

Chapter 2

Introduction to Atmospheric Physics

Fundamental Concepts

The book of nature is written in the language of mathematics.

—Galileo Galilei (15–16th century A.D.)

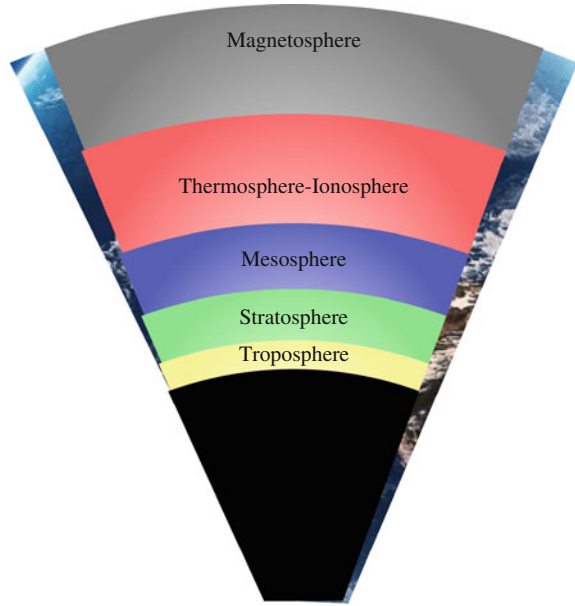
Abstract A brief introduction to the physics of the neutral atmospheres is given by reviewing some fundamental concepts. Some fluid mechanical principles of atmospheric physics are highlighted. To a good approximation, Earth's atmosphere and most other planetary atmospheres can be treated as an ideal gas that is in hydrostatic equilibrium. Thermodynamical laws are powerful in describing the basics of energy conservation in the atmosphere. Transport phenomena, such as diffusion, conduction, viscosity, and radiation are discussed briefly in order to provide some insight in the mechanisms of energy and momentum transfer processes in planetary atmospheres.

Keywords Continuum hypothesis • Geopotential • Potential temperature • Transport processes • Thermodynamic laws • Hydrostatic equilibrium

2.1 Introduction to Earth's Atmosphere

Earth's atmosphere, commonly known as “air” in daily life, is a thin layer of gas that surrounds the lithosphere, i.e., the upper layer of solid Earth. The atmosphere is pervaded by a complex geomagnetic field that is produced internally within Earth's core by an active magnetohydrodynamic (MHD) dynamo. This intrinsic geomagnetic field extends into the outer space by several Earth radii and largely shields Earth from high-energetic particles from the Sun and other astronomical sources, such as, the galactic cosmic rays. Existence of air with a significant amount of oxygen content with the right pressure permits the existence of life on our planet Earth. The

Fig. 2.1 Vertical atmospheric layers: Troposphere, Stratosphere, Mesosphere, Thermosphere, which coexists with the ionosphere. Not to scale



atmosphere also protects life on the surface from Sun's dangerous radiation. The study of the structure and composition of planetary atmospheres is of great relevance to the subject of habitability.

Figure 2.1 demonstrates a simple illustration of the vertical layers of Earth's atmosphere. The lower portion of the atmosphere is called the *troposphere* and contains approximately 90 % of the total atmospheric mass and is the region where the meteorological processes (i.e., weather) occur. These processes affect the day-to-day human life. With increasing altitude, the density drops off rapidly, causing the increase in the intensity of dynamical interaction processes occurring at higher altitudes. This is to say that a given amount of energy deposition would produce larger effects, e.g., heating, in the upper atmosphere than at lower altitudes.

Compared to the mean radius of Earth ($r_e \approx 6378$ km), the atmosphere is relatively thin. Its overall structure is controlled by a complex interplay of chemical and physical processes that couple the different atmospheric layers downward and upward over a large range of altitudes. If only in-situ processes are considered, the lower atmosphere (troposphere) is dominated primarily by meteorological processes while the upper portion of the atmosphere (*thermosphere-ionosphere*) is subject to electromagnetic and electrodynamical processes of solar and geomagnetic origin. The middle atmosphere (*stratosphere-mesosphere*) is the interface region between the lower and upper atmosphere that is influenced from below and above. The ozone layer, the protective shield from Sun's EUV radiation, is situated within the stratosphere. The physical structure of this layer has influence on the penetration of radiation and thus has importance for human health. Small- and large-scale waves that are produced by

Table 2.1 Some planetary parameters for Mercury, Venus, Earth, and Mars

	Radius [km]	Mean sun-planet distance [AU]	Rotational period [hr]
Mercury	2436	0.387	1403.3
Venus	6052	0.723	5832.2
Earth	6378	1	23.93
Mars	3396	1.523	24.6

1AU $\approx 1.496 \times 10^8$ km

various sources shape the circulation of the middle atmosphere. The importance of the lower atmosphere in influencing the upper atmosphere across the middle atmosphere is being increasingly acknowledged by the research community, following the recent progress in modeling and observations of vertical coupling processes.

Depending primarily on the solar and geomagnetic activity, the atmosphere extends from the surface up to 500–900 km. In particular, the 11-year solar activity cycle and geomagnetic processes, such as magnetic storms associated with solar flares and coronal mass ejections (CMEs), are dominant geophysical processes that largely control the overall morphology of the upper layers of the atmosphere, i.e., thermosphere-ionosphere. The vertical structure of the atmosphere-ionosphere system will be presented in more detail in Chap. 4.

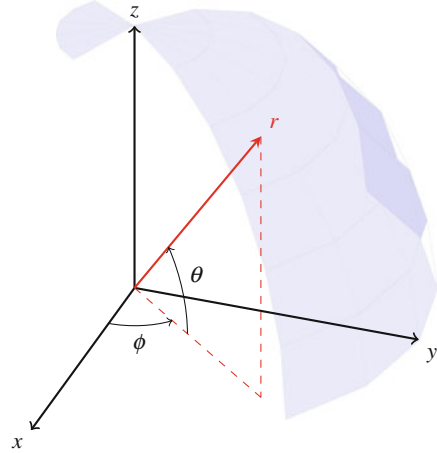
The essential planetary parameters of Earth are described in Table 2.1 along with their values for the innermost planet Mercury, and the two with respect to Earth neighboring terrestrial-like planets, Mars and Venus. It is noteworthy that Earth has a rotational period similar to Mars and a planetary radius similar to Venus. With a mean distance of about 0.3 AU Earth is closer to Venus than to Mars at a mean distance of 0.5 AU, where 1 AU is approximately 149.6 million kilometers. Mercury’s radius is about 2.5 times smaller than Earth’s.

In this chapter, my goal is to present a brief overview of some fundamental concepts of atmospheric dynamics and thermodynamics that are useful in the description of planetary atmospheres. Later chapters will survey governing equations of atmospheric physics (Chap. 3), the geospace environment and the vertical structure of Earth’s atmosphere (Chap. 4), atmospheric waves and their effects (Chap. 5), and the general circulation of the atmosphere (Chap. 6). In particular, Chaps. 5 and 6 will include state-of-the-art research topics in atmospheric waves and dynamics.

2.2 Basic Atmospheric Parameters and Coordinate System

The atmosphere is a thermodynamic fluid and can therefore be described by a set of thermodynamic parameters. Often the expressions of “parameter” and “variable” are used interchangeably. Some fundamental atmospheric parameters are temperature T , pressure p , and volume V . Physical variables, such as, temperature, mass density

Fig. 2.2 Illustration of the Cartesian and spherical geometries, where x , y , and z are the rectangular coordinates; and r , ϕ , and θ are the radial distance, azimuthal angle, and latitude, respectively. Latitude and longitude are in radians



ρ , pressure, and humidity, express the physical state of the atmosphere. *Kinematic variables*, such as wind velocity $\mathbf{u} = (u, v, w)$ with zonal (u , East-West), meridional (v , North-South) and vertical (w , upward-downward) wind components, describe the motion of air particles or systems of air particles. *Direct parameters*, such as, p , T , surface wind, and relative humidity, can be directly measured; parameters, such as, momentum \mathbf{p} , ρ , and potential temperature θ_T (see Sect. 2.11) are evaluated using equations of physics and are thus called derived parameters.

In a realistic atmosphere, all properties are time-dependent and vary in three-dimensions represented by the position vector \mathbf{r} . Time is denoted by the variable t , while the space variables depend on the choice of the coordinate system. In the rectangular coordinate system or the Cartesian¹ coordinate system, we have $\mathbf{r} = (x, y, z) = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$, where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are the associated unit vectors. Sometimes unit vectors are denoted by $\hat{\mathbf{e}}$, for example, as $\hat{\mathbf{e}}_x$, $\hat{\mathbf{e}}_y$, and $\hat{\mathbf{e}}_z$. In spherical polar geometry, $\mathbf{r} = (r, \phi, \theta) = r\hat{\mathbf{r}} + \phi\hat{\boldsymbol{\phi}} + \theta\hat{\boldsymbol{\theta}}$, where r is the radial distance, ϕ is the azimuthal angle, and θ is the latitude, as illustrated in Fig. 2.2.

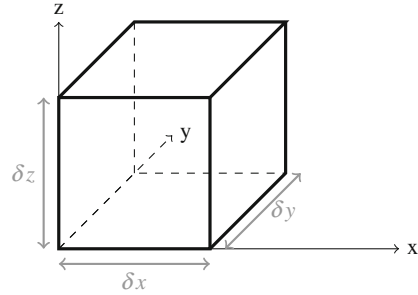
The amount of a given thermodynamic quantity per unit time (second) and per area element m^{-2} is understood as the flux of the quantity through a given area. For example, in atmospheric physics, one often speaks of momentum, energy, mass, and particle flux that provide important diagnostic information.

2.3 Continuum Hypothesis

Fluid mechanics is concerned with the behaviour of matter on a macroscopic scale, i.e., a scale that is much larger than the distances between individual molecules. Microscopic scale is related to atomic scales. Very often the molecular structure of

¹This coordinate system is called in honor of the mathematician René Descartes.

Fig. 2.3 A volume element $\delta V = \delta x \delta y \delta z$ in Cartesian coordinates



matter does not need to be taken into account. This assumption is justified provided that the flow length scales are much larger than the molecular mean free path. An atmospheric air parcel is typically represented by a volume element δV , which is assumed to be infinitesimally small and contain many atoms and molecules. The word “hypothesis” means originally a basis or a basis of an argument. In a scientific context, it is a tentative explanation of a phenomenon. A “continuum” implies continuity. Therefore, the “continuum hypothesis” states that the macroscopic characteristics of a fluid matter containing many molecules is the same and perfectly continuous in structure. The momentum and energy of particles contained in a given air parcel are then uniformly distributed within the volume element. Such an infinitesimally small volume element is often imagined simply in terms of the Cartesian coordinates as $\delta V = \delta x \delta y \delta z$ illustrated in Fig. 2.3. This simplification is very convenient in deriving conservation laws of physics.

2.4 Atmospheric Composition

The atmosphere, a gaseous matter, is composed of atoms and molecules. An atom is made up of electron(s) and a nucleus that includes the proton(s) and the neutron(s). Complex organization of these subatomic particles in the presence of various fundamental forces gives rise to a spectrum of atoms listed in the periodic table. The lightest atom is the hydrogen atom, ${}^1_1\text{H}$, composed of one electron and one proton, where the proton mass is $m_p = 1.672 \times 10^{-27}$ kg and the electron mass is $m_e = 9.109 \times 10^{-31}$ kg, and thus $m_p \gg m_e$. While the strong force keeps the nucleus together, the electromagnetic force is responsible for interactions between charged particles. Molecules are composed of two or more atoms.

Earth’s lower and middle atmosphere is mixed by turbulence. Table 2.2 lists the major and minor atmospheric constituents according to their abundance from left to right. About 78% of the atmosphere consists of Nitrogen (N_2), while only about 21% is the molecular Oxygen (O_2). Minor constituents such as Carbon Dioxide (CO_2), Argon (Ar), and water vapor (H_2O) together make up about 1%. 0.03% of CO_2 fractional composition is sometimes expressed in terms of

Table 2.2 Fractional composition of the terrestrial atmosphere in terms of volume mixing ratios in percentage

O ₂	N ₂	CO ₂	Ar	H ₂ O
21	78	0.03	0.9	0–2

particle per million (ppm) and corresponds to 300 ppm. In addition to the water vapor, CO₂ molecule is an important greenhouse gas. The variation of its concentration has wide-reaching impact on Earth's climate and is a central topic of research in climate sciences.

The atmosphere including the troposphere, stratosphere, and mesosphere, where the neutral species are well-mixed by turbulence, is often referred to as the *homosphere*. Above the homosphere in the thermosphere, diffusive separation gradually overtakes and the concentrations of the different species drop off differently with increasing altitude, depending on their weights. In the upper atmosphere, neutral mass density can demonstrate significant geographical and temporal variations.

2.5 Universal Gravity

The law of universal gravitation introduced by Isaac Newton in 17th cc states that every matter in the universe attracts every other matter with a force F_{12} that is proportional to the product of their masses m_1 and m_2 and inversely proportional to the square of the distance r between them. In other words, “everything pulls everything else”. This law is given mathematically by

$$F_{12} \propto \frac{m_1 m_2}{r^2}, \quad (2.1)$$

where

$$F_{12} = -F_{21}, \quad (2.2)$$

meaning that the force exerted by m_1 on m_2 , F_{12} , and the force exerted by m_2 on m_1 , F_{21} , have the same magnitude but opposite directions. The constant of proportionality is the G ,

$$F_{12} = -G \frac{m_1 m_2}{r^2}. \quad (2.3)$$

The numerical value of G can be determined experimentally by measuring the force between two test masses with a known separation distance. Although for small bodies the gravitational force is very small, the Cavendish balance can be used to determine it. With this method the constant is found to be $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$. The value of G has great significance for the formation of the universe after the Big Bang. A slightly smaller value could have meant that there could not have been enough

gravitational force between matter to clump together to form more complex matter. A stronger G could have implied that the expansion of the universe could have not taken place.

In terms of fundamental particles, you can consider the gravitational attraction between a proton and an electron, which amounts to $\sim 10^{-47}$ N at an atomic distance of 1 Å. In planetary atmospheres, the most obvious effect of gravity is that everything is accelerated toward the center of the planet and it is a body force in the equation of motion. The associated acceleration of the mass m in a planet of mass M due to the force F_{Mm} exerted by the planet on m is

$$g = \frac{F_{Mm}}{m} = G \frac{M}{r^2}, \quad (2.4)$$

where the distance $r = r_p + z$ is the sum of planetary radius and the altitude from the surface. For Earth, with $M_e = 5.97 \times 10^{24}$ kg the mean gravitational acceleration on the surface is $g_0 \approx 9.80 \text{ m s}^{-2}$. For Mars, with $M_m = 0.1 M_e$ and $r_m = r_e/2$ the mean surface gravitation acceleration is 3.92 m s^{-2} , which is 2.5 smaller than on Earth. The altitude variation of the gravitational acceleration is approximately given by

$$g(z) = \frac{g_0}{1 + \frac{z}{r_E}}, \quad (2.5)$$

For $z \ll r_E$ we have $g \approx g_0$.

2.6 Equation of State: Ideal Gas Law

A relation between the fundamental thermodynamic variables, such as, pressure p , temperature T , volume V , and number of particles (molecules) N in a thermodynamic system is given by an equation of state f as

$$f(p, T, V, N) = 0. \quad (2.6)$$

This property states that only a certain number of the properties of a substance can be given arbitrary values in a thermodynamic system. The specific form of an equation of state depends on the substance.

An ideal gas is defined in an atomistic view as a collection of gas of any species in which no forces operate between the individual gas particles. It has been experimentally found that there is a distinct relationship between the pressure and the temperature of an ideal gas. Namely, pressure times molal specific volume ν (i.e., volume per mole V/n) divided by temperature is equal to a constant R

$$\frac{p\nu}{T} = R, \quad (2.7)$$

where the universal gas constant R is

$$R = 8.317 \text{ J mole}^{-1} \text{ K}^{-1}. \quad (2.8)$$

Because there are a large number of atoms in a given volume, chemists have introduced the concept of “mole” in order to count more easily. One mole contains $N_A = 6.02 \times 10^{23}$ particles, where N_A is known as Avogadro’s number² and is defined by the number of carbon-12 atoms contained exactly in 12 g of carbon-12 (^{12}C). So the amount of matter in a volume element can be measured in terms of moles as well. For example, a mole of oxygen $^{16}_8\text{O}$ is 16 g. The number of atoms in a mole times the Boltzmann constant yields the universal gas constant R

$$R = N_A k_b, \quad (2.9)$$

where the Boltzmann constant $k_b = 1.381 \times 10^{-23} \text{ J K}^{-1}$. In turn, at a fixed temperature and volume, the total number of atoms can be determined.

The ideal gas law in terms of the volume and the number of moles is then

$$pV = nRT. \quad (2.10)$$

Equation (2.10) can be expressed in terms of the density ρ , which is of interest in planetary atmospheres. With $V = m/\rho$ combined with an expression for the total mass m in terms of the mean molecular mass m_m as $n = m/m_m$, we can express volume

$$V = \frac{nm_m}{\rho}, \quad (2.11)$$

and substitute in (2.10), which yields

$$p = \rho \frac{R}{m_m} T. \quad (2.12)$$

The mean molecular mass varies with altitude in planetary atmospheres. On Earth, $m_m \approx 28.8 \text{ g mol}^{-1}$ in the lower and middle atmosphere and decreases in the thermosphere. For practical calculations a typical value of m_m can be assumed to define

$$R_* = \frac{R}{m_m} \approx 288.7 \text{ J kg}^{-1} \text{ K}^{-1}, \quad (2.13)$$

yielding a simplified equation of state for an ideal gas

$$p = \rho R_* T. \quad (2.14)$$

²Named after the Italian physicist Amedeo Avogadro.

The ideal gas law is a limiting case for the behaviour of a real gas, which is described by the Van der Waals law. For planetary atmospheres, the ideal gas law is a very good approximation.

At constant temperature, pressure is inversely proportional to volume

$$p \propto V^{-1}, \quad (2.15)$$

called the Boyle-Mariotte law. At constant volume, pressure is proportional to temperature

$$p \propto T, \quad (2.16)$$

referred to as the Guy-Lussac law.

2.7 Thermodynamic Laws

Thermodynamics is the branch of physics that studies the relationship among the various properties of matter, without the knowledge of internal structure of the matter. The *laws of thermodynamics* contain qualitatively all the principles of thermodynamics. The first law of thermodynamics states that the conservation of energy given by

$$\Delta U = \Delta Q + \Delta W, \quad (2.17)$$

where ΔU , ΔQ , and ΔW are the changes in internal energy, heat flow, and in work done. This law states that the total change in the internal energy of a system is given by the sums of changes in heat flow and the work done. The differential form of (2.17) is then

$$dU = dQ + dW, \quad (2.18)$$

where $dW = -pdV$ is the work done on the system.

The second law of thermodynamics originally hypothesized by Carnot implies that heat on its own cannot flow from a cold to a warm object. This is the concept that is familiar to us from the day-to-day life. In a hot summer day, if you leave the balcony door open, then your room that had been cooled by an air conditioner will warm up by heat flow from outside into the room. Specifically, the entropy (disorder) in an isolated system either increases or remains constant.

A thermodynamical system can be taken from one state to another state by various thermal and/or dynamical processes. This change can take place in different ways. For example, in adiabatic processes, the temperature of a system is independent of its environment and the kinetic and potential energy of the system do not change.

2.8 Heat Capacity

Heat capacity C is an important concept in atmospheric thermodynamics. It expresses the amount of energy required to raise the temperature of a substance by 1 K. In general, we have

$$C \equiv \frac{dQ}{dT} \cdot \left[\frac{\text{J}}{\text{K}} \right] \quad (2.19)$$

The first law (2.18) can be written in terms of the specific quantities

$$d\varepsilon = dq_s - p d\nu, \quad (2.20)$$

where ε is the internal energy density (that is, internal energy per unit mass), q_s is the heat per unit mass, and $\nu = \rho^{-1}$ is the specific volume. Additionally, let us expand the change in the internal energy density $\varepsilon = \varepsilon(T, \nu)$ in terms of the changes in temperature and specific volume

$$d\varepsilon = \left(\frac{\partial \varepsilon}{\partial T} \right)_\nu dT + \left(\frac{\partial \varepsilon}{\partial \nu} \right)_T d\nu, \quad (2.21)$$

where the subscripts ν and T indicate that the partial derivatives are taken at constant specific volume and constant temperature, respectively. Inserting (2.21) into (2.20) yields

$$dq_s = \left(\frac{\partial \varepsilon}{\partial T} \right)_\nu dT + \left[\left(\frac{\partial \varepsilon}{\partial \nu} \right)_T + p \right] d\nu, \quad (2.22)$$

In general, heat flow can occur at constant volume or constant pressure. In the case of constant volume ($d\nu = 0$), we get the specific heat capacity at constant volume c_v

$$dq_s = \left(\frac{\partial \varepsilon}{\partial T} \right)_\nu dT \iff dq_s = c_v dT. \quad (2.23)$$

At constant pressure ($dp = 0$), one has

$$dq_s = c_p dT. \quad (2.24)$$

Note that the specific heat capacity has the unit of $\text{J kg}^{-1} \text{K}^{-1}$. On Earth, for dry air, $c_p = 1004 \text{ J kg}^{-1} \text{K}^{-1}$ and $c_v = 717 \text{ J kg}^{-1} \text{K}^{-1}$. In comparison, water has a specific capacity of $c_p = 4180 \text{ J kg}^{-1} \text{K}^{-1}$. The ratio of the specific heat capacities is given by

$$\gamma \equiv \frac{c_p}{c_v}, \quad (2.25)$$

where γ is sometimes called the specific heat ratio. For monoatomic gases, such as, He and Ar $\gamma = 1.67$ and for diatomic gases, e.g., for N_2 and O_2 , $\gamma = 1.4$.

2.9 Geopotential

Consider a volume of air at rest on the surface of Earth. Then, the fluid volume is subject to two types of forces: (i) pressure gradient force due to the exponentially decreasing pressure with altitude; (ii) Coriolis forces due to the rotation of Earth plus the gravitational force. The conservative body force that is associated with a potential Φ is called the geopotential. It is composed of Earth's gravitational potential and the centrifugal potential related to Earth's rotation. Geopotential is the amount of energy required to move a unit mass vertically upward from the surface to a reference level z .

$$\Phi(z) = \int_0^z g(z') dz' \quad [\text{m}^2 \text{s}^{-2}] \quad (2.26)$$

If the sea on Earth were at rest then the geopotential surface would coincide with the sea surface, which is taken as a reference level. For most calculations, the gravitational acceleration g is assumed to be constant as $g_0 = 9.8 \text{ m s}^{-2}$. Then,

$$\Phi(z) \approx g_0 z, \quad (2.27)$$

where the vertical coordinate z denotes the vertical distance from the reference level and is typically called the geopotential height,

$$Z = \frac{\Phi}{g_0}. \quad (2.28)$$

So, to move a mass of 1 kg over 1 m vertical distance in the gravitation field of Earth, one would need $\sim 9.8 \text{ J}$.

2.10 Hydrostatic Equilibrium

Under the effect of gravity, the atmospheric pressure p decreases exponentially with increasing height. In order to obtain an expression for the vertical variation of pressure, consider a column of air with altitude extent dz and bottom surface area of A . Then the pressure exerted by the air inside the column is given by the force divided by the area

$$dp = \frac{dF}{A}. \quad (2.29)$$

The infinitesimal force is $dF = -g dm = -g \rho dV$, with the volume element $dV = A dz$. The pressure dp becomes

$$dp = \frac{dF}{A} = -\frac{g \rho A dz}{A}. \quad (2.30)$$

The hydrostatic equilibrium (or balance) is then given by

$$\frac{dp}{dz} = -g\rho. \quad (2.31)$$

Equation (2.31) expresses that the downward gravitational acceleration is balanced by the upward directed acceleration associated with the negative upward pressure gradient. Without Earth's gravity, air would be accelerated into space. Hydrostatic acceleration implies that the vertical acceleration of air is negligible, though vertical air speed can be non-zero. In processes that may involve nonhydrostatic effects (e.g., Yiğit and Ridley 2011) the hydrostatic condition (2.31) does not apply.

From the hydrostatic relation, the vertical variation of pressure can be derived. Applying separation of variables in (2.31), representing the density with the ideal gas law (2.12) and then writing with the left-hand side and right-hand side with integrals with limits from some reference levels p_0 to p and z_0 to z , respectively, give

$$\int_{p_0}^p \frac{dp}{p} = - \int_{z_0}^z \frac{m(z) g(z)}{R T(z)} dz, \quad (2.32)$$

and finally integrating the left-hand side with respect to pressure yields

$$p(z) = p_0 \exp \left(- \int_{z_0}^z \frac{mg}{RT} dz \right), \quad (2.33)$$

in terms of the scale height H

$$H \equiv \frac{RT}{mg}, \quad (2.34)$$

we get

$$p(z) = p_0 \exp \left(- \int_{z_0}^z \frac{1}{H(z)} dz \right). \quad (2.35)$$

Over small vertical distances, the scale height may vary slowly in the lower atmosphere and can be assumed to be constant for practical purposes. Equation (2.35) can be approximated

$$p(z) = p_0 \exp \left(- \frac{z - z_0}{H} \right), \quad (2.36)$$

which shows that the pressure falls off exponentially with height. In the upper atmosphere, because of diffusive separation, one speaks of the individual scale heights of the different species. For a neutral species i with molecular mass m_i , we then have

$$H_i(z) = \frac{R T(z)}{m_i g(z)} \quad (2.37)$$

Overall, the scale height can vary significantly with height because of the large variations of temperature and the mean molecular mass with height. The concept of scale height also applies to ionized species.

2.11 Potential Temperature

The potential temperature is the temperature a parcel of air at pressure p and temperature T would obtain if it were expanded or compressed adiabatically to a standard pressure $p_s = 1000$ hPa.

$$\theta_T = T \left(\frac{p_s}{p} \right)^{R/c_p}, \quad (2.38)$$

where c_p is the specific heat at constant pressure and T is temperature. Equation (2.38) is known as Poisson equation as well. For dry adiabatic conditions, the potential temperature is conserved. Sometimes, instead of temperature and pressure, potential temperature and pressure are used as state variables. Also, the stability of the atmosphere is measured in terms of the potential temperature gradient.

2.12 Atmospheric Stability

The variation of neutral temperature with altitude in a planetary atmosphere is an important criterion that shapes the stability of the atmosphere.

2.12.1 Lapse Rate

Temperature lapse rate Γ describes the rate of temperature decrease with increasing altitude:

$$\Gamma = -\frac{\partial T}{\partial z} \quad (2.39)$$

Combining the Poisson equation (2.38), the ideal gas law and the law of hydrostatic equilibrium (2.31) yields

$$\frac{T}{\theta} \frac{\partial \theta_T}{\partial z} = \frac{\partial T}{\partial z} + \frac{g}{c_p} \quad (2.40)$$

Dry adiabatic lapse rate Γ_d is given when $\frac{\partial \theta_T}{\partial z} = 0$

$$-\frac{\partial T}{\partial z} = \frac{g}{c_p} \equiv \Gamma_d \quad (2.41)$$

2.12.2 Static Stability

In a realistic atmosphere the potential temperature varies with altitude. The actual lapse rate Γ would differ from the dry adiabatic lapse rate Γ_d . This difference is obtained by combining Eqs. (2.40)–(2.41):

$$\frac{T}{\theta} \frac{\partial \theta}{\partial z} = \Gamma_d - \Gamma \quad (2.42)$$

A parcel of air that undergoes adiabatic displacements can be either positively or negatively buoyant and tends to return to its equilibrium position. If the actual lapse rate is smaller than the dry adiabatic lapse rate, i.e., $\Gamma < \Gamma_d$, then the potential temperature increases with altitude ($\frac{\partial \theta}{\partial z} > 0$). The air parcel that is displaced adiabatically from its equilibrium position will be positively buoyant when displaced downward and negatively buoyant when displaced upward. Under these conditions Earth's atmosphere is stably stratified. Adiabatic oscillations of a fluid parcel around its equilibrium in a stably stratified atmosphere are called buoyancy oscillations. The frequency of these oscillations is given by the Brunt-Väisälä frequency N . In stably stratified atmospheres, gravity waves can propagate freely. If the atmospheric lapse rate exceeds the dry adiabatic lapse rate, i.e., $\Gamma > \Gamma_d$, it implies that ($\frac{\partial \theta}{\partial z} < 0$) and the atmosphere can become convectively unstable.

2.13 Transport Phenomena

Any non-equilibrium condition in a system involves some type of transport process. Basic transport processes within fluids are *diffusion*, *viscosity*, and thermal *conduction*. All transport processes have an associated transport coefficient that quantifies the degree of the transport. The coefficients of diffusion D , viscosity μ , and conduction κ are summarized along with their dimensions in Table 2.3.

Table 2.3 Transport processes and the associated transport coefficients

Transport process	Coefficient	Dimension
Diffusion	D	$\text{m}^2 \text{s}^{-1}$
Viscosity	μ	$\text{kg m}^{-1} \text{s}^{-1}$
Conduction	κ	$\text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}$

D is the diffusion coefficient, μ is the coefficient of viscosity, and κ is the thermal conductivity coefficient

2.13.1 Diffusion

Diffusion involves molecular motion. It is a consequence of frequent, stochastically distributed collisions between particles. Imagine a container of a background gas in thermal equilibrium. We then introduce a small amount of a different gas in a random place in the container. The particles of the new gas then spread gradually through the background gas, colliding with the particles of the background gas. This new gas is said to “diffuse” through the background gas and the process is called diffusion.

In a system of inhomogeneously distributed particles, Fick’s law expresses the net particle flux Φ , which is proportional to the negative gradient of the particle number density. In 1–D it is given by

$$\Phi = -D \frac{dn}{dx}, \quad (2.43)$$

where D is the diffusion coefficient and n is the number density, that is, number of particles per unit volume, of particles. If we consider three-dimensional diffusion, then the most generic form of particle flux is

$$\Phi = -\mathbf{D} \cdot \nabla n, \quad (2.44)$$

where \mathbf{D} is the anisotropic diffusion tensor and the Φ is the particle flux vector. Assuming $n = n(x, t)$, particle flux can be expressed considering the continuity (Sect. 3.7) requirement

$$\frac{\partial n}{\partial t} + \frac{\partial(nu)}{\partial x} = 0, \quad (2.45)$$

where u is the speed in the positive x -direction and nu is the particle flux in the x -direction. Combination of (2.43) with (2.45) yields the diffusion equation.

$$\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2}, \quad (2.46)$$

assuming isotropic homogeneous diffusion. In three dimensions one gets

$$\frac{\partial n}{\partial t} = D \nabla^2 n, \quad (2.47)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator in Cartesian coordinates.

In processes that involve advective processes as well, the diffusion equation can be extended to the advective-diffusion equations. More on the formalism of advection and diffusion can be found in the work by Medvedev and Greatbatch (2004).

2.13.2 Viscosity

Viscosity can be considered as a frictional force. If there is a velocity shear (gradient) perpendicular to the flow in a fluid, then transport of momentum occurs within the flow in order to balance the velocity gradient. Therefore, in any nonuniform fluid flow, we expect viscosity to be present. The flow is said to possess viscosity because of the thermal motion of particles perpendicular to the flow direction.

Let us imagine a one-dimensional shear flow directed in the positive x -direction (zonal direction) given by the speed u , that is, $\mathbf{u} = (u(y), 0, 0)$. Let the flow possess an increasing gradient of velocity in the y -direction (meridional direction), as illustrated in Fig. 2.4. Then neighbouring fluid layers exert some stress, that is, force per unit contact area, on each other. This configuration leads to the transport of y -momentum in x -direction denoted by τ_{xy} and is experimentally found to be proportional to the gradient of the x -component of the flow in the y -direction

$$\tau_{xy} = -\mu \frac{\partial u}{\partial y}, \quad (2.48)$$

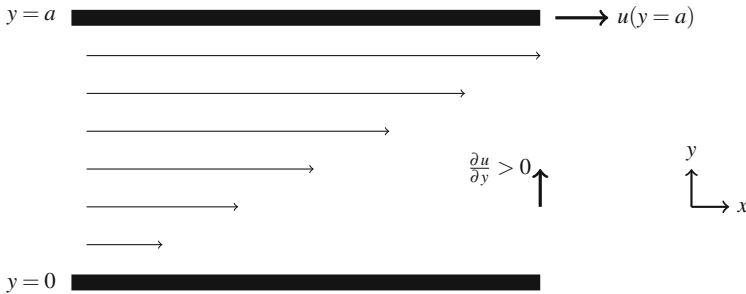


Fig. 2.4 Illustration of viscosity in the x - y plane with u denoting the x -component of the velocity. Thick lines denote the *top* and the *bottom* boundaries, while the *thin arrow* show the velocity components in away from the boundaries, shown here declining in the negative y -direction and increasing in the positive y -direction.

where the proportionality factor μ is the coefficient of viscosity, which is given in terms of the kinematic viscosity $\mu = \nu \rho$. In the absence of velocity shear there is no induced stress because of viscosity. In other words, the stress arising from the presence of viscosity acting perpendicular to the flow is called *shear stress*. As the shear stress is proportional to the velocity gradient, the fluid said to be *Newtonian*. The physical significance of shear stresses in a fluid arises from the fact that any imbalance in stress can lead to a net body force on the fluid. In our example, τ_{xy} expresses the transfer of y-momentum in the x-direction, resulting from a velocity shear in the y-direction. In a three-dimensional case, additional contribution to momentum flux in x-direction can come from a velocity shear in the z-direction, that is, $\tau_{xz} > 0$ in the presence of shear in the z-direction. In general, the force per unit mass in the i-th direction resulting from viscous processes is

$$a_i = \frac{F_i}{m} = \frac{1}{\rho} \frac{\partial \tau_{ij}}{\partial x_j} = -\frac{1}{\rho} \frac{\partial}{\partial x_j} \left(\mu \frac{\partial u_i}{\partial x_j} \right), \quad (2.49)$$

where τ_{ij} is the flux of x_j -component of momentum in the x_i -direction. It is important to note that the total stress σ_{ij} arising in a fluid is composed of pressure (normal stress) and shear stress τ_{ij}

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij}, \quad (2.50)$$

where δ_{ij} is the Kronecker delta with the property

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \quad (2.51)$$

Viscous force arising from the relative motion of different fluid layers is always a frictional force that counteracts the velocity variations in the flow.

2.13.3 Conduction

Conduction describes the flow of thermal energy per unit area per unit time, i.e., heat flux. In response to a temperature gradient, heat is conducted. Heat flows in the direction of decreasing temperature, that is, from hot to cold regions in accordance with the second law of thermodynamics. In a uniform temperature distribution, heat conduction would be zero. Fourier's law describes the heat conduction in a stationary medium

$$q = -\kappa \frac{\partial T}{\partial x}, \quad [\text{J s}^{-1} \text{ m}^{-2}] \quad (2.52)$$

where q is heat flux, κ is the thermal conductivity, and T is temperature. Heat flux describes the transfer of heat in terms of energy per unit area per unit time. The negative sign is because the heat flow is in the opposite direction to the temperature

gradient. Fourier law of heat conduction assumes a steady-state heat conduction, meaning that the temperature distribution in the system is independent of time.

If we assume that the temperature distribution in a closed system is changing with time, for example, due to time-varying boundary conditions, then we speak of an unsteady (time-dependent) heat conduction process. The change of the internal energy per unit mass ε is then

$$-\frac{\partial \varepsilon}{\partial t} = \frac{1}{\rho} \frac{\partial q}{\partial x} = -\frac{\partial}{\partial x} \left(\kappa \frac{\partial T}{\partial x} \right). \quad (2.53)$$

With $\varepsilon = c_v T$ and assuming constant thermal conductivity, we get the *unsteady heat conduction equation*

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}. \quad (2.54)$$

In three dimensions, we have

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T, \quad (2.55)$$

where $\alpha = \kappa (\rho c_v)^{-1}$ is the thermal diffusivity.

2.13.4 Radiation

Radiation is transport of energy (heat) by means of electromagnetic wave propagation. Sun's energy reaches us via radiation. Every matter in nature emits energy in form of radiation. The characteristics of the emission depends on the temperature of the body. At higher temperatures higher frequency (shorter wavelength) emission takes place. Specifically, the rate of emission (energy radiation) associated with a body is proportional to the surface area A and to the fourth power of temperature T^4 . The Stefan-Boltzmann law is given by

$$\frac{dQ}{dt} = A \varepsilon_r \sigma_b T^4, \quad [\text{W}] \quad (2.56)$$

where dQ/dt is the heat flow, the proportionality constants ε_r is the radiative emissivity and $\sigma_b = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ is the Stefan-Boltzmann constant. The emissivity varies between 0 and 1 and is determined with respect to an ideal radiating body with a given surface and at a given temperature. The net heat flow depends on the temperature difference between the emitting body and its surroundings. For example, the human body is about 10°C warmer than the average room temperature. An emissivity of unity and body area of 1 m^2 yield a net heat flow of about 60 W from the human body.

The radiation field is described in terms of the amount of radiant energy dE_ν in a given frequency interval $(\nu, \nu + d\nu)$, which is transported across an area element dA in the direction confined to an element of the solid angle $d\omega$, during an time interval of dt (Chandrasekhar 1960). In terms of the intensity I_ν associated with the frequency ν , the infinitesimal radiant energy is expressed as

$$dE_\nu = I_\nu \cos \alpha \, dA \, d\omega \, d\nu \, dt. \quad (2.57)$$

Integration over all frequency bands yields the total radiant energy.

2.14 Richardson Number

The Richardson number R_i is a measure of evolution of turbulence in the atmosphere and describes atmospheric stability. It is given by

$$R_i = \frac{N^2}{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2}, \quad (2.58)$$

where N is the buoyancy frequency, u and v are the zonal and meridional components of the horizontal flow, respectively. The Richardson number expresses the importance of vertical variations in a horizontal shear flow. If $R_i > 0.25$, the flow is stable, that is, if the fluid is vertically displaced by a disturbance, it will either return to its initial position or perform periodic oscillations around the equilibrium position. For $R_i < 0.25$, the flow undergoes dynamical instability and if the $R_i < 0$ then the flow is convectively unstable. Another interpretation of R_i is as the ratio of production of turbulence by buoyancy to the production of turbulence by wind shear (Nappo 2002). Typically, dynamical instability preceeds convective instability. Horizontal flow with large wind shear is more prone to instability than the flow with small wind shear.

2.15 Reynolds Number

The Reynolds number \Re is an important dimensionless parameter that characterizes the nature of flow patterns in a fluid. It is defined as the ratio of inertial forces to viscous forces

$$\Re = \frac{\rho |\mathbf{u}| L}{\mu} = \frac{u L}{\nu}, \quad (2.59)$$

where L is the length scale of the flow, ρ is the mass density, and μ is the viscosity coefficient (or dynamic viscosity). \Re can be used to identify whether the flow is

laminar or turbulent. In a dynamic fluid \mathfrak{R} is variable. Increasing \mathfrak{R} means that inertial effects are becoming gradually more dominant over viscous effect and the flow thus has a tendency to transition to a turbulent state.

References

- Chandrasekhar S (1960) Radiative transfer. Dover books on physics, Dover
- Medvedev AS, Greatbatch RJ (2004) On advection and diffusion in the mesosphere and lower thermosphere: the role of rotational fluxes. J Geophys Res 109:D07104. doi:[10.1029/2003JD003931](https://doi.org/10.1029/2003JD003931)
- Nappo CJ (2002) An introduction to atmospheric gravity waves, International geophysics series, vol 85. Academic Press, Amsterdam
- Yiğit E, Ridley AJ (2011) Role of variability in determining the vertical wind speeds and structure. J Geophys Res 116:A12305. doi:[10.1029/2011JA016714](https://doi.org/10.1029/2011JA016714)

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