

Chapter 2

Coherent Manipulation and Interferometry

The interferometric laser cooling and composite pulse schemes explored in this thesis are based on coherent transitions in a two-level atomic system, and we realise this experimentally by inducing stimulated Raman transitions between the ground hyperfine states of rubidium-85. Broadly speaking, a stimulated Raman transition is a two-photon process, whereby two counter-propagating laser beams, far-detuned from resonance with a radiative upper level and with a frequency difference equal typically to the ground hyperfine interval, induce coherent population transfer between (meta)stable states.

This chapter details the theory governing velocity-sensitive stimulated Raman transitions. Firstly, we introduce the stimulated Raman system, and derive the equations of motion of the state amplitudes. We then describe the Bloch sphere picture for visualisation of coherent pulses, and discuss the velocity-sensitive Ramsey interferometer which forms the basis of the cooling schemes of Chap. 7. We then give a brief summary of the atomic structure of rubidium-85, and finish by presenting a vector formalism for stimulated Raman transitions.

2.1 Raman Transitions

In this section we derive the general equations of motion for the hyperfine state amplitudes during interaction with two counter-propagating laser beams, which are far-detuned from atomic resonance. We begin with a three-level system, then apply a series of transformations and approximations to derive an effective two-level Hamiltonian for the system, from which the equations of motion are obtained. A particularly detailed treatment of velocity-selective stimulated Raman theory in the context of cold atoms can be found in [1]. For the interested reader, and in order to explain particular aspects of the theory not discussed in the main text, we give a summary of the theory governing two-level atom-laser interactions in Appendix A.1.

2.1.1 3-Level System and the Rabi Frequency

We start by considering a three-level atomic system, as shown in Fig. 2.1, in which there are two lower levels, labelled (ignoring their momenta, for now) $|1\rangle$ and $|2\rangle$ separated by the difference between their atomic frequencies $\omega_{12} \equiv \omega_2 - \omega_1$, and an upper level with atomic frequency ω_3 , labelled $|3\rangle$. The lower levels represent ground atomic hyperfine states, from which population cannot spontaneously decay. The upper level represents an *intermediate* state, from which spontaneous decay is possible, at an optical interval from the lower levels. Our atom is irradiated by two laser beams of frequencies ω_{L1} and ω_{L2} , which are counter-propagating along the z -axis. The electric field of the laser beams is described by

$$\mathbf{E} = \frac{1}{2}\mathbf{E}_1 e^{i(\mathbf{k}_{L1} \cdot \mathbf{z} - \omega_{L1}t + \phi_{L1})} + \frac{1}{2}\mathbf{E}_2 e^{i(\mathbf{k}_{L2} \cdot \mathbf{z} + \omega_{L2}t + \phi_{L2})} + c.c. \quad (2.1)$$

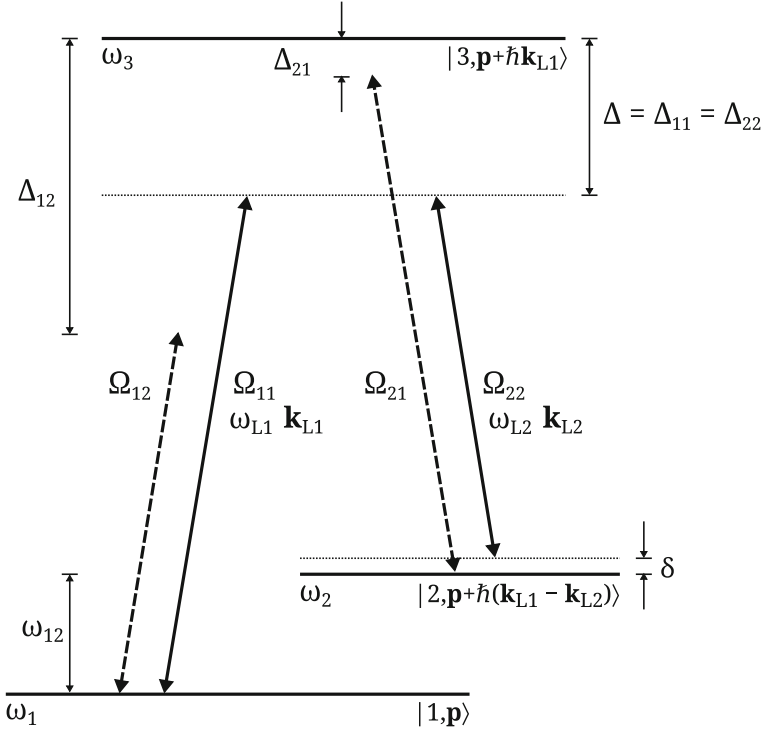


Fig. 2.1 The 3-level Raman system. The arrows indicate the two Raman field components, each with its associated frequency ω_{La} and wavevector $\mathbf{k}_{La}$. The *solid* (*dashed*) arrows indicate interaction of level n with laser a , where $n = a$ ($n \neq a$). The Rabi frequencies are denoted Ω_{na} , and the single-photon detunings are Δ_{na}

where $\mathbf{k}_{L1} \simeq -\mathbf{k}_{L2}$. By the dipole interaction, the laser ω_{La} , where $a = 1, 2$, couples the state $|n\rangle$, where $n = 1, 2$, to $|3\rangle$. The two beams are detuned by a large amount Δ_{na} from single photon resonance with $|3\rangle$, thus making transfer of population to the upper level negligible, and allowing for the pair of components with a common detuning $\Delta_{nn} = \Delta$ to induce coherent transitions between $|1\rangle$ and $|2\rangle$. This type of two-photon transition is known as a stimulated *Raman* transition. A photon carries momentum $\hbar\mathbf{k}$, and we expect the transitions to occur within a closed basis, which we therefore preemptively define as $|1, \mathbf{p}\rangle$, $|3, \mathbf{p} + \hbar\mathbf{k}_{L1}\rangle$ and $|2, \mathbf{p} + \hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2})\rangle$ (this choice of basis will become clear in Sect. 2.1.2), where $\mathbf{p}$ is the initial atomic momentum. The atomic wavefunction can be written in terms of the amplitudes associated with these basis states:

$$|\Psi(t)\rangle = a_1(t)|1, \mathbf{p}\rangle + a_3(t)|3, \mathbf{p} + \hbar\mathbf{k}_{L1}\rangle + a_2(t)|2, \mathbf{p} + \hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2})\rangle = \begin{pmatrix} a_1(t) \\ a_2(t) \\ a_3(t) \end{pmatrix}, \quad (2.2)$$

and this will evolve according to the time dependent Schrödinger equation (TDSE)

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle, \quad (2.3)$$

where the Hamiltonian $\hat{H} = \hat{H}_A + V$ is the sum of the *atomic* Hamiltonian $\hat{H}_A = \sum_n \omega_n |n\rangle\langle n| + \hat{\mathbf{p}}^2/2M$ (where M is the atom's mass) and the *interaction* Hamiltonian $V = -\hat{\mathbf{d}} \cdot \mathbf{E}$. The Hamiltonian can be written explicitly as [2]

$$\hat{H} = \frac{\hat{\mathbf{p}}^2}{2M} + \hbar\omega_1|1\rangle\langle 1| + \hbar\omega_3|3\rangle\langle 3| + \hbar\omega_2|2\rangle\langle 2| - \hat{\mathbf{d}} \cdot \mathbf{E}, \quad (2.4)$$

where the $\hat{\mathbf{p}}$ term acts on the momentum portion of the basis states. The interaction term $V = -\hat{\mathbf{d}} \cdot \mathbf{E} = -e\hat{\mathbf{r}} \cdot \mathbf{E}$ represents the atom-field *dipole* interaction, where $\mathbf{r}$ is the vector defining the position of the electron relative to the atomic nucleus. The matrix elements of V are defined as

$$V_{n3} = \langle n|V|3\rangle = \sum_{a=1,2} \frac{\hbar\Omega_{na}}{2} \left(e^{i(\mathbf{k}_{La} \cdot \mathbf{z} + (-1)^a \omega_{La} t + \phi_{La})} + c.c. \right) |3\rangle\langle n|, \quad n = 1, 2 \quad (2.5)$$

where we introduce the *Rabi frequency* Ω_{na} for coupling of level $|n\rangle$ to level $|3\rangle$ by laser a :

$$\Omega_{na} = -\frac{\langle n|\hat{\mathbf{d}} \cdot \mathbf{E}_a|3\rangle}{\hbar}, \quad n = 1, 2. \quad (2.6)$$

The Rabi frequency is the angular frequency at which population will ‘Rabi flop’ between two states due to interaction with a resonant laser.

2.1.2 The Momentum Operator

We now explain the reasoning behind the choice of momentum-inclusive basis. The matrix elements V_{n3} contain the terms $e^{\pm i\mathbf{k}_{La} \cdot \mathbf{z}}$ which, when we align $\mathbf{k}_{La}$ parallel to $\mathbf{z}$, become $e^{\pm ik_{La}z}$. We can re-write this with the closure relation $\int dp |p\rangle\langle p| = 1$ as

$$1 \times e^{\pm ik_{La}z} = \int dp e^{\pm ik_{La}z} |p\rangle\langle p|, \quad (2.7)$$

where the momentum-state wavefunction $|p\rangle$ can be expressed as

$$|p\rangle = \int_{-\infty}^{\infty} dz |z\rangle\langle z|p\rangle = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dz e^{ipz/\hbar} |z\rangle. \quad (2.8)$$

Substituting Eq. 2.8 into Eq. 2.7 gives

$$\begin{aligned} e^{ik_{La}z} &= \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} dz e^{i(p \pm \hbar k_{La})z/\hbar} |z\rangle\langle p| \\ &= \int dp |p \pm \hbar k_{La}\rangle\langle p|. \end{aligned} \quad (2.9)$$

This shows that the $e^{\pm i\mathbf{k}_{La} \cdot \mathbf{z}}$ part of the interaction term V_{n3} acting on the momentum portion $|\mathbf{p}\rangle$ of the atomic state gives rise to a change in the atomic momentum by $\pm \hbar \mathbf{k}_{La}$. Therefore for an atom initially in the state $|1, \mathbf{p}\rangle$, coupling to $|3, \mathbf{p} + \hbar \mathbf{k}_{L1}\rangle$ can occur by absorption of a photon at ω_{L1} (where $\omega_{La} = c/k_{La}$), and subsequently¹ coupling from $|3, \mathbf{p} + \hbar \mathbf{k}_{L1}\rangle$ to $|2, \mathbf{p} + \hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2})\rangle$ can occur by the *stimulated* emission of a photon at ω_{L2} . This justifies the choice of momentum-inclusive basis made in the previous subsection.

2.1.3 The 3-Level Hamiltonian

If we apply the rotating wave approximation (which removes the off-resonant terms in the interaction Hamiltonian, as described in Appendix A.1.1), we can write out the full Hamiltonian for the three-level system in the absence of spontaneous emission from $|3\rangle$ [2]

$$\hat{H} = \begin{pmatrix} \frac{\mathbf{p}^2}{2M} + \hbar\omega_1 & 0 & \frac{\hbar\Omega_{11}}{2} e^{i(\omega_{L1}t - \phi_{L1})} \\ 0 & \frac{(\mathbf{p} + \hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2}))^2}{2M} + \hbar\omega_2 & \frac{\hbar\Omega_{22}}{2} e^{i(\omega_{L2}t + \phi_{L2})} \\ \frac{\hbar\Omega_{11}^*}{2} e^{-i(\omega_{L1}t - \phi_{L1})} & \frac{\hbar\Omega_{22}^*}{2} e^{-i(\omega_{L2}t + \phi_{L2})} & \frac{(\mathbf{p} + \hbar\mathbf{k}_{L1})^2}{2M} + \hbar\omega_3 \end{pmatrix}. \quad (2.10)$$

¹It is important to note that the Raman transition is not a sequential absorption *then* stimulated emission process (as is, for example, the case for STIRAP [3]), but is closer to a simultaneous absorption *and* stimulated emission process.

It is important to be aware of the absent terms here: we have considered only $a = n$ couplings, that is, where the field at frequency ω_{La} only interacts with the ‘correct’ level $|a\rangle$. In reality, both lasers will couple to *both* levels, as illustrated in Fig. 2.1 and described by Eq. 2.5. A treatment considering all combinations of a and n is given in Appendix A.2, and as we discuss there, the result is simply an additional term in the light shift of each level, which we re-insert later on in this section.

With the Hamiltonian given in Eq. 2.10, the TDSE exhibits fast oscillations of $a_1(t)$, $a_2(t)$ and $a_3(t)$ at frequencies specified by their respective atomic frequencies and momenta, alongside relatively slow oscillations due to interaction with the laser. In the following we take steps to transform this Hamiltonian into a more manageable form.

We have the freedom to make the transformation $|\Psi'\rangle = \hat{O}|\Psi\rangle$, where $\hat{O}$ is some operator ($\hat{O}^\dagger \hat{O} = \mathbb{I}$) and $|\psi'\rangle$ is a transformed state vector. With such a substitution, the TDSE becomes

$$i\hbar \frac{\partial}{\partial t} (\hat{O}^\dagger |\Psi'\rangle) = \hat{H} (\hat{O}^\dagger |\Psi'\rangle), \quad (2.11)$$

and if we apply the product rule to the left hand side, operate with $\hat{O}$ from the left, and re-arrange, we find

$$i\hbar \frac{\partial}{\partial t} |\Psi'\rangle = \left(\hat{O} \hat{H} \hat{O}^\dagger - i\hbar \hat{O} \frac{\partial}{\partial t} \hat{O}^\dagger \right) |\Psi'\rangle, \quad (2.12)$$

where the term inside the brackets is the transformed Hamiltonian. Using the above, we can factor out the fast oscillations in the TDSE by making the substitutions

$$a_1(t) = b_1(t) e^{-i \left(\frac{\mathbf{p}^2}{2M} + \hbar\omega_1 \right) t / \hbar} \quad (2.13a)$$

$$a_2(t) = b_2(t) e^{-i \left(\frac{(\mathbf{p} + \hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2}))^2}{2M} + \hbar\omega_2 \right) t / \hbar} \quad (2.13b)$$

$$a_3(t) = b_3(t) e^{-i \left(\frac{(\mathbf{p} + \hbar\mathbf{k}_{L1})^2}{2M} + \hbar\omega_3 \right) t / \hbar}, \quad (2.13c)$$

which is equivalent to the transformation $|\Psi'\rangle = \hat{T}_1 |\Psi\rangle$ with the diagonal matrix

$$\hat{T}_1 = \begin{pmatrix} e^{i \left[\frac{\mathbf{p}^2}{2M} + \hbar\omega_1 \right] t / \hbar} & 0 & 0 \\ 0 & e^{i \left[\frac{(\mathbf{p} + \hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2}))^2}{2M} + \hbar\omega_2 \right] t / \hbar} & 0 \\ 0 & 0 & e^{i \left[\frac{(\mathbf{p} + \hbar\mathbf{k}_{L1})^2}{2M} + \hbar\omega_3 \right] t / \hbar} \end{pmatrix}, \quad (2.14)$$

where $|\Psi'\rangle$ is the wavefunction described by the new amplitudes $b_n(t)$. By Eq. 2.12, the transformed Hamiltonian then takes the more convenient form

$$\hat{H} = \begin{pmatrix} 0 & 0 & \frac{\hbar\Omega_{11}}{2}e^{-i[\Delta t + \phi_{L1}]} \\ 0 & 0 & \frac{\hbar\Omega_{22}}{2}e^{-i[(\Delta + \delta)t - \phi_{L2}]} \\ \frac{\hbar\Omega_{11}^*}{2}e^{i[\Delta t + \phi_{L1}]} & \frac{\hbar\Omega_{22}^*}{2}e^{i[(\Delta + \delta)t - \phi_{L2}]} & 0 \end{pmatrix} \quad (2.15)$$

where Δ and δ are defined overleaf, and if we write out the transformed TDSE for each level we obtain three coupled differential equations describing the time-evolution of the amplitudes of each state:

$$i\hbar \frac{\partial}{\partial t} b_1(t) = \frac{\hbar\Omega_{11}}{2} e^{-i[\Delta t + \phi_{L1}]} b_3(t) \quad (2.16a)$$

$$i\hbar \frac{\partial}{\partial t} b_2(t) = \frac{\hbar\Omega_{22}}{2} e^{-i[(\Delta + \delta)t - \phi_{L2}]} b_3(t) \quad (2.16b)$$

$$i\hbar \frac{\partial}{\partial t} b_3(t) = \frac{\hbar\Omega_{11}^*}{2} e^{i[\Delta t + \phi_{L1}]} b_1(t) + \frac{\hbar\Omega_{22}^*}{2} e^{i[(\Delta + \delta)t - \phi_{L2}]} b_2(t). \quad (2.16c)$$

In the above equations, we have defined the detuning Δ of the lasers ω_{Ln} from *single-photon* resonance with the transition $|n\rangle$ to $|3\rangle$ as

$$\Delta = \left(\frac{(\mathbf{p} + \hbar\mathbf{k}_{L1})^2}{2M\hbar} + \omega_3 \right) - \left(\frac{\mathbf{p}^2}{2M\hbar} + \omega_1 \right) - \omega_{L1}, \quad (2.17)$$

and the detuning δ from the *two-photon* Raman resonance, which is applied to laser ω_{L2} , as

$$\delta = \left(\frac{(\mathbf{p} + \hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2}))^2}{2M\hbar} + \omega_2 \right) - \left(\frac{\mathbf{p}^2}{2M\hbar} + \omega_1 \right) - (\omega_{L2} - \omega_{L1}). \quad (2.18)$$

Whilst Δ is large and therefore relatively insensitive to the momentum terms, δ is much smaller, and is highly sensitive to the atomic momentum. At this point we define the *laser detuning* from the rest-frame resonance as

$$\delta_L = (\omega_1 - \omega_2) - (\omega_{L1} - \omega_{L2}). \quad (2.19)$$

The above transformation represents a move to the *interaction picture* (we began in the Schrödinger picture), in which some of the time evolution of the state amplitudes is ‘shunted’ into the transformation operators.

2.1.4 Effective Two-Level System

In the Raman arrangement the detuning Δ is large, such that $\Delta \gg |\Omega_{11}|, |\Omega_{22}|, |\delta|$, and therefore the population of $|3\rangle$, and the associated spontaneous emission from $|3\rangle$, is negligible. In order to simplify our equations further, we make the assumption that the populations $b_1(t)$ and $b_2(t)$ oscillate (via the Raman transition) at a frequency much smaller than the detuning Δ . In this case, the exponential terms in Eq. 2.16c oscillate much faster than $b_1(t)$ and $b_2(t)$ (and $\Delta \gg \delta$), and we can therefore integrate 2.16c, assuming $b_1(t)$ and $b_2(t)$ to be effectively constant:

$$\begin{aligned} i\hbar \int_{t_0}^{t_0+t} \frac{\partial}{\partial t'} b_3(t') dt' &= i\hbar [-b_3(t_0) + b_3(t_0 + t)] \\ &= i\hbar \left[-b_3(t_0) - \frac{\Omega_{11}^*}{2\Delta} e^{i[\Delta t + \phi_{L1}]} b_1(t) - \frac{\Omega_{22}^*}{2\Delta} e^{i[(\Delta + \delta)t + \phi_{L2}]} b_2(t) \right]. \end{aligned} \quad (2.20)$$

We then substitute this into Eqs. 2.13 which respectively become

$$i\hbar \frac{\partial}{\partial t} b_1(t) = -\frac{\hbar |\Omega_{11}|^2}{4\Delta} b_1(t) - \frac{\hbar \Omega_{11} \Omega_{22}^*}{4\Delta} e^{i(\delta t - \phi_L)} b_2(t) \quad (2.21a)$$

$$i\hbar \frac{\partial}{\partial t} b_2(t) = -\frac{\hbar \Omega_{11}^* \Omega_{22}}{4\Delta} e^{-i(\delta t - \phi_L)} b_1(t) - \frac{\hbar |\Omega_{22}|^2}{4\Delta} b_2(t), \quad (2.21b)$$

$$i\hbar \frac{\partial}{\partial t} b_3(t) = \frac{\hbar \Omega_{11}^*}{2} e^{i[\Delta t + \phi_{L1}]} b_1(t) + \frac{\hbar \Omega_{22}^*}{2} e^{i[(\Delta + \delta)t - \phi_{L2}]} b_2(t), \quad (2.21c)$$

where we have defined the *effective* laser phase $\phi_L = \phi_{L1} + \phi_{L2}$, which is the two-photon analog to the individual laser phase. Equation 2.21c exhibits only fast oscillations $e^{\pm i\Delta t}$ (where $\Delta \gg \delta$) of the intermediate state amplitude $b_3(t)$. These oscillations quickly average out to zero, and can be omitted [2] from the TDSE, leaving an effective two-level system.

We can now write down the simplified, *two-level* Hamiltonian [4]

$$\hat{H} = -\hbar \begin{pmatrix} \Omega_1^{AC} & \frac{\Omega_R}{2} e^{i(\delta t - \phi_L)} \\ \frac{\Omega_R^*}{2} e^{-i(\delta t - \phi_L)} & \Omega_2^{AC} \end{pmatrix}, \quad (2.22)$$

which acts to couple the states $|1, \mathbf{p}\rangle$ and $|2, \mathbf{p} + \hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2})\rangle$, whilst the population in $|3, \mathbf{p} + \hbar\mathbf{k}_{L1}\rangle$ remains zero. In Eq. 2.22, the diagonal terms represent the light shift of the associated level. As stated previously, this treatment has omitted couplings where $a \neq n$, since they act only to alter the light shifts. The full two-level Hamiltonian *including* these terms is given in Appendix A.2, and at this point we re-introduce them in the definition of the light shift of level n :

$$\Omega_n^{AC} = \sum_{a=1,2} \frac{|\Omega_{na}|^2}{4\Delta_{na}}, \quad n = 1, 2. \quad (2.23)$$

These are consistent with the far-detuned light shifts obtained with single-photon couplings in the dressed state picture, as described in Appendix Sect. A.1.3. The *two-photon* Rabi frequency, the rate at which population ‘Rabi flops’ between $|1\rangle$ and $|2\rangle$ on resonance, is defined as

$$\Omega_R = \frac{\Omega_{11}^* \Omega_{22}}{2\Delta}. \quad (2.24)$$

2.1.5 The Dressed State Approach

The time-dependent Schrödinger equation for this effective two-level system can be solved if we shunt the remaining time-dependent terms into transformation operators, and convert to what is known as the *dressed state* picture, as follows. We first apply a uniform shift of $-(\Omega_1^{AC} + \Omega_2^{AC})\hbar/2$ to the energy scale. This is equivalent to a transformation $(|\Psi''\rangle = \hat{T}_2|\Psi'\rangle)$, as in Eqs. 2.11 and 2.12) using the diagonal matrix $\hat{T}_2 = e^{-i(\Omega_1^{AC} + \Omega_2^{AC})t/2}\mathbb{I}$. The transformed Hamiltonian has diagonal elements $\pm\hbar\delta^{AC}/2$, where

$$\delta^{AC} = (\Omega_1^{AC} - \Omega_2^{AC}) \quad (2.25)$$

is the relative light-shift between the two levels. As in Appendix A.1.1, we can remove the time-dependent off-diagonal terms in H if we make a transformation $|\Psi\rangle_R = \hat{R}|\Psi\rangle = |1\rangle_R + |2\rangle_R$ with the *rotation* operator

$$\hat{R} = e^{i\sigma_z\delta t/2} = \begin{pmatrix} e^{-i\delta t/2} & 0 \\ 0 & e^{i\delta t/2} \end{pmatrix}, \quad (2.26)$$

where σ_z is a Pauli spin matrix. For reference, the exponent of a Pauli matrix σ is given by $e^{ia\sigma} = \mathbb{I} \cos a + i\sigma \sin a$. This acts to rotate the frame of reference at a rate δ about the z-axis. Substituting $\hat{R}$ into Eq. 2.12 gives a transformed time-independent Hamiltonian (with known solutions) in the *rotating frame*:

$$\hat{H}_R = -\hbar \begin{pmatrix} \frac{(\delta^{AC} - \delta)}{2} & \frac{\Omega_R}{2} e^{-i\phi_L} \\ \frac{\Omega_R^*}{2} e^{i\phi_L} & -\frac{(\delta^{AC} - \delta)}{2} \end{pmatrix}. \quad (2.27)$$

The eigenvalues of this matrix are $\lambda_{\pm} = \pm \frac{\hbar}{2} \tilde{\Omega}_R$, where we define the *generalised*, off-resonant two-photon Rabi frequency as

$$\tilde{\Omega}_R = \sqrt{|\Omega_R|^2 + (\delta^{AC} - \delta)^2}. \quad (2.28)$$

This gives the angular frequency at which the atom will oscillate between $|1\rangle$ and $|2\rangle$. The procedure for finding the eigenvectors of a matrix of the form $\hat{H}_R$ are given in Appendix A.1.2. Here, we simply write them down as

$$|\lambda_+\rangle = \cos \frac{\Theta}{2} |1\rangle_R e^{-i\phi_L/2} - \sin \frac{\Theta}{2} |2\rangle_R e^{i\phi_L/2} \quad (2.29)$$

$$|\lambda_-\rangle = \sin \frac{\Theta}{2} |1\rangle_R e^{-i\phi_L/2} + \cos \frac{\Theta}{2} |2\rangle_R e^{i\phi_L/2}. \quad (2.30)$$

These are known as the dressed-state eigenvectors, where we have defined

$$\tan \Theta = \frac{-\Omega_r}{(\delta^{AC} - \delta)} \quad (2.31a)$$

$$\cos \Theta = \frac{-(\delta^{AC} - \delta)}{\tilde{\Omega}_R} \quad (2.31b)$$

$$\sin \Theta = \frac{\Omega_R}{\tilde{\Omega}_R}. \quad (2.31c)$$

2.1.6 Solutions to the Two-Level TDSE

Now that we have a time-independent Hamiltonian, for which we have defined energy eigenvalues and eigenvectors, we are able to derive the solutions to the time-dependent Schrödinger equation in the interaction picture. The following describes the procedure from starting-state to solutions, as adapted from [4].

We begin at time t_0 with our effective two-level atom in the state $|\Psi'(t_0)\rangle = c_1(t_0)|1\rangle + c_2(t_0)|2\rangle$, where we have already made the transformation with a two-level version of $\hat{T}_1$. We first make the transformation $|\Psi''(t_0)\rangle = \hat{T}_2|\Psi'(t_0)\rangle$ to antisymmetrise the Hamiltonian as in Sect. 2.1.5. Next we transform $|\Psi''(t_0)\rangle$ to the rotating frame, in which we have defined the eigenvectors $|\lambda_{\pm}\rangle$, by acting on it with the operator $\hat{R}(t_0)$ from Eq. 2.26. The wavefunction in the rotating frame becomes

$$|\Psi(t_0)\rangle_R = c_1(t_0)e^{-i(\Omega_1^{AC} + \Omega_2^{AC})t/2}e^{-i\delta t_0/2}|1\rangle + c_2(t_0)e^{-i(\Omega_1^{AC} + \Omega_2^{AC})t/2}e^{i\delta t_0/2}|2\rangle. \quad (2.32)$$

During an interaction with the laser of duration t , the atom will evolve according to the TDSE with the Hamiltonian $\hat{H}_R$, which can be solved in terms of the time-evolution operator $U(t_0, t)$:

$$|\Psi(t_0 + t)\rangle_R = U(t_0, t)|\Psi(t_0)\rangle_R, \text{ where } U(t_0, t) = e^{-i\hat{H}_R(t-t_0)/\hbar}. \quad (2.33)$$

Given that we have a set of eigenvalues $\lambda_{\pm}$ and eigenvectors $|\lambda_{\pm}\rangle$, which are represented in the eigenvalue equations $\hat{H}_R|\lambda_{\pm}\rangle = \lambda_{\pm}|\lambda_{\pm}\rangle$, a function $f(\hat{H}_R)$ of the Hamiltonian (an operator) will obey the equation $f(\hat{H}_R)|\lambda_{\pm}\rangle = f(\lambda_{\pm})|\lambda_{\pm}\rangle$.

This can be applied to the exponential function $U(t_0, t)$ in Eq. 2.33 along with the closure relation $|\lambda_+\rangle\langle\lambda_+| + |\lambda_-\rangle\langle\lambda_-| = \mathbb{1}$ in our two-eigenvector basis to give

$$\begin{aligned} U(t_0, t) &= e^{-i\hat{H}_R(t-t_0)/\hbar} (|\lambda_+\rangle\langle\lambda_+| + |\lambda_-\rangle\langle\lambda_-|) \\ &= e^{i\lambda_+(t-t_0)/\hbar} |\lambda_+\rangle\langle\lambda_+| + e^{i\lambda_-(t-t_0)/\hbar} |\lambda_-\rangle\langle\lambda_-|. \end{aligned} \quad (2.34)$$

We can then use $\hat{R}^\dagger(t_0 + t)$ and $\hat{T}_2^\dagger(t_0 + t)$ to convert back to the initial basis:

$$|\Psi^\dagger(t_0 + t)\rangle = \hat{T}_2^\dagger(t_0 + t) \hat{R}^\dagger(t_0 + t) |\Psi(t_0 + t)\rangle_R \quad (2.35)$$

From this we obtain the amplitudes $c_1(t_0 + t)$ and $c_2(t_0 + t)$ after an interaction of duration t with the laser field (see, for example [2, 4], or earlier work by Ramsey [5]):

$$\begin{aligned} c_1(t_0 + t) &= e^{i(\Omega_1^{AC} + \Omega_2^{AC})t/2} e^{i\delta t/2} \left(c_2(t_0) e^{i(\delta t_0 - \phi_L)} \left[i \sin \Theta \sin \left(\frac{\tilde{\Omega}_R t}{2} \right) \right] \right. \\ &\quad \left. + c_1(t_0) \left[\cos \left(\frac{\tilde{\Omega}_R t}{2} \right) - i \cos \Theta \sin \left(\frac{\tilde{\Omega}_R t}{2} \right) \right] \right) \end{aligned} \quad (2.36a)$$

$$\begin{aligned} c_2(t_0 + t) &= e^{i(\Omega_1^{AC} + \Omega_2^{AC})t/2} e^{-i\delta t/2} \left(c_2(t_0) \left[\cos \left(\frac{\tilde{\Omega}_R t}{2} \right) + i \cos \Theta \sin \left(\frac{\tilde{\Omega}_R t}{2} \right) \right] \right. \\ &\quad \left. + c_1(t_0) e^{-i(\delta t_0 - \phi_L)} \left[i \sin \Theta \sin \left(\frac{\tilde{\Omega}_R t}{2} \right) \right] \right). \end{aligned} \quad (2.36b)$$

These equations are the work horses of all numerical simulations in this thesis. The amplitudes $c_1(t)$ and $c_2(t)$ can be numerically calculated and squared to give the resultant populations (or probabilities) $|c_1(t)|^2$ and $|c_2(t)|^2$ for any pulse or pulse sequence with a given laser intensity, laser detuning (both Δ and δ), laser phase, atomic dipole coupling strength and atomic momentum. Whilst useful for simulations, Eqs. 2.36 offer little intuitive insight into the population dynamics of coherent Raman pulses. The next section presents a more visually-intuitive approach to coherent transitions, and provides a toolkit for understanding the key coherent pulses in atom interferometry.

2.2 The Bloch Sphere Picture

A useful method for visualisation of coherent pulses, and the topic of this section, is the Bloch sphere picture. It allows us to visually map out pulses as rotations of a state vector around the surface of the unit sphere, and gives us a more intuitive understanding of what occurs during a Raman pulse when, for example, the laser interaction time, phase or detuning is changed. This treatment was adapted by Feynman [6] from early nuclear magnetic resonance work by Rabi et al. [7], and (earlier still) Bloch [8]. A particularly good description of the Bloch sphere picture is given by Shor in [9].

2.2.1 Time Evolution of the Density Matrix

We begin our introduction to the Bloch sphere picture by defining the density matrix $\hat{\rho}$, which describes a many-level quantum system as a statistical ensemble in a mixed state:

$$\hat{\rho} = \sum_{n,m} c_n c_m^* |n\rangle \langle m|, \quad (2.37)$$

where c_n is the amplitude of the pure state $|n\rangle$. In an effective two-level system, the density matrix can be written as

$$\hat{\rho} = |\Psi'\rangle \langle \Psi'| = \begin{pmatrix} |c_1|^2 & c_1 c_2^* \\ c_2 c_1^* & |c_2|^2 \end{pmatrix} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}, \quad (2.38)$$

where we have dropped the time-dependence notation for simplicity. In the density matrix, the off-diagonal terms ρ_{12} and ρ_{21} are known as *coherences*, and represent the level of coherence between the states, which is maximised where $\rho_{nm} \rho_{mn} = \rho_{nn} \rho_{mm}$. The diagonal terms ρ_{11} and ρ_{22} are the *populations*, which are simply the probabilities of the atom being found in the associated state. We can write the TDSE in its *Liouville variant* form:

$$i\hbar \frac{\partial \hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}], \quad (2.39)$$

which for the Hamiltonian in Eq. 2.22 gives the equations of motion for the elements of the density matrix as

$$i\hbar \frac{\partial \rho_{11}}{\partial t} = -\frac{\hbar \Omega_R}{2} e^{i(\delta t - \phi_L)} \rho_{21} + \frac{\hbar \Omega_R^*}{2} e^{-i(\delta t - \phi_L)} \rho_{12} \quad (2.40a)$$

$$i\hbar \frac{\partial \rho_{12}}{\partial t} = -\hbar(\Omega_1^{AC} - \Omega_2^{AC}) \rho_{12} + \frac{\hbar \Omega_R}{2} e^{i(\delta t - \phi_L)} (\rho_{11} - \rho_{22}) \quad (2.40b)$$

$$i\hbar \frac{\partial \rho_{21}}{\partial t} = \hbar(\Omega_1^{AC} - \Omega_2^{AC}) \rho_{21} - \frac{\hbar \Omega_R^*}{2} e^{-i(\delta t - \phi_L)} (\rho_{11} - \rho_{22}) \quad (2.40c)$$

$$i\hbar \frac{\partial \rho_{22}}{\partial t} = -\frac{\hbar\Omega_R^*}{2} e^{-i(\delta t - \phi_L)} \rho_{12} + \frac{\hbar\Omega_R}{2} e^{i(\delta t - \phi_L)} \rho_{21}. \quad (2.40d)$$

To simplify these equations, we transform to the rotating frame by making the substitutions

$$\tilde{c}_1 = c_1 e^{-i\delta t/2} \quad (2.41a)$$

$$\tilde{c}_2 = c_2 e^{i\delta t/2}, \quad (2.41b)$$

which transform the coherences of the density matrix to

$$\tilde{\rho}_{12} = \rho_{12} e^{-i\delta t} \quad (2.42a)$$

$$\tilde{\rho}_{21} = \rho_{21} e^{i\delta t}, \quad (2.42b)$$

and leave the populations unchanged ($\tilde{\rho}_{11} = \rho_{11}$ and $\tilde{\rho}_{22} = \rho_{22}$). We can then re-write Eqs. 2.40 as

$$i\hbar \frac{\partial \tilde{\rho}_{11}}{\partial t} = -\frac{\hbar\Omega_R}{2} e^{-i\phi_L} \tilde{\rho}_{21} + \frac{\hbar\Omega_R^*}{2} e^{i\phi_L} \tilde{\rho}_{12} \quad (2.43a)$$

$$i\hbar \frac{\partial \tilde{\rho}_{12}}{\partial t} = \hbar(\delta - \delta^{AC}) \tilde{\rho}_{12} + \frac{\hbar\Omega_R}{2} e^{-i\phi_L} (\tilde{\rho}_{11} - \tilde{\rho}_{22}) \quad (2.43b)$$

$$i\hbar \frac{\partial \tilde{\rho}_{21}}{\partial t} = -\hbar(\delta - \delta^{AC}) \tilde{\rho}_{21} - \frac{\hbar\Omega_R^*}{2} e^{i\phi_L} (\tilde{\rho}_{11} - \tilde{\rho}_{22}) \quad (2.43c)$$

$$i\hbar \frac{\partial \tilde{\rho}_{22}}{\partial t} = -\frac{\hbar\Omega_R^*}{2} e^{i\phi_L} \tilde{\rho}_{12} + \frac{\hbar\Omega_R}{2} e^{-i\phi_L} \tilde{\rho}_{21}, \quad (2.43d)$$

where we have made the substitution $\delta^{AC} = \Omega_1^{AC} - \Omega_2^{AC}$.

2.2.2 The Optical Bloch Equations

The expectation value of an operator $\hat{A}$ can be written in terms of the density matrix as $\langle \hat{A} \rangle = \text{Tr}(A\rho)$ [10]. Considering the expectation values of the Pauli spin matrices, which are projections of the state vector on to each of the three Cartesian axes x, y and z, we define the variables

$$u = \text{Tr}(\sigma_x \rho) = \tilde{\rho}_{12} + \tilde{\rho}_{21} \quad (2.44a)$$

$$v = \text{Tr}(\sigma_y \rho) = -i(\tilde{\rho}_{21} - \tilde{\rho}_{12}) \quad (2.44b)$$

$$w = \text{Tr}(\sigma_z \rho) = \tilde{\rho}_{11} - \tilde{\rho}_{22}. \quad (2.44c)$$

Eqs. 2.44 are then substituted into Eqs. 2.43 to give three coupled differential equations:

$$\frac{\partial u}{\partial t} = -(\delta - \delta^{AC})v - \frac{i}{2}(\Omega_R e^{-i\phi_L} - \Omega_R^* e^{i\phi_L})w \quad (2.45a)$$

$$\frac{\partial v}{\partial t} = (\delta - \delta^{AC})u + \frac{1}{2}(\Omega_R e^{-i\phi_L} + \Omega_R^* e^{i\phi_L})w \quad (2.45b)$$

$$\frac{\partial w}{\partial t} = -i(\Omega_R^* e^{i\phi_L} \tilde{\rho}_{12} - \Omega_R e^{-i\phi_L} \tilde{\rho}_{21}). \quad (2.45c)$$

These are a simplified form of the *optical Bloch equations* (OBEs). The full, general form incorporates loss due to spontaneous emission, however we choose to neglect such effects here. In the case where Ω_R is real ($\Omega_R = \Omega_R^*$), the OBEs become

$$\frac{\partial u}{\partial t} = -(\delta - \delta^{AC})v - \Omega_R \sin(\phi_L)w \quad (2.46a)$$

$$\frac{\partial v}{\partial t} = (\delta - \delta^{AC})u + \Omega_R \cos(\phi_L)w \quad (2.46b)$$

$$\frac{\partial w}{\partial t} = -\Omega_R \cos(\phi_L)v + \Omega_R \sin(\phi_L)u. \quad (2.46c)$$

2.2.3 The Bloch Vector

We can now define the *Bloch vector* $\mathbf{R}$, of which u , v and w are the amplitudes along x , y and z respectively:

$$\mathbf{R} = u\hat{\mathbf{x}} + v\hat{\mathbf{y}} + w\hat{\mathbf{z}}. \quad (2.47)$$

Differentiating $\mathbf{R}$ gives the time-evolution of the Bloch vector, which can be represented as a *torque* on $\mathbf{R}$ caused by the *field vector* $\mathbf{\Omega}$:

$$\frac{d\mathbf{R}}{dt} = \mathbf{R} \times \mathbf{\Omega}, \quad (2.48)$$

where by inspection we find

$$\mathbf{\Omega} = \Omega_R \cos \phi_L \hat{\mathbf{x}} + \Omega_R \sin \phi_L \hat{\mathbf{y}} - (\delta - \delta^{AC}) \hat{\mathbf{z}}. \quad (2.49)$$

Given that $d\mathbf{R}/dt$ and $\mathbf{\Omega}$ are orthogonal, we can see that $\mathbf{R}$ rotates *around* the axis defined by $\mathbf{\Omega}$, and that (in the absence of any loss processes, such as spontaneous

emission) $\mathbf{R}$ maintains a constant length. $\mathbf{\Omega}$ has horizontal components Ω_R and a vertical component $\delta - \delta^{AC}$, hence it is inclined out of the x-y plane by an angle $\arctan((\delta - \delta^{AC})/\Omega_R)$. Furthermore, we note here that the field vector lies along the eigenvectors given in Eqs. 2.29 and 2.30. The sphere on whose surface the Bloch vector $\mathbf{R}$ exists is known as the *Bloch sphere*. We can also see from Eq. 2.48 that the angular frequency at which $\mathbf{R}$ rotates around $\mathbf{\Omega}$ is given by

$$|\mathbf{\Omega}| = \sqrt{\Omega_R^2 + (\delta - \delta^{AC})^2}, \quad (2.50)$$

which is identical to the generalised, two-photon Rabi frequency $\tilde{\Omega}_R$ given in Eq. 2.28. We can gain some insight into the behaviour of our coherent Raman pulses by plotting the evolution of the Bloch vector as a function of pulse time for a range of pulse parameters. The detuning of the laser with respect to the atomic transition, which we characterise by the term $\delta - \delta^{AC}$, affects the trajectory of the Bloch vector as depicted in Fig. 2.2. A *resonant* pulse ($\delta - \delta^{AC} = 0$), as shown in Fig. 2.2a, exhibits (for $\phi_L = 0$) a field vector along the x axis. The Bloch vector therefore traces out a great circle in the y-z plane at an angular frequency $\tilde{\Omega}_R = \Omega_R$. An *off-resonant* pulse

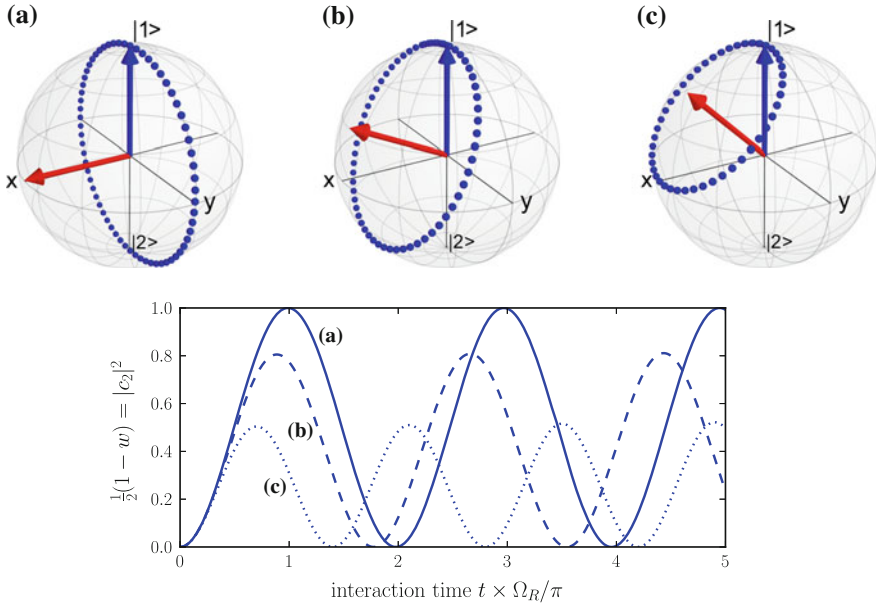


Fig. 2.2 *Top* Bloch sphere plots showing the initial Bloch vector (blue arrow), the field vector (red arrow), and the rotation trajectory of the Bloch vector (blue dots), for **a** a resonant pulse with $\delta - \delta^{AC} = 0$, **b** an off-resonant pulse with $(\delta - \delta^{AC})/\Omega_R = 0.5$, and **c** an off-resonant pulse with $(\delta - \delta^{AC})/\Omega_R = 1.0$. In all cases $\phi_L = 0$. *Bottom* The excited state population as a function of the normalised atom-laser interaction time for the parameters given in the upper plots. **a** solid line, **b** dashed line, and **c** dotted line

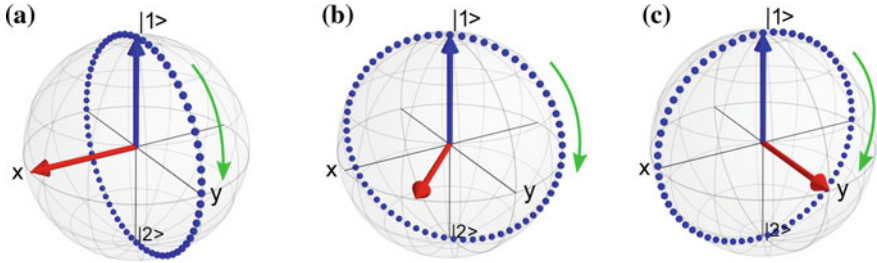


Fig. 2.3 Bloch sphere plots showing the initial Bloch vector (*blue arrow*), the field vector (*red arrow*), and the rotation trajectory of the Bloch vector (*blue dots*), for resonant ($\delta - \delta^{AC} = 0$) pulses with **a** laser phase $\phi_L = 0$, **b** $\phi_L = \pi/4$, and **c** $\phi_L = \pi/2$. The *green arrow* in each case shows the direction of rotation

($|\delta - \delta^{AC}| > 0$) is characterised by a field vector which does not lie in the x-y plane. In this case the Bloch vector traces out a tilted, *small* circle on the Bloch sphere as shown in Fig. 2.2b, c, at an increased angular frequency $\tilde{\Omega}_R = \sqrt{\Omega_R^2 + (\delta - \delta^{AC})^2}$. It is useful to note that the *linear speed* along the sphere's surface remains the same, as defined by Ω_R , regardless of the detuning. The lower plot in the figure shows the normalised temporal evolution of $(1 - w)/2$, which is equal to the population in state $|2\rangle$, corresponding to the upper plots. We can see that only in the on-resonance case can we achieve complete population inversion.

Bloch vector trajectories for different values of the Raman beam phase ϕ_L on resonance are shown in Fig. 2.3. Altering ϕ_L simply acts to rotate the field vector around the z-axis by the angle ϕ_L . We can make the Bloch vector rotate in the x-z plane by setting $\phi_L = \pi/2$, for example, and we can reverse its direction of rotation by shifting ϕ_L by π . This freedom to control the axis of rotation is essential in composite pulses (Chap. 6) and atom interferometry (Chap. 7) experiments.

2.2.4 The Coherent Pulse Toolkit

To end this section, we introduce the key operations in atom interferometry. By tailoring the duration t of our coherent pulses we are able to achieve rotations by well-defined angles around the Bloch sphere. The two most useful rotations in atom interferometry are the *resonant* $\pi/2$ and π , which are shown in Fig. 2.4a, b respectively. To achieve a $\pi/2$ pulse, we set $\Omega_R t = \pi/2$. This gives rise to a rotation by an angle $\pi/2$ around the field vector, and (for a state initially at the pole) induces an equal superposition of the two atomic states. Since the two states constituting this superposition have different momenta and therefore follow different paths in space, the $\pi/2$ pulse is analogous to a *beam splitter* in an optical interferometer. To achieve a π pulse, we set $\Omega_R t = \pi$. This induces population inversion, which is represented by a π rotation around the field vector. The π pulse is analogous to a *mirror* in an

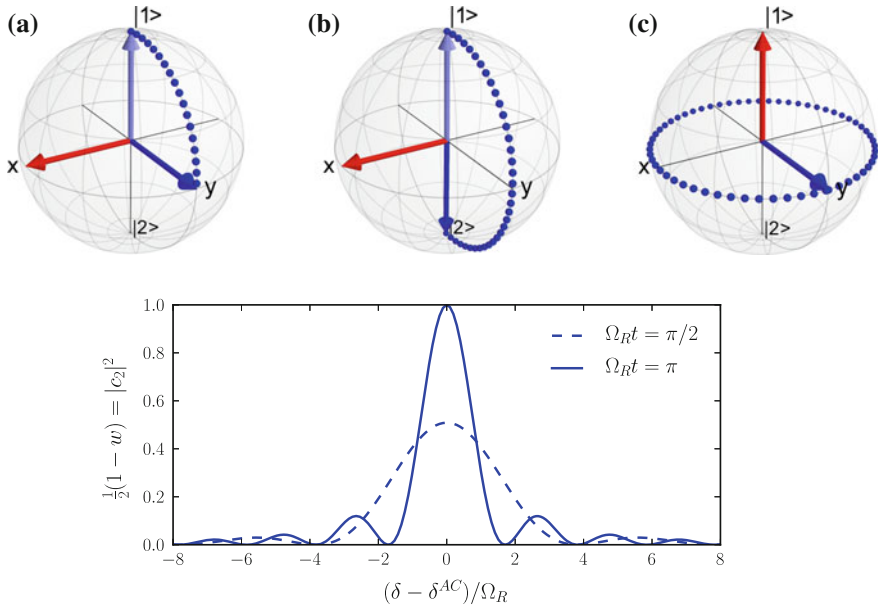


Fig. 2.4 Top Bloch sphere plots showing the three main operations in atom interferometry. Plots (a) and (b) show a $\pi/2$ and π pulse, respectively, with the field vector (red arrow), the initial/final Bloch vectors (light/dark blue arrows), and the Bloch vector trajectory (blue dots). Plot (c) shows *Free evolution* where $\Omega_R = 0$ and the Bloch vector precesses about the z-axis. Bottom Resultant excited state population as a function of the normalised detuning parameter for a $\pi/2$ pulse (dashed line) and a π pulse (solid line)

optical interferometer, since it physically deflects the atomic wavepacket, and does so in opposite directions for the two states, as if approaching a mirror that bisects the angle between them. The third and final operation in the toolkit is *free evolution*, which is simply the evolution of the Bloch vector in the absence of any driving field, as shown in Fig. 2.4c. If the detuning $\delta - \delta^{AC}$ is non-zero, the Bloch vector will precess around the z-axis at a rate equal to this detuning.

The lower plot in the figure shows the population in state $|2\rangle$ as a function of the *normalised* detuning parameter $(\delta - \delta^{AC})/\Omega_R$, which may be non-zero due to a non-zero velocity or laser detuning, for pulse durations corresponding to a resonant $\pi/2$ and a resonant π pulse. At zero detuning, we find their excited state populations $|c_2|^2$ to be 0.5 and 1.0, respectively. The curves exhibit a ‘sinc-squared’ line-shape, whose spectral width ($\text{FWHM} = \pi/2$) is linearly dependent on the two-photon Rabi frequency Ω_R . The increase in spectral width with increasing Ω_R is known as *power-broadening*. These key operations will be combined in the next section, which discusses velocity-selective atom interferometry.

2.3 Velocity-Selective Atom Interferometry

This toolkit of coherent pulses allows the construction of a light-pulse atom interferometer. The interferometer schemes of interest in this work are those which are velocity-sensitive, that is, where the phase accrued between the two paths in the interferometer depends on the initial velocity of the atom. One example of this is the most basic interferometer sequence: $\frac{\pi}{2} - \frac{\pi}{2}$. This is commonly referred to as the *Ramsey* sequence, since it is inspired by Ramsey's early work on his method of separated oscillatory fields [5]. In the following we give a short treatment of the phase accrued within this interferometer, and discuss its dependence on the atomic velocity.

2.3.1 The Ramsey Interferometer

This sequence is summarised in Fig. 2.5. We begin with an atom in the state $|1, \mathbf{p}\rangle$, such that $c_1(t_0) = 1$ and $c_2(t_0) = 0$. A Raman $\frac{\pi}{2}$ pulse of length $\tau = \frac{\pi}{2\Omega_R}$ at $t_0 = 0$ puts the atom into an equal superposition of the states $|1, \mathbf{p}\rangle$ and $|2, \mathbf{p} + \hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2})\rangle$.

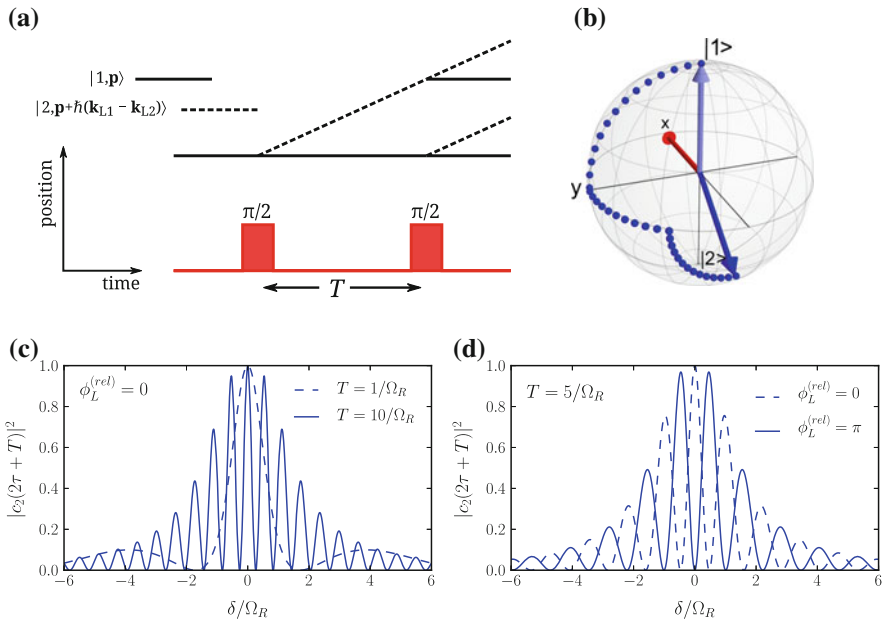


Fig. 2.5 The $\frac{\pi}{2} - \frac{\pi}{2}$ (Ramsey) interferometer. **a** Wavepacket position-time diagrams for the interferometer, in the absence of external potentials. **b** An example Bloch sphere representation of the $\frac{\pi}{2} - \frac{\pi}{2}$ sequence, in which the phase of both pulses is $\phi_L = 0$. The light (dark) blue arrow is the initial (final) state vector, and the red arrow is the field vector. **c** Resultant upper state populations (Eq. 2.55) as a function of detuning δ for fixed relative pulse phase and different pulse spacings T , given in the legend. **d** As (c), with fixed T and different relative phases

This pulse has an effective phase $\phi_L^{(1)}$. Application of Eqs. 2.36 to find the state at the end of the $\frac{\pi}{2}$ yields

$$\begin{aligned} c_1(\tau) &= e^{i(\Omega_1^{AC} + \Omega_2^{AC})\tau/2} e^{i\delta\tau/2} \frac{1}{\sqrt{2}} (1 - i \cos \Theta) \\ c_2(\tau) &= e^{i(\Omega_1^{AC} + \Omega_2^{AC})\tau/2} e^{-i\delta\tau/2} \frac{1}{\sqrt{2}} \left(e^{i\phi_L^{(1)}} i \sin \Theta \right). \end{aligned} \quad (2.51)$$

The two coherent components of this superposition subsequently diverge in space, since they have different momenta. After the initial pulse, the atom is left to evolve for a time T with the Raman interaction switched off. During this period of free evolution $\Omega_1 = \Omega_2 = \Omega_R = 0$, hence $\tilde{\Omega}_R = -\delta$, $\cos \Theta = -1$ and $\sin \Theta = 0$ (the light-shift terms also disappear), and the Bloch vector precesses around the z-axis at the angular frequency δ . The state amplitudes become

$$\begin{aligned} c_1(\tau + T) &= e^{i\delta\tau/2} c_1(\tau) \left[\cos\left(\frac{-\delta T}{2}\right) + i \sin\left(\frac{-\delta T}{2}\right) \right] = c_1(\tau) \\ c_2(\tau + T) &= e^{-i\delta\tau/2} c_2(\tau) \left[\cos\left(\frac{-\delta T}{2}\right) - i \sin\left(\frac{-\delta T}{2}\right) \right] = c_2(\tau), \end{aligned} \quad (2.52)$$

and we find, as expected since the state eigenenergies have been factored out in the interaction picture, that the amplitudes are unchanged during free evolution. Finally, another $\frac{\pi}{2}$ pulse is applied, with a laser phase $\phi_L^{(2)}$, after which the amplitude of the upper state $|2, \mathbf{p} + \hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2})\rangle$ is

$$\begin{aligned} c_2(2\tau + T) &= e^{i(\Omega_1^{AC} + \Omega_2^{AC})\tau} e^{-i\delta\tau} \\ &\quad \times \frac{1}{2} \sin \Theta \left[e^{i\phi_L^{(1)}} (i - \cos \Theta) + e^{i\phi_L^{(2)}} e^{-i\delta T} (i + \cos \Theta) \right]. \end{aligned} \quad (2.53)$$

We can simplify this by defining a *relative* phase $\phi_L^{(\text{rel})} = \phi_L^{(2)} - \phi_L^{(1)}$ between the two $\frac{\pi}{2}$ pulses, and taking $e^{i\phi_L^{(1)}}$ outside the square brackets:

$$\begin{aligned} c_2(2\tau + T) &= e^{i(\Omega_1^{AC} + \Omega_2^{AC})\tau} e^{-i\delta\tau} e^{-i\delta T/2} e^{i\phi_L^{(1)}} \\ &\quad \times \frac{1}{2} \sin \Theta \left[(i - \cos \Theta) + e^{-i(\delta T - \phi_L^{(\text{rel})})} (i + \cos \Theta) \right]. \end{aligned} \quad (2.54)$$

To clarify, this relative phase is distinct from the effective phase introduced in Sect. 2.1.4. The amplitude $c_2(2\tau + T)$ is squared to give the population of the state

$$|c_2(2\tau + T)|^2 = \sin^2 \Theta \left[\cos\left(\frac{\delta T - \phi_L^{(\text{rel})}}{2}\right) - \cos \Theta \sin\left(\frac{\delta T - \phi_L^{(\text{rel})}}{2}\right) \right]^2, \quad (2.55)$$

which in the case where the laser is close to resonance ($\delta^{AC} - \delta \ll \Omega_R$, and therefore $\cos \Theta \simeq 0$ and $\sin \Theta \simeq 1$) becomes

$$|c_2(2\tau + T)|^2 \simeq \frac{1}{2} \left[1 + \cos(\delta T - \phi_L^{(\text{rel})}) \right]. \quad (2.56)$$

The resultant upper state population follows an oscillation whose period is dependent on the time T between the $\frac{\pi}{2}$ pulses, as illustrated in Fig. 2.5c where we plot Eq. 2.55 as a function of the detuning δ for a fixed relative laser phase. We find an increased period at smaller values of T . Figure 2.5d shows the effects of the relative phases of the two $\frac{\pi}{2}$ pulses. Shifting $\phi_L^{(\text{rel})}$ by π simply shifts the maxima of the Ramsey pattern along by half of one oscillation period. In both Fig. 2.5c, d the light-shift $\delta^{AC} = 0$. The curves are enclosed within a Gaussian envelope, which we can shift left or right by making δ^{AC} non-zero. It is important to note that an interference pattern is only observed if the two wavepackets are still spatially overlapping as the second pulse is applied. The spatial extent of a wavepacket, otherwise known as its *coherence length*, is limited by the position-momentum uncertainty principle $l_{\text{coh}} = \hbar/2\Delta p \geq \Delta z$, and therefore fringes are observed if the momentum-spread Δp of the wavepacket is sufficiently small.

2.3.2 Velocity-Dependent Phase

As we have seen in Fig. 2.5, the output state of the interferometer, as determined by the interferometer *phase*, depends on the value δ attributed to the atom. Returning to the definition of δ given in Eq. 2.18, we can write the momentum-inclusive detuning as

$$\delta = -\delta_L + \frac{\mathbf{p} \cdot (\mathbf{k}_{L1} - \mathbf{k}_{L2})}{M} + \frac{\hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2})^2}{2M}. \quad (2.57)$$

The first term on the right hand side is, as described, the laser detuning. The second is the velocity-dependent, or *Doppler shift* term which can be written as $\mathbf{v} \cdot (\mathbf{k}_{L1} - \mathbf{k}_{L2})$ where $\mathbf{v}$ is the atomic velocity. For a cold atom moving at 50 mm/s and interacting with counter-propagating ($\mathbf{k}_{L1} = -\mathbf{k}_{L2}$) 780 nm beams, this term is equal to around $2\pi \times 128$ kHz. It is useful to define at this stage the *Raman recoil velocity*,² i.e. the change in atomic velocity upon undergoing the Raman transition, as

$$\mathbf{v}_R = \frac{\hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2})}{M}. \quad (2.58)$$

²As opposed to the single-photon recoil velocity, which is half of this value.

We can therefore label the third term in Eq. 2.57 as the Raman *recoil shift* $M\mathbf{v}_R^2/2\hbar$, which for a rubidium-85 atom (mass number 85) interacting again with 780 nm light is approximately $2\pi \times 15.4$ kHz. The corresponding Raman recoil velocity is $v_R = 12$ mm/s.

We can see from the above that the output state of the $\frac{\pi}{2} - \frac{\pi}{2}$ interferometer is dependent on the initial velocity of the atom. Since the output state determines whether or not the atom has absorbed a photon (or two in the case of Raman transitions), this represents a velocity-dependent *impulse*. Chapter 7 explores this principle, and its use in engineering atomic *cooling* forces.

2.4 Rubidium-85

In the previous sections we have discussed Raman transitions and interferometry based on a system of three atomic levels, namely $|1, \mathbf{p}\rangle$, $|3, \mathbf{p} + \hbar\mathbf{k}_{L1}\rangle$ and $|2, \mathbf{p} + \hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2})\rangle$. This section describes an atomic system of such form in rubidium-85, which we use in experiments in the lab.

We begin this section with a brief summary of atomic structure. Note that detailed explanations can be found in many textbooks, including Foot [11] and Atkins [12].

2.4.1 Atomic Structure

We describe the state of an atom by its corresponding set of *quantum numbers*. It is important to note that here we consider only the (hydrogen-like) alkali metals, which exhibit a single outer electron. Ignoring the electron spin for now, the necessary quantum numbers are n , l and m_l , and the electron wavefunction, described by these quantum numbers, can be written as the product of a radial and an angular part:

$$\psi_{n,l,m_l}(r, \theta, \phi) = R_{n,l}(r)Y_{l,m_l}(\theta, \phi). \quad (2.59)$$

The radial part of the wavefunction is represented by the Laguerre polynomials $R_{n,l}(r)$. The *principal* quantum number $n = 1, 2, 3, \dots$, which originates from the Bohr model of the atom, specifies the orbital ‘shell’ in which the outermost electron is situated, and describes the electronic energy according to $E_n \propto 1/n^2$.

The angular part of the wavefunction is described by the spherical harmonics $Y_{l,m_l}(\theta, \phi)$, which satisfy the simultaneous eigenvalue equations $\mathbf{l}^2 Y_{l,m_l} = l(l+1)\hbar^2 Y_{l,m_l}$ and $l_z Y_{l,m_l} = \hbar m_l Y_{l,m_l}$, where $\mathbf{l}$ is the angular momentum operator, whose z-component (where we arbitrarily choose z as the quantisation axis) is $l_z = -i\hbar \frac{\partial}{\partial \phi}$. The *orbital angular momentum* quantum number, l , can take the values $l = 0, 1, 2, \dots, n-1$; the *magnetic* quantum number, m_l , describes the projection of l along the z-axis, and can have the values $m_l = -l, -l+1, \dots, l$.

The electronic *spin* angular momentum s , described by the quantum numbers $s = \frac{1}{2}$ and $m_s = \pm\frac{1}{2}$ (electrons are spin- $\frac{1}{2}$ particles), gives rise to an intrinsic electron magnetic moment $\mu_s = -\mu_B g_s \mathbf{s}/\hbar$, where μ_B is the Bohr magneton and $g_s \approx 2$ is the electron g-factor. The interaction between μ_s and the magnetic field created by the orbiting electron is known as the *spin-orbit interaction*. For the alkalis, the *total* orbital and spin angular momentum quantum numbers, L and S respectively, are equal to their single electron counterparts, that is, $L = l$ and $S = s$. The spin-orbit interaction leads to the definition of the total electronic angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$, where the quantum numbers J and m_J , which respectively describe the total electronic angular momentum and its projection along the z axis, take the values $J = |L - S|, |L - S + 1|, \dots, L + S$, and $m_J = -J, -J + 1, \dots, J$. The spin-orbit interaction acts to split the levels into states separated typically by many THz, giving the atom so-called *fine structure*.

The atomic nucleus also exhibits spin angular momentum, $\mathbf{I}$, specified by the quantum number I . This gives rise to an intrinsic nuclear magnetic moment, which in turn interacts with the magnetic fields generated by the electron. In light of this we define the total angular momentum $\mathbf{F} = \mathbf{J} + \mathbf{I}$, which we specify by the total atomic angular momentum quantum number $F = |J - I|, |J - I + 1|, \dots, J + I$ and its magnetic counterpart $m_F = -F, -F + 1, \dots, F$. These interactions with the nuclear spin give rise to energy level structure typically on the scale of GHz, known as *hyperfine structure*. In our experiments, we induce coherent transitions between levels within this hyperfine structure.

Each hyperfine state F is split into $2F + 1$ magnetic-sensitive substates labelled m_F . We refer to these sublevels as *Zeeman* sublevels, since the magnetic fields considered in our experiments induce Zeeman-like (weak field) splitting. The energy shift of a Zeeman sublevel m_F due to an external magnetic field of magnitude B is given by

$$\Delta E_{\text{Zeeman}} = \mu_B g_F m_F B, \quad (2.60)$$

where μ_B is the Bohr magneton and g_F is the Landé g-factor, which can be measured experimentally.

The set of quantum numbers given in the above allows us to write the full spectroscopic notation, or term symbol, for an atomic state as

$$n^{2S+1}L_J(F, m_F). \quad (2.61)$$

Conventionally we replace the numbers $L = 0, 1, 2, \dots$ with the letters $S, P, D, \dots$, giving rise to the notion of ‘s-orbitals’, ‘p-orbitals’, etc.

2.4.1.1 Rubidium-85 Structure

The structure of rubidium-85 (^{85}Rb , atomic number $Z = 37$), relevant to the experiments in this work, is shown in Fig. 2.6. This particular sub-section of the structure is known as the rubidium D_2 line, which is represented by fine structure transitions

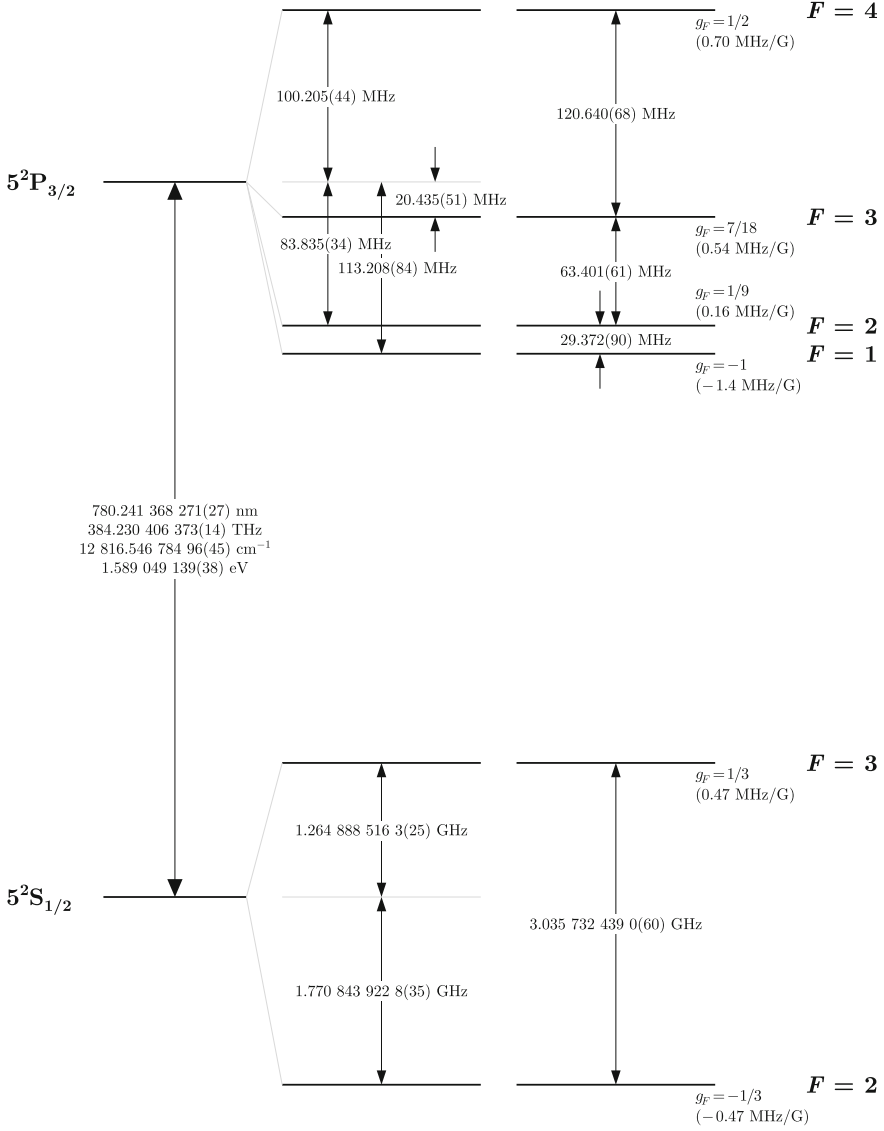


Fig. 2.6 (Taken from [13]) Rubidium 85 energy level diagram

within the manifold defined by the levels $|5^2S_{1/2}\rangle$ and $|5^2P_{3/2}\rangle$, and can be addressed by tuning our laser to around 780 nm. Within these two levels, the nuclear ($I = 5/2$) and electronic (lower, upper $J = 1/2, 3/2$ respectively) angular momenta interact to give a hyperfine level splitting defined by the quantum numbers F and m_F as described previously. As shown in the figure, the lower level is split into two hyperfine states $|5^2S_{1/2}, F = 2, 3\rangle$, separated by 3.036 GHz, and the upper level is split

into four $|5^2P_{3/2}, F = 1, 2, 3, 4\rangle$, separated by smaller intervals. In our experiments, we drive Raman transitions between the levels $|5^2S_{1/2}, F = 2\rangle$ and $|5^2S_{1/2}, F = 3\rangle$ which are respectively split into 5 and 7 magnetic-sensitive Zeeman sublevels. The Landé g-factors for each hyperfine level are given in Fig. 2.6, along with the coefficient $\mu_B g_F$, which is quoted in MHz/G.

2.4.2 Dipole Transitions

Broadly speaking, dipole transitions between atomic levels occur when the frequency of an applied alternating electric field coincides with an inter-level frequency interval, or atomic *resonance*. They obey selection rules governed by the conservation of angular momentum, which to some extent we are able to visualise in terms of an *overlap* between the atomic wavefunctions. For a dipole transition from state $|\alpha, F, m_F\rangle$ to $|\alpha', F', m'_F\rangle$, where α describes all quantum numbers other than F and m_F , the coupling strength, or Rabi frequency, is given by the overlap integral

$$\Omega_{|F, m_F\rangle, |F', m'_F\rangle}^{(a)} = -\frac{1}{\hbar} \langle \alpha', F', m'_F | \mathbf{d} \cdot \mathbf{E}_a | \alpha, F, m_F \rangle, \quad (2.62)$$

where $\mathbf{E}_a$ ($a = 1, 2$ in the Raman system indicates the particular beam) is the electric field vector, and $\mathbf{d} = e\mathbf{r}$ is the atomic *dipole moment*, as determined by the electron's charge e and position $\mathbf{r}$ relative to the nucleus. We can decompose both $\mathbf{r}$ and $\mathbf{E}_a$ into irreducible components within the spherical basis defined by the unit vectors

$$\hat{\mathbf{e}}_{+1} = -(\hat{\mathbf{x}} + i\hat{\mathbf{y}})/\sqrt{2}, \quad (2.63a)$$

$$\hat{\mathbf{e}}_0 = \hat{\mathbf{z}}, \quad (2.63b)$$

$$\hat{\mathbf{e}}_{-1} = (\hat{\mathbf{x}} - i\hat{\mathbf{y}})/\sqrt{2}, \quad (2.63c)$$

in which the scalar product is given by (Appendix F in [14])

$$\mathbf{d} \cdot \mathbf{E}_a = eE_a \sum_q (-1)^q r_q \epsilon_{-q}, \quad (2.64)$$

where $q = -1, 0, 1$ and ϵ_q describe the polarisation of the electric field, in order to re-write Eq. 2.62 as

$$\Omega_{|F, m_F\rangle, |F', m'_F\rangle}^{(a)} = -\frac{eE_a}{\hbar} \sum_q (-1)^q \epsilon_{-q} \langle J', F', m'_F | r_q | J, F, m_F \rangle. \quad (2.65)$$

We are able to simplify this by application of the Wigner-Eckart theorem, which states that for an irreducible tensor operator T_q^k , the overlap integral

$\langle \alpha', F', m'_F | T_q^k | \alpha, F, m_F \rangle$, where α describes all quantum numbers other than F and m_F , is proportional to the Clebsch-Gordan coefficient

$$\langle F', m'_F | F, k, m_F, q \rangle = (-1)^{F'-k+m_F} \sqrt{2F+1} \begin{pmatrix} F' & k & F \\ m'_F & q & -m_F \end{pmatrix} \quad (2.66)$$

in which k and q depend on the properties of the operator and the term in parentheses is a Wigner 3-j symbol [15], with a constant of proportionality $\langle \alpha', F' \| T^k \| \alpha, F \rangle$, known as the *reduced* matrix element. The reduced matrix element is independent of m_F and q , and we are able to factor out its dependence on F , leaving a reduced matrix element that depends only on the initial and final n , L , and J quantum numbers by writing

$$\begin{aligned} \langle \alpha', F' \| T^k \| \alpha, F \rangle &= \langle \alpha', J' \| T^k \| \alpha, J \rangle (-1)^{F'+J+k+I} \\ &\times \sqrt{(2F'+1)(2J+1)} \begin{Bmatrix} J & J' & k \\ F' & F & I \end{Bmatrix}, \end{aligned} \quad (2.67)$$

in which the term in braces is a Wigner 6-j symbol. Since it is a vector, the position operator $\hat{\mathbf{r}}$ has rank $k = 1$, hence by applying the Wigner-Eckart theorem to Eq. 2.65 we arrive at the final form for the Rabi frequency

$$\Omega_{|F, m_F\rangle, |F', m'_F\rangle}^{(a)} = -\frac{eE_a}{\hbar} \langle J' \| \mathbf{d} \| J \rangle \sum_q (-1)^q \epsilon_{-q} G(F', F, J', J, I, m'_F, m_F, q). \quad (2.68)$$

where we have separated the experimental, empirically soluble parameters from the purely geometrical term

$$\begin{aligned} G(F', F, J', J, I, m'_F, m_F, q) &= (-1)^{2F'+J+I+m_F} \sqrt{(2F+1)(2F'+1)(2J+1)} \times \\ &\begin{pmatrix} F' & 1 & F \\ m'_F & q & -m_F \end{pmatrix} \begin{Bmatrix} J & J' & 1 \\ F' & F & I \end{Bmatrix}. \end{aligned} \quad (2.69)$$

The Wigner 3-j and 6-j symbols can be calculated using the `ThreeJSymbol` and `SixJSymbol` functions in Mathematica [16], and the reduced dipole matrix element for the $5S_{1/2} \rightarrow 5P_{3/2}$ transition as found in Steck's alkali D line data [13] (along with tabulated values of G) is $3.58425(74) \times 10^{-29}$ C m.

The 3-j and 6-j symbols in the above equations impose a neat statement of the selection rules based on the associated transition quantum numbers: the product of the symbols is equal to zero unless:

1. $\Delta J = J' - J = 0, \pm 1$ except for $J = 0 \rightarrow J' = 0$.
2. $\Delta F = F' - F = 0, \pm 1$ except for $F = 0 \rightarrow F' = 0$.
3. $\Delta m_F = m'_F - m_F = -q$.

These selection rules reflect angular momentum conservation. The angular momentum of the photon is added to (subtracted from) the atom in any absorption (emission) event, and this imposes restrictions on the types of transitions allowed, dependent on the photon polarisation and the initial and final m_F states. A *linearly* polarised photon whose wavevector is perpendicular to the quantisation axis ($\mathbf{k} \perp \hat{\mathbf{e}}_0$) and whose polarisation vector is parallel to it ($\epsilon \parallel \hat{\mathbf{e}}_0$) drives a π^0 transition, whereby $\Delta m_F = 0$. A *circularly* polarised photon whose wavevector is *parallel* to the quantisation axis ($\mathbf{k} \parallel \hat{\mathbf{e}}_0$, moving in the positive $\hat{\mathbf{e}}_0$ direction) and whose polarisation vector rotates counter-clockwise as observed from the receiver is denoted LCP (left-circularly-polarised, helicity $+1$), and drives σ^+ transitions, whereby $\Delta m_F = +1$. This is represented by the $q = -1$ component of Eq. 2.68. Similarly, an RCP (right-circularly-polarised, helicity -1) photon, *ceteris paribus*, drives σ^- transitions, whereby $\Delta m_F = -1$, as represented by the $q = 1$ component of Eq. 2.68. A good description of the circular polarisation and corresponding helicity states is given in [17].

Furthermore, a linearly polarised photon whose electric field vector is *perpendicular* to $\hat{\mathbf{e}}_0$ (i.e. along $\hat{\mathbf{x}} = -(\hat{\mathbf{e}}_{+1} - \hat{\mathbf{e}}_{-1})/\sqrt{2}$ or $\hat{\mathbf{y}} = i(\hat{\mathbf{e}}_{+1} + \hat{\mathbf{e}}_{-1})/\sqrt{2}$) drives an equal superposition of σ^+ and σ^- transitions. These are labelled $\pi^+ = (\sigma^+ + \sigma^-)/\sqrt{2}$ for polarisation parallel to $\hat{\mathbf{y}}$ and $\pi^- = (\sigma^+ - \sigma^-)/\sqrt{2}$ for polarisation parallel to $\hat{\mathbf{x}}$. All of the above polarisation states and their associated labels and m_F selection rules are summarised in Table 2.1.

2.4.3 Raman Transitions

In order to create a Raman system in ^{85}Rb we require two laser beams at $780.243 + \frac{2\pi c}{\Delta}$ nm, separated in frequency by the lower hyperfine splitting $\omega_{\text{HFS}} = 2\pi \times 3.036$ GHz. With this we can drive Raman transitions between $|5S_{1/2}, F = 2\rangle = |1, \mathbf{p}\rangle$ and $|5S_{1/2}, F = 3\rangle = |2, \mathbf{p} + \hbar(\mathbf{k}_{L1} - \mathbf{k}_{L2})\rangle$ via the upper states $|5P_{3/2}, F'\rangle = |3, \mathbf{p} + \hbar\mathbf{k}_{L1}\rangle$, provided that the detuning Δ from single-photon resonance is large. The coupling strength for the Raman transition (Eq. 2.24) is determined by the prod-

Table 2.1 Table showing the possible polarisation states of the driving field photon, and their associated dipole transitions, where $\hat{\mathbf{e}}_0 = \hat{\mathbf{z}}$ is the quantisation axis of the transition, and ϵ is the polarisation vector of the photon

Polarisation	Transition	Δm_F
Left circular (LCP) $\mathbf{k} \parallel \hat{\mathbf{e}}_0$	σ^+	$+1$
Right circular (RCP) $\mathbf{k} \parallel \hat{\mathbf{e}}_0$	σ^-	-1
Linear $\epsilon \parallel \hat{\mathbf{e}}_0$ ($\epsilon \parallel \hat{\mathbf{z}}$)	π^0	0
Linear $\epsilon \parallel \hat{\mathbf{y}}$	$\pi^+ = (\sigma^+ + \sigma^-)/\sqrt{2}$	± 1
Linear $\epsilon \parallel \hat{\mathbf{x}}$	$\pi^- = (\sigma^+ - \sigma^-)/\sqrt{2}$	± 1

For the circularly polarised cases, the photon is considered to be moving in the positive $\hat{\mathbf{e}}_0$ direction

uct of the two single-photon transition coupling strengths, and by inspection of the dipole selection rules given in the previous section, we find that Raman transitions may occur via more than one upper F' hyperfine level. This makes the problem of describing the system rather more complex, however a useful formalism for describing Raman transitions via multiple atomic levels, which we adopt here, is given by Bateman et al. in [18]. We discuss the general concepts of this formalism in the following.

2.4.3.1 Vector Formalism for Multiple Raman Routes

In the regime where the hyperfine splitting of the upper state levels is much smaller than the single-photon detuning Δ we can make the approximation $\Delta_{|F,m_F\rangle,|1,m'_F\rangle} \approx \Delta_{|F,m_F\rangle,|2,m'_F\rangle} \approx \dots \approx \Delta$, and treat all transitions as having the same Δ .

Considering the system consisting of two lower and four upper hyperfine levels ($F = 2, 3$ and $F' = 1, 2, 3, 4$), we can write the single-photon couplings (Eq. 2.62) associated with the two Raman beams as components of two vectors:

$$\mathbf{\Omega}_{1a} = \left(\Omega_{|2,m_F\rangle,|1,m'_F\rangle}^{(a)}, \Omega_{|2,m_F\rangle,|2,m'_F\rangle}^{(a)}, \Omega_{|2,m_F\rangle,|3,m'_F\rangle}^{(a)}, \Omega_{|2,m_F\rangle,|4,m'_F\rangle}^{(a)} \right) \quad (2.70a)$$

$$\mathbf{\Omega}_{2a} = \left(\Omega_{|3,m_F\rangle,|1,m'_F\rangle}^{(a)}, \Omega_{|3,m_F\rangle,|2,m'_F\rangle}^{(a)}, \Omega_{|3,m_F\rangle,|3,m'_F\rangle}^{(a)}, \Omega_{|3,m_F\rangle,|4,m'_F\rangle}^{(a)} \right). \quad (2.70b)$$

Next, the two-photon Rabi frequency is re-written for the system of multiple upper levels as the scalar product³ of these two vectors where $a = n$

$$\Omega_R = \frac{|\mathbf{\Omega}_{11} \cdot \mathbf{\Omega}_{22}|}{2\Delta}, \quad (2.71)$$

(we omitted the terms Ω_{an} with $a \neq n$ earlier due to their fast oscillatory nature) and this can be expressed in terms of the reduced matrix element, the electric field strengths, and the detuning Δ as

$$\Omega_R = \frac{|\langle J' || \mathbf{d} || J \rangle|^2}{2\Delta\hbar^2} E_1^* E_2 |\mathbf{G}_{11} \cdot \mathbf{G}_{22}|, \quad (2.72)$$

where the vectors $\mathbf{G}_{na}$ constitute the corresponding single-photon G coefficients (see Eq. 2.69) for coupling of level n by laser a . In this formalism, we have separated the experimental parameters from the purely geometrical term $|\mathbf{G}_{11} \cdot \mathbf{G}_{22}|$, which describes the dipole matrix elements of the Raman transition.

By inspecting Eqs. 2.70 and considering the selection rule $\Delta F = 0, \pm 1$, we can see that both the fourth component of $\mathbf{\Omega}_{11}$ and the first component of $\mathbf{\Omega}_{22}$ (and

³Note that this vector formalism is a subset of a more general *tensor* notation for transitions between multiple levels, and for systems with larger numbers of levels, tensor products must be considered when calculating the coupling strengths.

their corresponding components in $\mathbf{G}_{11,22}$) are equal to zero. This makes the term $|\mathbf{G}_{11} \cdot \mathbf{G}_{22}|$ only dependent on the middle two components, i.e. our Raman transitions occur via $F' = 2, 3$.

This vector formalism is also useful for expressing light-shifts in the Raman system. In order to incorporate the effect due to all upper levels we can re-write the light shift term as

$$\delta^{AC} = \sum_{a=1,2} \left(\frac{\|\boldsymbol{\Omega}_{1a}\|^2}{4\Delta_{1a}} - \frac{\|\boldsymbol{\Omega}_{2a}\|^2}{4\Delta_{2a}} \right), \quad (2.73)$$

where we have used double-bars to denote the vector norm.

Finally, for completeness we express the generalised, off-resonant two-photon Rabi frequency in the vector formalism as

$$\tilde{\Omega}_R = \sqrt{\left(\frac{|\boldsymbol{\Omega}_{11} \cdot \boldsymbol{\Omega}_{22}|}{2\Delta} \right)^2 + \left[\sum_{a=1,2} \left(\frac{\|\boldsymbol{\Omega}_{1a}\|^2}{4\Delta_{1a}} - \frac{\|\boldsymbol{\Omega}_{2a}\|^2}{4\Delta_{2a}} \right) - \delta \right]^2}. \quad (2.74)$$

2.4.3.2 Raman Polarisation Arrangements

The polarisations of the two Raman beams determine the allowed transitions between the various m_F states. For example, with two counter-propagating opposite-circularly polarised photons, one with LCP (helicity -1) propagating in the positive $\hat{\mathbf{e}}_0$ direction and the other with RCP (helicity $+1$) propagating along $-\hat{\mathbf{e}}_0$, we can drive Raman transitions between the lower levels $|2, m_F\rangle$ and $|3, m_F\rangle$ via the two upper levels $|F', m_F + 1\rangle$, where $F' = 2, 3$. This is the relatively simple arrangement which we label, for obvious reasons, $\sigma^+ - \sigma^+$. As another example, if we have two orthogonal linearly polarised photons whose electric field vectors are both perpendicular to the quantisation axis $\hat{\mathbf{e}}_0 = \hat{\mathbf{z}}$, i.e. $\epsilon_1 \parallel \hat{\mathbf{y}}$ and $\epsilon_2 \parallel \hat{\mathbf{x}}$, then we drive Raman transitions along four distinct routes, namely: $|2, m_F\rangle \leftrightarrow |3, m_F\rangle$ via $|F', m_F \pm 1\rangle$, where $F' = 2, 3$. This arrangement is labelled $\pi^+ - \pi^-$, and is commonly referred to as ‘lin-perp-lin’. The Raman routes associated with the two aforementioned polarisation arrangements are depicted in Fig. 2.7, and we give a summary of these and other polarisation combinations and their allowed Raman routes in Table 2.2.

It is interesting to note that the $\pi^+ - \pi^+$, $\pi^- - \pi^-$, $\pi^0 - \pi^0$ and $\sigma^\pm - \sigma^\mp$ arrangements (i.e. parallel linear and orthogonal circular) do not drive Raman transitions, because the dipole matrix elements for their allowed Raman routes via $F' = 2$ and $F' = 3$ cancel each other out. As an example of this, we look at the $m_F = 1$ sublevel in the $\pi^0 - \pi^0$ arrangement. In this case $q_1 = q_2 = 0$, and the scalar product of the G-coefficient vectors is

$$|\mathbf{G}_1 \cdot \mathbf{G}_2| = (\sqrt{3/20}, \sqrt{7/108}, -\sqrt{16/135}, 0) \cdot (0, \sqrt{8/189}, \sqrt{5/216}, -\sqrt{15/56}), \quad (2.75)$$

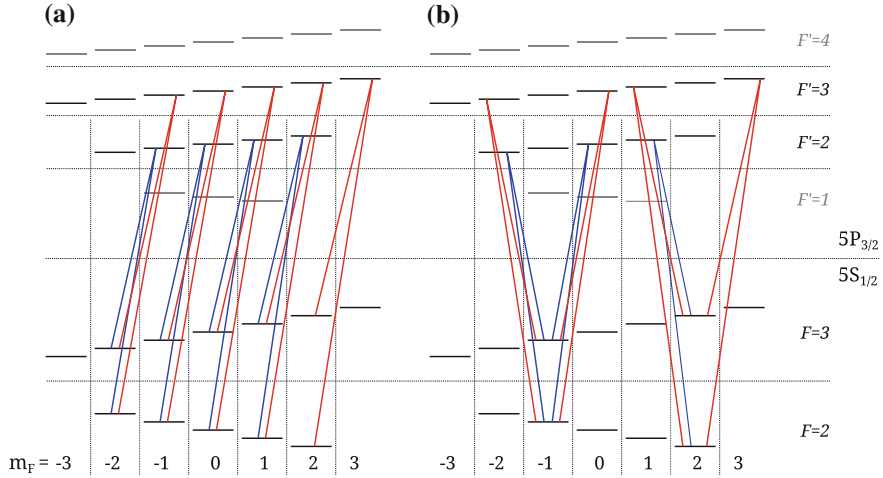


Fig. 2.7 Allowed Raman routes on the ^{85}Rb D_2 line in: **a** the $\sigma^+ - \sigma^+$ polarisation arrangement; and **b** the $\pi^+ - \pi^-$ polarisation arrangement. The *blue (red)* lines indicate Raman routes via $F' = 2$ (3). In **b**, only Raman routes from two starting m_F states are shown in order to avoid overcrowding. The splittings of the levels are not shown to scale

Table 2.2 Table summarising the polarisation states of the two Raman driving field photons and their associated dipole-allowed Raman routes

Transitions	Raman routes	via
$\sigma^+ - \sigma^+$	$ 2, m_F\rangle \leftrightarrow 3, m_F\rangle$	$ F', m_F + 1\rangle$
$\sigma^- - \sigma^-$	$ 2, m_F\rangle \leftrightarrow 3, m_F\rangle$	$ F', m_F - 1\rangle$
$\pi^0 - \sigma^+$	$ 2, m_F\rangle \leftrightarrow 3, m_F - 1\rangle$	$ F', m_F\rangle$
$\pi^0 - \sigma^-$	$ 2, m_F\rangle \leftrightarrow 3, m_F + 1\rangle$	$ F', m_F\rangle$
$\pi^0 - \pi^+$	$ 2, m_F\rangle \leftrightarrow 3, m_F + 1\rangle$	$ F', m_F\rangle$
	$ 2, m_F\rangle \leftrightarrow 3, m_F - 1\rangle$	$ F', m_F\rangle$
$\pi^+ - \pi^-$	$ 2, m_F\rangle \leftrightarrow 3, m_F\rangle$	$ F', m_F + 1\rangle$
	$ 2, m_F\rangle \leftrightarrow 3, m_F\rangle$	$ F', m_F - 1\rangle$

The two polarisation vectors are $\epsilon_{1,2}$, and the quantisation axis is $\hat{\mathbf{e}}_0 = \hat{\mathbf{z}}$. In all cases, $F' = 2, 3$

which is equal to zero. This is the same for all m_F levels in all of the above-mentioned arrangements, and provides the reasoning for the absence of $|2, m_F\rangle \leftrightarrow |3, m_F \pm 2\rangle$ (via $|F', m_F + 1\rangle$, where $F' = 2, 3$) transitions in the lin-perp-lin arrangement. This is only valid if we treat all the upper levels as having the same single-photon detuning Δ – the approximation we set out with.

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