

# Contents

<b>1</b>	<b>Risks and Insurance</b>	<b>1</b>
1.1	Introduction	1
1.2	“Risk”: Looking for Definitions	1
1.2.1	Some Preliminary Ideas	1
1.2.2	Transactions with Random Results	2
1.2.3	A Very Basic Insurable Risk	4
1.2.4	Random Number of Events and Random Amounts	4
1.2.5	Risks Inherent in the Individual Lifetime	9
1.3	Managing Risks	14
1.3.1	General Aspects	14
1.3.2	The Risk Management (RM) Process	15
1.3.3	Risk Identification	17
1.3.4	Risk Assessment and Impact Assessment	17
1.3.5	Risk Management Actions	18
1.3.6	Self-insurance versus Insurance	20
1.3.7	Counterparts in a Risk Transfer Deal	22
1.3.8	Monitoring and the Risk Management Cycle	23
1.3.9	More on Hedging	23
1.4	Risk and Impact Assessment: Some Models	25
1.4.1	Some Preliminary Ideas	25
1.4.2	A Very Basic Model	26
1.4.3	Random Number of Events and Random Amounts	27
1.4.4	Random Sums: A Critical Assumption	33
1.4.5	Introducing Time into Valuations	34
1.4.6	Comparing Random Yields	36
1.4.7	Risk-Adjusted Valuations	38
1.5	Risk Measures	42
1.5.1	Some Preliminary Ideas	42
1.5.2	Traditional Risk Measures	43
1.5.3	Downside Risk Measures	44
1.5.4	Risk Measures and Capital Requirements	46

1.6	Transferring Risks . . . . .	49
1.6.1	Building Up a Pool. . . . .	49
1.6.2	Financing the Pool . . . . .	52
1.6.3	The Role of the Insurer. . . . .	57
1.6.4	The Risk Transformation. . . . .	59
1.7	Insurance Products . . . . .	60
1.7.1	The Insurance Cover. Policy Conditions . . . . .	60
1.7.2	Some Examples . . . . .	61
1.7.3	Pricing Insurance Products. . . . .	63
1.7.4	Premium Calculation. . . . .	64
1.7.5	Technical Bases . . . . .	71
1.7.6	Reserving . . . . .	72
1.8	References and Suggestions for Further Reading . . . . .	74
<b>2</b>	<b>Managing a Portfolio of Risks. . . . .</b>	<b>75</b>
2.1	Introduction . . . . .	75
2.2	Rating: The Basics . . . . .	75
2.2.1	Some Preliminary Ideas. . . . .	75
2.2.2	The Portfolio Structure . . . . .	76
2.2.3	Homogeneous Risks . . . . .	77
2.2.4	Non-homogeneous Risks . . . . .	77
2.2.5	A More General Rating System . . . . .	80
2.2.6	Rating Systems and Technical Equilibrium . . . . .	81
2.2.7	From Risk Factors to Rating Classes. . . . .	84
2.2.8	Cross-Subsidy: Mutuality and Solidarity . . . . .	86
2.3	Facing Portfolio Riskiness . . . . .	87
2.3.1	Expected Outgo versus Actual Outgo . . . . .	87
2.3.2	Risk Components and Risk Factors. . . . .	88
2.3.3	Risk Assessment . . . . .	89
2.3.4	The Risk Index . . . . .	91
2.3.5	The Probability Distribution of the Total Payment . . . . .	93
2.3.6	The Safety Loading . . . . .	97
2.3.7	Capital Allocation and Beyond. . . . .	101
2.3.8	Solvency . . . . .	104
2.3.9	Creating Value. . . . .	105
2.3.10	Risk Management and Risk Analysis: Some Remarks. . . . .	107
2.3.11	The “Uncertainty Risk”. . . . .	108
2.4	Reinsurance: The Basics . . . . .	112
2.4.1	General Aspects . . . . .	112
2.4.2	Stop-Loss Reinsurance . . . . .	113
2.4.3	From Portfolios to Contracts . . . . .	115
2.4.4	Two Reinsurance Arrangements . . . . .	118

2.4.5	Examples . . . . .	120
2.4.6	Optimal Reinsurance Policy . . . . .	122
2.5	Reinsurance: Further Aspects . . . . .	125
2.5.1	Reinsurance Arrangements . . . . .	125
2.5.2	Random Claim Sizes: XL Reinsurance . . . . .	126
2.5.3	Catastrophe Reinsurance . . . . .	129
2.5.4	Purposes of Reinsurance . . . . .	131
2.5.5	Insurance–Reinsurance Networks . . . . .	132
2.5.6	Reinsurance Treaties, Reinsurance Programmes . . . . .	133
2.6	Alternative Risk Transfers . . . . .	135
2.6.1	Some Preliminary Ideas . . . . .	135
2.6.2	Securitization and the Role of Capital Markets . . . . .	136
2.6.3	Organizing a Securitization Transaction . . . . .	139
2.6.4	An Example: The Mortality Bonds . . . . .	140
2.7	The Time Dimension . . . . .	143
2.7.1	General Aspects . . . . .	143
2.7.2	Premiums, Payments, Portfolio Fund . . . . .	144
2.7.3	Solvency and Capital Requirements . . . . .	146
2.7.4	Generalizing the Model . . . . .	149
2.7.5	Solvency and Capital Flows . . . . .	151
2.8	References and Suggestions for Further Reading . . . . .	152
<b>3</b>	<b>Life Insurance: Modeling the Lifetime . . . . .</b>	<b>157</b>
3.1	Introduction . . . . .	157
3.2	Life Tables . . . . .	157
3.2.1	Elements of a Life Table . . . . .	157
3.2.2	Cohort Tables and Period Tables . . . . .	158
3.2.3	Construction of a Period Life Table . . . . .	161
3.2.4	“Population” Tables versus “Market” Tables . . . . .	163
3.2.5	The Life Table as a Probabilistic Model . . . . .	164
3.2.6	One-Year Measures of Mortality . . . . .	165
3.2.7	A More Formal Setting: The Random Lifetime . . . . .	169
3.3	A Mortality “Law” . . . . .	170
3.3.1	From Tables to Parameters . . . . .	170
3.3.2	The Heligman–Pollard Law . . . . .	170
3.4	Summarizing a Life Table . . . . .	172
3.4.1	The Life Expectancy . . . . .	173
3.4.2	Other Markers . . . . .	176
3.5	From the Basic Model to More General Models . . . . .	177
3.6	Heterogeneity . . . . .	177
3.6.1	Some Preliminary Ideas . . . . .	177
3.6.2	Rating Classes . . . . .	178
3.6.3	Sub-standard Risks . . . . .	180
3.6.4	The “Factor Formula” . . . . .	182

3.7	Mortality by Age and Duration . . . . .	183
3.7.1	Some Preliminary Ideas. . . . .	183
3.7.2	Select Tables and Ultimate Tables . . . . .	184
3.7.3	A Practical Issue . . . . .	186
3.8	Mortality Dynamics . . . . .	187
3.8.1	Mortality Trends. . . . .	187
3.8.2	Representing Mortality Dynamics . . . . .	190
3.8.3	Probabilities and Life Expectancy in a Dynamic Context . . . . .	192
3.8.4	Approaches to Mortality Forecasts . . . . .	192
3.8.5	Extrapolation via Exponential Formulae . . . . .	195
3.8.6	Mortality Forecasts Allowing for Random Fluctuations . . . . .	196
3.9	Moving to a Time-Continuous Context . . . . .	198
3.9.1	The Survival Function. . . . .	198
3.9.2	Other Related Functions . . . . .	199
3.9.3	The Force of Mortality . . . . .	201
3.9.4	Markers. . . . .	203
3.9.5	Parametric Models . . . . .	205
3.9.6	A Time-Continuous Dynamic Context. . . . .	207
3.10	Stochastic Mortality . . . . .	207
3.10.1	Number of People Alive in a Cohort. . . . .	207
3.10.2	Deterministic Models versus Stochastic Models . . . . .	207
3.10.3	Random Fluctuations in Mortality. . . . .	210
3.10.4	Systematic Deviations in Mortality . . . . .	212
3.10.5	The Impact of Mortality/Longevity Risk on Life Insurance . . . . .	213
3.11	References and Suggestions for Further Reading . . . . .	214
<b>4</b>	<b>Life Insurance: Pricing.</b> . . . .	217
4.1	Life Insurance Products. . . . .	217
4.1.1	General Aspects . . . . .	217
4.1.2	Alterations of a Life Insurance Contract . . . . .	219
4.1.3	Insurances of the Person . . . . .	220
4.2	Discounting Cash flows. . . . .	222
4.2.1	Premiums, Benefits, Expenses . . . . .	222
4.2.2	A Lump Sum Benefit in the Case of Death . . . . .	223
4.2.3	A Lump Sum Benefit in the Case of Survival . . . . .	224
4.2.4	Combining Benefits . . . . .	224
4.2.5	Actuarial Values: Basic Terminology and Notation. . . . .	226
4.2.6	Actuarial Values for Varying Benefits. . . . .	229
4.2.7	Actuarial Values with Zero Interest Rate . . . . .	230
4.2.8	Actuarial Values: An Approximation. . . . .	231
4.2.9	Actuarial Values: Inequalities. . . . .	231

4.2.10	The Actuarial Discount Factor . . . . .	232
4.2.11	Actuarial Values: Further Relations. . . . .	233
4.2.12	Actuarial Values at Times Following the Policy Issue . . . . .	234
4.3	Single Premium . . . . .	235
4.3.1	The Equivalence Principle . . . . .	235
4.3.2	The Pure Endowment . . . . .	237
4.3.3	Life Annuities . . . . .	240
4.3.4	The Term Insurance . . . . .	242
4.3.5	The Whole Life Insurance . . . . .	244
4.3.6	Combining Survival and Death Benefits . . . . .	245
4.3.7	Endowment Insurance Products . . . . .	246
4.3.8	The Expected Profit: A First Insight . . . . .	249
4.4	Periodic Premiums . . . . .	250
4.4.1	An Example . . . . .	251
4.4.2	Level Premiums . . . . .	254
4.4.3	Natural Premiums. . . . .	256
4.4.4	Single Premium, Natural Premiums, and Level Premiums: Some Relations . . . . .	259
4.4.5	Single Recurrent Premiums . . . . .	261
4.4.6	Some Concluding Remarks . . . . .	264
4.5	Loading for Expenses . . . . .	265
4.5.1	Premium Components. . . . .	265
4.5.2	Expenses and Loading for Expenses . . . . .	266
4.5.3	The Expense-Loaded Premiums . . . . .	267
4.6	References and Suggestions for Further Reading . . . . .	270
<b>5</b>	<b>Life Insurance: Reserving.</b> . . . .	<b>271</b>
5.1	Introduction . . . . .	271
5.2	General Aspects . . . . .	271
5.3	The Policy Reserve. . . . .	273
5.3.1	Definition . . . . .	273
5.3.2	The Policy Reserve for Some Insurance Products . . . . .	274
5.3.3	The Time Profile of the Policy Reserve. . . . .	276
5.3.4	Change in the Technical Basis . . . . .	284
5.3.5	The Reserve at Fractional Durations . . . . .	288
5.3.6	The Retrospective Reserve. . . . .	292
5.3.7	The Actuarial Accumulation Process. . . . .	294
5.4	Risk and Savings . . . . .	295
5.4.1	A (Rather) General Insurance Product. . . . .	295
5.4.2	Recursive Equations . . . . .	296
5.4.3	Risk Premium and Savings Premium . . . . .	299
5.4.4	Life Insurance Products versus Financial Accumulation. . . . .	305

5.5	Expected Profits . . . . .	307
5.5.1	Expected Annual Profits . . . . .	308
5.5.2	Splitting the Annual Profit . . . . .	309
5.5.3	The Expected Total Profit . . . . .	311
5.5.4	Cash Flows, Profits, and Premium Margins . . . . .	314
5.5.5	Expected Profits According to Best-Estimate Reserving . . . . .	316
5.6	Reserving for Expenses . . . . .	317
5.7	Surrender Values and Paid-Up Values . . . . .	319
5.8	References and Suggestions for Further Reading . . . . .	321
<b>6</b>	<b>Reserves and Profits in a Life Insurance Portfolio . . . . .</b>	<b>323</b>
6.1	The Portfolio Reserve . . . . .	323
6.1.1	Future Portfolio Reserves . . . . .	324
6.1.2	Safe-Side Reserve versus Best Estimate Reserve . . . . .	325
6.1.3	The Risk Margin . . . . .	326
6.1.4	The Portfolio Liability and Beyond . . . . .	329
6.1.5	Risk Margin: The “Cost of Capital” Approach . . . . .	330
6.2	The Total Profit . . . . .	332
6.2.1	The Life Fund . . . . .	333
6.2.2	The Expected Life Fund and the Expected Total Profit . . . . .	334
6.2.3	The Total Profit: An Alternative Interpretation . . . . .	336
6.3	Expected Annual Profits . . . . .	337
6.3.1	The Expected Surplus and the Expected Annual Profits . . . . .	338
6.3.2	The Role of the Portfolio Reserve . . . . .	341
6.4	Expected Annual Profits: A More General Setting . . . . .	345
6.5	References and Suggestions for Further Reading . . . . .	351
<b>7</b>	<b>Linking Life Insurance Benefits to the Investment Performance . . .</b>	<b>353</b>
7.1	Introduction . . . . .	353
7.2	Adjusting Benefits . . . . .	355
7.2.1	The General Case . . . . .	355
7.2.2	Addressing Specific Insurance Products . . . . .	358
7.2.3	Implementing Solutions . . . . .	364
7.2.4	The Yield to Maturity for the Policyholder . . . . .	368
7.3	Participating Policies . . . . .	372
7.3.1	Participating Policies with a Guaranteed Annual Return . . . . .	372
7.3.2	Participating Policies: Comparing Guarantee Structures . . . . .	378

7.4	Unit-Linked Policies . . . . .	386
7.4.1	Definition of Unit-Linked Benefits . . . . .	386
7.4.2	Unit-Linked Policies Without Guarantees. . . . .	387
7.4.3	Unit-Linked Policies with Financial Guarantees . . . . .	392
7.5	Financial Options in Unit-Linked and Participating Policies . . . .	395
7.5.1	The Structure of Minimum Guarantees . . . . .	396
7.5.2	The Valuation of Financial Options in a Unit-Linked Policy . . . . .	397
7.6	Hybrid Products . . . . .	399
7.7	With-Profit Policies . . . . .	402
7.8	Index-Linked Policies . . . . .	405
7.9	Universal Life Policies . . . . .	407
7.10	Variable Annuities . . . . .	409
7.11	References and Suggestions for Further Reading . . . . .	416
<b>8</b>	<b>Pension Plans: Technical and Financial Perspectives . . . . .</b>	<b>417</b>
8.1	Introduction . . . . .	417
8.2	Pension Programmes. . . . .	418
8.2.1	Individual and Group Pension Plans . . . . .	418
8.2.2	Benefits and Contributions. . . . .	420
8.2.3	Timing of the Funding . . . . .	422
8.3	Transferring Risks to the Provider . . . . .	423
8.4	Pension Savings Before Retirement. . . . .	425
8.5	Arranging the Post-retirement Income . . . . .	425
8.5.1	Some Basic Features of Life Annuities . . . . .	425
8.5.2	Packaging Benefits into the Life Annuity Product. . . . .	426
8.5.3	Life Annuities versus Income Drawdown . . . . .	428
8.5.4	Phased Retirement . . . . .	431
8.6	Risks for the Provider. . . . .	433
8.7	References and Suggestions for Further Reading . . . . .	437
<b>9</b>	<b>Non-life Insurance: Pricing and Reserving. . . . .</b>	<b>439</b>
9.1	Introduction . . . . .	439
9.2	Non-life Insurance Products. . . . .	440
9.2.1	General Aspects . . . . .	440
9.2.2	Main Categories of Non-life Insurance Products . . . . .	440
9.3	Loss and Claim Amount . . . . .	442
9.4	The Equivalence Premium. . . . .	446
9.4.1	The Items of the Equivalence Premium . . . . .	446
9.4.2	The Time-Pattern of a Claim . . . . .	447
9.4.3	The Expected Aggregate Claim Amount . . . . .	447
9.5	The Net Premium. . . . .	449
9.6	The Expense-Loaded Premium. . . . .	452
9.7	Statistical Data for the Equivalence Premium . . . . .	454

9.7.1	Risk Premium, Claim Frequency, Loss Severity . . . . .	454
9.7.2	Units of Exposure: The Case of Heterogeneous Portfolios. . . . .	457
9.7.3	Units of Exposure: The Number of Policy Years . . . . .	458
9.7.4	Updating the Risk Premium to Portfolio Experience . . .	461
9.8	Stochastic Modeling of the Aggregate Claim Amount . . . . .	465
9.8.1	Modeling the Claim Frequency . . . . .	465
9.8.2	Modeling the Claim Severity . . . . .	469
9.8.3	Modeling the Aggregate Claim Amount . . . . .	470
9.9	Risk Classification and Experience-Rating . . . . .	473
9.9.1	Risk Classes and Rating Classes. . . . .	473
9.9.2	Risk Classification at Issue . . . . .	474
9.9.3	Risk Classification at Renewal Times: Individual Experience Rating . . . . .	475
9.10	Technical Reserves: An Introduction . . . . .	479
9.11	Earned Premiums, Incurred Claim Amounts and Profit Assessment. . . . .	483
9.12	Deterministic Models for Claim Reserves . . . . .	486
9.12.1	Run-Off Triangles . . . . .	486
9.12.2	The Expected Loss Ratio Method . . . . .	488
9.12.3	The Chain-Ladder Method . . . . .	489
9.12.4	The Bornhuetter-Ferguson Method . . . . .	491
9.12.5	Further Aspects . . . . .	493
9.13	References and Suggestions for Further Reading . . . . .	493
<b>References . . . . .</b>		<b>495</b>
<b>Index . . . . .</b>		<b>499</b>



<http://www.springer.com/978-3-319-21376-7>

Introduction to Insurance Mathematics

Technical and Financial Features of Risk Transfers

Olivieri, A.; Pitacco, E.

2015, XVIII, 508 p. 195 illus., 21 illus. in color., Softcover

ISBN: 978-3-319-21376-7