass}()) <: \text{eval}(\text{Class.forName}("A"))$$

$$\mathcal{T} \vdash e : A, \mathcal{T} \vdash D <: A,$$
$$\mathcal{T} \vdash \text{eval}(e.\text{getClass}()) \not<: \text{eval}(\text{Class.forName}("D"))$$

$$\mathcal{T} \vdash \text{eval}((D)e) : \text{ClassCastException}$$

PARAMETRIC TYPES

$$\begin{array}{l} \mathcal{T} \vdash T <: \text{Object} \Rightarrow \\ \mathcal{T} \vdash \mathbf{class} \ C < T > \{ \text{fields}(C); \text{methods}(C) \} : \text{Class}, \\ \mathcal{T} \vdash D <: \text{Object} \end{array}$$

$$\mathcal{T} \vdash C < D > <: \text{Object}$$

$$\begin{array}{l} \mathcal{T} \vdash T <: \text{Object} \Rightarrow \\ \mathcal{T} \vdash \mathbf{class} \ C < T > \{ \text{fields}(C); \text{methods}(C) \} : \text{Class}, \\ \mathcal{T} \vdash D <: \text{Object} \end{array}$$

$$\begin{array}{l} \mathcal{T} \vdash \text{Sig}(C < D >) = \\ \{ \text{fields}(C)[D/T]; \text{methods}(C)[D/T] \} \end{array}$$

UNIVERSAL INSTANTIATION

```
Sig(Collection<T>) =  
< T[] elements; boolean belongs(T);  
    void add(T); void remove(T) >.
```

```
Sig(Collection<Employee>) =  
< Employee[] elements; boolean belongs(Employee);  
    void add(Employee); void remove(Employee) >.
```

$$\begin{array}{c} \mathcal{T} \vdash B <: \text{Object}, \\ \mathcal{T} \vdash T <: B \Rightarrow \\ \mathcal{T} \vdash \text{class } C < T \text{ extends } B > \{ \text{fields}(C); \text{methods}(C) \} : \text{Class}, \\ \mathcal{T} \vdash D <: B[D/T] \\ \hline \mathcal{T} \vdash C < D > <: \text{Object} \end{array}$$

BOUNDED INSTANTIATION

```
Sig(OrderedCollection<T extends Comparable< T >>)  
< T[] elements; boolean belongs(T);  
void add(T); void remove(T) >.
```

```
Employee <: Comparable<Employee>
```

```
Sig(OrderedCollection<Employee>) =  
< Employee[] elements; boolean belongs(Employee);  
void add(Employee); void remove(Employee) >.
```

TYPE ERASURE: COLLECTION CLASS

```
class Collection {  
    private Object[] elements;  
    public boolean belongs(Object x){...}  
    public int size(){...}  
    public void add(Object x){...}  
    public void remove(Object x) {...}  
}
```

TYPE ERASURE: PARAMETRIC CLASS

```
class Collection<T> {  
    private T[] elements;  
    public boolean belongs(T x){...}  
    public int size(){...}  
    public void add(T x){...}  
    public void remove(T x) {...}  
}
```

TYPE ERASURE: SIGNATURES

```
boolean belongs(Object x)  
void add(Object x)  
void remove(Object x)
```

```
boolean belongs(Employee x)  
void add(Employee x)  
void remove(Employee x)
```

```
Collection ✂: Collection<Employee>  
Collection<Employee> ✂: Collection
```

TYPE ERASURE: LEGACY CLASSES

```
public class LegacyClass {  
    public void addEmployees(Collection c) {  
        for (int i=0; i < 5; i++)  
            c.add(new Employee());  
    }  
}  
  
public class TestLegacy {  
    public static void main(String[] args) {  
        Collection<Department> c =  
            new LinkedList<Department>();  
            new LegacyClass().addEmployees(c);  
    }  
}
```

TYPE ERASURE: MESSAGE PASSING

```
public class MessagePassing {  
    public static void main(String[] args) {  
        LinkedList<Employee> employees =  
            new LinkedList<Employee>();  
        LinkedList<Department> departments =  
            new LinkedList<Department>();  
        updateCollection(employees,departments);  
    }  
    public static void updateCollection  
        (LinkedList collectionA, LinkedList collectionB) {  
        collectionA.addAll(collectionB);  
    }  
}
```

TYPE ERASURE: OVERLOADING

```
public class UpdateTransaction {  
    public void update(Collection<Employee> employees) {...}  
    public void update(Collection<Department> departments) {...}  
}
```

TYPE ERASURE: OVERLOADING

```
public interface EmployeeView {  
    public Collection<Employee>  
        select(Collection<Employee> x);  
}  
public interface DepartmentView  
    extends EmployeeView {  
    public Collection<Department>  
        select(Collection<Department> x);  
}  
public class Company  
    implements DepartmentView {  
    public Collection<Employee>  
        select(Collection<Employee> x) {...}  
    // public Collection<Department>  
        // select(Collection<Department> x) {...}  
}
```


TYPE ERASURE: ASSIGNMENTS

```
public class EmployeesAreDepartments {  
    public static void main(String[] args) {  
        LinkedList<Department> departments =  
            new LinkedList<Department>();  
        LinkedList<Employee> employees =  
            new LinkedList<Employee>();  
        LinkedList objects = employees;  
        departments = objects;  
    }  
}
```

TYPE ERASURE: SUBTYPING

```
public class UpdateTransaction {  
    public static void main(String[] args) {  
        LinkedList<Employee> employees =  
            new LinkedList<Employee>();  
        employees.add(new Employee("Joe"));  
        LinkedList objects = new LinkedList();  
        objects.add(new Object());  
        updateCollection(employees);  
        updateDepartment(objects);  
    }  
    public static void updateCollection(  
        LinkedList collection) {  
        collection.add(new Department());  
    }  
    public static void updateDepartment(  
        LinkedList<Department> collection) {  
        Department d = collection.getFirst();  
    }  
}
```

TYPE ERASURE: DYNAMIC TYPECHECKING

```
public class UpdateTransaction {  
    public static void main(String[] args) {  
        LinkedList<Employee> employees =  
            new LinkedList<Employee>();  
        updateCollection(employees);  
        Employee emp = employees.remove();  
        // runtime exception here  
    }  
    public static void updateCollection(Object collection) {  
        ((LinkedList<Department>)collection).add(new Department());  
    }  
}
```

TYPE ERASURE: SERIALIZABILITY

```
public class StoreCollectionObject {  
    public static void main(String[] args)  
        throws Exception {  
        FileOutputStream fileout =  
            new FileOutputStream("employees");  
        ObjectOutputStream out =  
            new ObjectOutputStream(fileout);  
        Collection<Employee> employees =  
            new LinkedList<Employee>();  
        employees.add(new Employee());  
        out.writeObject(employees);  
    }  
}
```

TYPE ERASURE: SERIALIZABILITY

```
public class ReadCollectionObject {  
    public static void main(String[] args)  
        throws Exception {  
        FileInputStream filein =  
            new FileInputStream("employees");  
        ObjectInputStream in =  
            new ObjectInputStream(filein);  
        Collection<Department> departments = null;  
        try { departments =  
            (Collection<Department>) in.readObject();  
        } catch (ClassCastException e) {exception handling }  
    }
```

```
Iterator<Department> it = departments.iterator();  
Department d = it.next(); ???
```

TYPE ERASURE: REFLECTION

```
public class ReflectiveTransaction {  
    public void updateEmployees(  
        LinkedList<Employee> collection) {  
        Employee e = collection.remove();  
        // unexpected ClassCastException!? }  
    public static void main(String[] args) {  
        ReflectiveTransaction trans =  
            new ReflectiveTransaction();  
        Collection<Department> departments =  
            new LinkedList<Department>();  
        departments.add(new Department());  
        try {  
            Method method =  
                trans.getClass().getMethod  
                ("updateEmployees", departments.getClass());  
            method.invoke(trans, departments);  
        } catch (Exception e) {exception handling }  
    }  
}
```