# Chapter 2
# Does Model Specification Matter?

**Abstract** Multiregional demography stresses the importance of identifying the proper flows to enter as numerators in constant coefficient account models, and of relating these numerators to appropriate denominators measuring population stocks. When applied in demographic definitional and structural equations, such procedures lead to correctly specified "incidence" rates and the subpopulations at risk of experiencing the changes brought about by these particular rates. In this context, models of the determinants and consequences of migration that rely on inmigration rates and net migration rates are misspecified. So too are models, for example, that rely on the "labor force participation rate." In both instances the denominators of the rates do not correspond to the subpopulations that are at risk of experiencing the events represented in the numerators. Demographic innumeracy produces a biased model.

**Keywords** Model specification · Net migration rates · Incidence rates · Prevalence rates · Urbanization

## 2.1 Introduction

The principal arguments of this chapter are developed with the aid of a prototype biregional baseline model with constant rates, and in which simple contrived numbers are used to demonstrate the arguments. (After all, most of us learned how to solve quadratic equations by using only integer numbers.) However, an empirical illustration is also included later in the chapter for completeness.

The baseline model is used to generate spatial population dynamics that are described by a set of conventional indices, measured over time until stability. Sections 2.2 and 2.3 focus on the *net migration rate* as a representation of spatial population flows, and show how that introduces a projection bias. Section 2.4 uses the concept of Simpson's Paradox to illustrate how the introduction of age-specific migration rates can produce counterintuitive reversals. Section 2.5 examines the



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proximate sources of urban population growth. Finally, Sect. 2.6 concludes the chapter with a discussion of the principal results and an assessment of their significance for the modeling of migration.

## 2.2 The Net Migration Rate: How It Creates a Projection Bias

Net changes in regional population stocks are often dwarfed by the gross migration flows that help to produce them, hiding the spatial dynamics that are at work; a modest net contribution to regional population growth by migration may be generated by large gross flows in both directions. Demographers generally focus on demographic rates rather than counts, because analyses based on rates are superior to those based on counts. Demographic rates exhibit strong age-specific regularities and temporal stabilities that a projection based on rates can exploit to generate events through demographic accounting identities; a projection based on a count of events ignores this information. Moreover, it is much easier to assess and interpret the reasonableness of results produced by forecasted rates. For instance, a set of one hundred numbers representing deaths by single year of age of decedent is not very informative; nor is a collection of thirty numbers representing number of births by single year of age of mother. Yet the meanings of the expectation of life at birth and the net reproduction rate implied by these two sets of numbers (both calculated using age-specific rates) are readily grasped, and unrealistic values for these two variables suggest possible sources of error in the data or in the forecasting procedure.

The net migration rate, $m_j$, for a particular region $j$ is defined as the difference between the region's inmigration rate, $i_j$, and its outmigration rate, $o_j$. The outmigration rate is defined as a true rate because it divides the number of times that an event, outmigration, occurred during a year, say, by the number of persons exposed to the risk of experiencing that event. The inmigration rate, on the other hand, is a measure of *prevalence* rather than of propensity. It too has a numerator that is an occurrence count of a particular event, inmigration in this case, but its denominator is not a count of the number of persons that could have experienced the event. Rather, its denominator is the population in the region of destination that was at risk of experiencing the *outmigration* event. Since the net migration rate is the difference between a measure of prevalence and a true rate or propensity, its interpretation is necessarily ambiguous.

For each set of fixed outmigration rates, different spatial distributions of a population will give rise to different values of the net migration rate to a region. Also, for each fixed initial spatial distribution of a population, a given value of the net migration rate can be generated by a wide range of inmigration and outmigration rates; but the long-term implications for the geography of the population may be quite different. Thus one must be wary of cross-sectional comparisons of net migration rates of different regions as well as comparisons of such rates for the same region over time. In both instances the net migration rate will embody the influences of spatial population distribution along with those of movement propensities.

Consider, for example, how projections of urbanization might be carried out with uniregional (net migration) and multiregional (gross directional migration) models. In a uniregional model, the urban population is the central focus of interest and all rural-to-urban migration flows are assessed only with respect to the population in the region of destination, that is, the urban population. Changes in the population at the region of origin are totally ignored, with potentially serious consequences. For example, the rural population ultimately may be reduced to near zero levels, but a fixed and positive net migration into urban areas will nevertheless continue to be generated by the uniregional model.

To see the source of the problem more clearly, consider how the rural-urban migration specification is altered when a biregional model of urban and rural population growth is transformed into a uniregional model (Rogers 1990). Let urban population growth be described by the equation

$$P_u(t+1) = (1 + b_u - d_u - o_u)P_u(t) + o_v P_v(t) \tag{2.1}$$

Equation (2.1) states that next year's urban population total, $P_u(t+1)$, may be calculated by adding to this year's urban population $[P_u(t)]$ the increment due to the excess of births over deaths, that is, urban natural increase $[(b_u - d_u)\ P_u(t)]$, the decrement due to urban outmigration to rural ($v =$ village) areas $[o_u P_u(t)]$, and the increment due to rural-to-urban migration $[o_v P_v(t)]$.

Now, multiplying the last term in the Eq. (2.1) by unity expressed as $P_u(t)/P_u(t)$ transforms that equation into its uniregional counterpart

$$\begin{aligned} P_u(t+1) &= (1 + b_u - d_u - o_u)P_u(t) + o_v P_u(t) \\ &= (1 + b_u - d_u - o_u + i_u)P_u(t) \\ &= (1 + b_u - d_u + m_u)P_u(t) = (1 + r_u)P_u(t) \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} i_u &= o_v \left[\frac{P_v(t)}{P_u(t)}\right] = o_v \left[\frac{1 - U(t)}{U(t)}\right], \\ m_u &= i_u - o_u, \end{aligned}$$

and U($t$) is the fraction of the total national population that is urban at the time $t$. If all annual rates are assumed to be fixed in the biregional projection, then in the uniregional model $i_u$, and therefore also $m_u$, depend on U($t$), which varies in the course of projection, thereby introducing a bias. The dependence of the urban net migration rate $m_u$ on the level of urbanization at the time $t$ means that $m_u$ *must decrease as the level of urbanization increases.* Consequently, it seems inappropriate to use such a model to answer, for example, the question whether it is natural increase or net migration that is the principal source of urban population growth over time, as did Keyfitz (1980).

## 2.3 The Uniregional Fallacy and Bias

The notion that the spatial dynamics of a system of multiple interacting regional populations can be analyzed profitably by a set of *independent* uniregional models, which apply net migration rates to each regional population, dies hard. The biases and inconsistencies that are created by such decompositions of multiregional population projection models are generally ignored. Thus despite decades of published work on multiregional demography exposing the "uniregional fallacy," it is unfortunately still common to find articles in prominent journals that ignore this literature.

### *2.3.1 Aggregation Bias*

Imagine, once again, a closed two-region population system consisting of an urban population, $P_u(t)$, and a rural population, $P_v(t)$. In its discrete-time formulation

$$\begin{aligned} P(t) &= P_u(t) + P_v(t) \\ &= (1+r_u)^t P_u(0) + (1+r_v)^t P_v(0) \\ &= (1+r_u)P_u(t-1) + (1+r_v)P_v(t-1) \end{aligned} \tag{2.3}$$

and

$$r(t) = U(t-1)r_u + [1 - U(t-1)]r_v \tag{2.4}$$

where, as before, $U(t) = P_u(t)/P(t)$ is the fraction urban at time $t$.

By definition, the urban growth rate, $r_u$, is equal to the birth rate, $b_u$, minus the death rate, $d_u$, minus the outmigration rate, $o_u$, plus the inmigration rate, $i_u$:

$$r_u = b_u - d_u - o_u + i_u. \tag{2.5}$$

If $r_u$ is to remain constant, then the component rates on the right-hand side of the Eq. (2.5) must sum to a constant, and

$$P_u(t) = (1+r_u)^t P_u(0) \tag{2.6}$$

Note that instead of a chained multiplication of one-year at a time, one can simply raise the quantity in the parentheses to the power $t$. But we have earlier shown that

$$i_u = o_v\left[\frac{P_v(t)}{P_u(t)}\right] = o_v\left[\frac{1-U(t)}{U(t)}\right] \tag{2.7}$$

which means that $i_u$ (and therefore $r_u$) *changes over time as urbanization proceeds*. Bias and inconsistency are therefore the probable result of viewing this biregional population system through a uniregional perspective. Changes in magnitude of migration flows may occur apart from changes in the propensity to move. A biregional perspective can be used to distinguish between changes in rates that reflect actual changes in propensity from changes in rates that are merely a consequence of changes in compositions. The uniregional perspective does not have this ability. To see this, assume a behaviorally fixed and totally *homogeneous* population in Eq. (2.5),

$$\begin{aligned} r_u(t) &= b - d - o + o\left[\frac{1 - U(t)}{U(t)}\right] \\ &= b - d + \left[\frac{1 - 2U(t)}{U(t)}\right]o = n + A(t)o \end{aligned} \tag{2.8}$$

and, similarly,

$$\begin{aligned} r_v(t) &= b - d - o + o\left[\frac{U(t)}{1 - U(t)}\right] \\ &= b - d + \left[\frac{2U(t) - 1}{1 - U(t)}\right]o = n + B(t)o \end{aligned} \tag{2.9}$$

where natural increase, $n = b - d$, $A(t) = [1 - 2U(t)]/U(t)$, and $B(t) = [2U(t) - 1]/[1 - U(t)]$.

Since all members of the population exhibit identical and constant behavior, one might expect both regional growth rates $r_u(t)$ and $r_v(t)$, to be identical and to remain fixed at the value of the natural increase rate $n = b - d$; but this will only occur either if (i) the two regional populations do not interact with each other via migration (that is, $o = 0$), or (ii) the entire population is currently experiencing stable growth, a condition that in this illustration can only arise if the two regional populations happen to be identical in size (that is, $U(t) = ½$, whence $A(t) = B(t) = 0$).

Unlike the case of the "perfect aggregation," total homogeneity is not a sufficient condition for "perfect deconsolidation," that is, for avoiding a bias in transformations of multiregional models to uniregional ones; indeed homogeneity is irrelevant and distributional stability is essential (Rogers 1969).

### 2.3.2 Decomposition Bias

The transformation of a multiregional model that describes interregional migrations between the constituent regions of the population system into the corresponding separate uniregional models can be viewed as a process of compensated decomposition in which net migration rates carry out the "compensation." Before such a transformation, the population of the $j$th region, $P_j(t + 1)$ for example, can be defined as

$$P_j(t+1) = (1 + b_j - d_j - o_j)P_j(t) + \sum_{i \neq j} o_{ij} P_i(t). \tag{2.10}$$

Denoting $1 + b_j - d_j - o_j$ by $o_{jj}$, and multiplying the last term in the equation by unity, in the form of $P_j(t)/P_j(t)$, gives

$$\begin{aligned} P_j(t+1) &= o_{jj}P_j(t) + \left[\sum_{i \neq j} o_{ij}\frac{P_i(t)}{P_j(t)}\right]P_j(t) \\ &= \left[o_{jj}(t) + \sum_{i \neq j} o_{ij}\frac{P_i(t)}{P_j(t)}\right]P_j(t) \end{aligned} \tag{2.11}$$

### 2.3.3 *Numerical Illustration: A Simple Projection Model of Urbanization*

The urban population of the Pacific island of Mora-Bora increased by three quarters last year ($r_u = \frac{3}{4}$), while the rural population grew by an eighth ($r_v = \frac{1}{8}$). At the start of the year the two populations were enumerated to be 16 and 32 thousand, respectively. During the course of the year a half of the rural population migrated to urban areas ($o_v = \frac{1}{2}$), while a fourth of the urban population moved to the rural areas ($o_u = \frac{1}{4}$). Given these rates and the initial populations, it is a simple matter to define the growth process that will project the island's biregional population forward two consecutive years. The island's population increases from 48 thousand to 64 thousand after a year, and then it grows to 82 thousand after the following year. The demographic accounting equations for the first year are:

$$\begin{aligned} P_u(1) &= [1 + (b_u - d_u) - o_u]P_u(0) + o_vP_v(0) \\ &= (1 - n_{\mathrm{u}} - o_u)P_u(0) + o_vP_v(0) \\ &= (1 + 0 - \frac{1}{4})16 + (\frac{1}{2})32 = 28 \end{aligned}$$

and

$$\begin{aligned} P_v(1) &= o_uP_u(0) + (1 + n_{\mathrm{v}} - o_v)P_v(0) \\ &= (\frac{1}{4})16 + (1 + \frac{1}{2} - \frac{1}{2})32 = 36 \end{aligned}$$

where $n$ denotes the natural increase rate. Notice that the urban population is experiencing replacement level fertility, that is, $b_u = d_u$, and the rate of natural increase, $n_u$, is zero. The natural increase rate of the rural population is a half.

The above disaggregated model produces a projected evolution of the national population that is: 48, 64, 82, …. Notice that the corresponding consolidated uniregional model for the national total [that is, $P(1) = \frac{4}{3}P(0)$] leads to a *higher*, not lower, projected set of totals: 48, 64, $85\frac{1}{3}$,…. Hence, Keyfitz's (1977, p. 16) proof of a guaranteed overprojection by the more disaggregated model does not apply in this case.The above two fundamental equations define the biregional model. The corresponding uniregional models may be obtained by a compensated decomposition. In that event, the net migration rate for the urban region is

$$m_u = i_u - o_u = o_v\left[\frac{P_v(0)}{P_u(0)}\right] - o_u$$
$$= \frac{1}{2}[2] - \frac{1}{4} = \frac{3}{4}$$

and correspondingly,

$$m_v = -\frac{3}{8}$$

Thus

$$P_u(1) = 28 = [1 + 0 + \frac{3}{4}]16$$

and

$$P_v(1) = 36 = [1 + \frac{1}{2} - \frac{3}{8}]32$$

Similarly

$$P_u(2) = 49 = (\frac{7}{4})28$$

and

$$P_v(2) = 40\frac{1}{2} = (\frac{9}{8})36$$

The island's population after the initial "calibration" period is overprojected by seven and a half thousand people relative to the biregional projection, with the rural population's *underprojection* of two and a half thousand being over-compensated by the urban population's *overprojection* of ten thousand. Notice that the former was losing net migrants, whereas the latter was gaining net migrants.

Finally, consider the same growth process as before, but now imagine that the initial national population of 48 thousand is distributed equally among the two regions. Then the biregional projections give

$$P_u(1) = 30 = (\frac{3}{4})24 + (\frac{1}{2})24$$

and

$$P_v(1) = 30 = (\frac{1}{4})24 + (1)24$$

Similarly

$$P_u(2) = 37\frac{1}{2} = (\frac{3}{4})30 + (\frac{1}{2})30$$

and

$$P_v(2) = 37\frac{1}{2} = (\frac{1}{4})30 + (1)30$$

The relevant net migration rates now are $m_u = \frac{1}{4}$ and $m_v = -\frac{1}{4}$, and the corresponding uniregional projection becomes

$$P_u(1) = 30 = (\frac{5}{4})24$$

and

$$P_v(1) = 30 = (\frac{5}{4})24$$

Similarly,

$$P_u(2) = 37\frac{1}{2} = (\frac{5}{4})30$$

and

$$P_v(2) = 37\frac{1}{2} = (\frac{5}{4})30$$

Because the initial population has a stable initial distribution, perfect decomposition results. No bias is introduced by shifting to a uniregional model by means of compensated decomposition.

### 2.3.4 The Simple Projection Model Expressed in Matrix Form

Matrix algebra provides a compact and useful means for studying the demographic evolution of multiple interacting populations. Matrix notation makes the projection process more transparent, and matrix theory brings to demographic analysis results that have direct application to population questions. Expressing the population projection process in matrix form also leads to the derivation of results that would be virtually impossible to establish otherwise.

The reader should confirm that the simple biregional projection of Mora Bora's urban and rural populations, described in Sect. 2.3.2 may be expressed in matrix form as

$$\begin{bmatrix} 28 \\ 36 \end{bmatrix} = \begin{bmatrix} 3/4 & 1/2 \\ 1/4 & 1 \end{bmatrix} \begin{bmatrix} 16 \\ 32 \end{bmatrix}$$

and

$$\begin{bmatrix} 39 \\ 43 \end{bmatrix} = \begin{bmatrix} 3/4 & 1/2 \\ 1/4 & 1 \end{bmatrix} \begin{bmatrix} 28 \\ 36 \end{bmatrix}$$

Recall that the multiplication rule in matrix algebra is "row times column."

A more transparent picture of the evolution to stable growth may be obtained by focusing on another numerical illustration in which an urban population of 24 million each year sends a fourth of its population to rural areas and receives, in exchange, one-half of the rural population, which initially is also taken to stand at 24 million persons. Assume that a zero population growth regime prevails, such that the annual increment due to births, in each region, is exactly offset by the annual decrement due to deaths. Then we have that

$$\mathbf{G} = \begin{bmatrix} 3/4 & 1/2 \\ 1/4 & 1/2 \end{bmatrix}$$

$$\{\mathbf{P}(t)\} = \begin{bmatrix} 24 \\ 24 \end{bmatrix}$$

and the projection to stability is

$$\begin{aligned}
\{\mathbf{P}(t+1)\} &= \begin{bmatrix} 30 \\ 18 \end{bmatrix} = \begin{bmatrix} 3/4 & 1/2 \\ 1/4 & 1/2 \end{bmatrix} \begin{bmatrix} 24 \\ 24 \end{bmatrix} \\
\{\mathbf{P}(t+2)\} &= \begin{bmatrix} 31\tfrac{1}{2} \\ 16\tfrac{1}{2} \end{bmatrix} = \begin{matrix} 3/4 & 1/2 \\ 1/4 & 1/2 \end{matrix} \begin{bmatrix} 30 \\ 18 \end{bmatrix} = \begin{bmatrix} 11/16 & 5/8 \\ 5/16 & 3/8 \end{bmatrix} \begin{bmatrix} 24 \\ 24 \end{bmatrix} \\
\{\mathbf{P}(t+3)\} &= \begin{bmatrix} 31\tfrac{7}{8} \\ 16\tfrac{1}{8} \end{bmatrix} = \begin{bmatrix} 3/4 & 1/2 \\ 1/4 & 1/2 \end{bmatrix} \begin{bmatrix} 31\tfrac{1}{2} \\ 16\tfrac{1}{2} \end{bmatrix} = \begin{bmatrix} 43/64 & 21/32 \\ 21/64 & 11/32 \end{bmatrix} \begin{bmatrix} 24 \\ 24 \end{bmatrix} \\
&\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots \\
\{\mathbf{P}(\infty)\} &= \begin{bmatrix} 32 \\ 16 \end{bmatrix} = \begin{bmatrix} 3/4 & 1/2 \\ 1/4 & 1/2 \end{bmatrix} \begin{bmatrix} 32 \\ 16 \end{bmatrix} = \begin{bmatrix} 2/3 & 2/3 \\ 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 24 \\ 24 \end{bmatrix}
\end{aligned}$$

Note that once the initial urbanization level of ½ grows to 2/3, it remains at that level forever. The population has achieved stable growth; each of its subgroups is increasing exponentially and at the same rate. Its urban and rural growth rates both are zero, and its stable distribution is forever fixed in the proportions 2/3 and 1/3. These two fundamental attributes of the process of projection to stability are augmented by a third; the independence of the stable growth results from the starting population distribution—a property of the process called "ergodicity."

That the stable or intrinsic growth rate and corresponding stable distribution are independent of the starting population distribution and depend only on the growth regime defined by the projection matrix, **G** may be illustrated by applying the same matrix to a different initial population distribution. For example the reader should confirm that

$$\begin{bmatrix} 34 \\ 14 \end{bmatrix} = \begin{bmatrix} 3/4 & 1/2 \\ 1/4 & 1/2 \end{bmatrix} \begin{bmatrix} 40 \\ 8 \end{bmatrix}$$

converges to the same stable state as was obtained before, and that the alternative projection matrix

$$\begin{bmatrix} 5/6 & 1/4 \\ 1/4 & 5/6 \end{bmatrix}$$

ultimately brings about a level of urbanization with half of the national population living in rural areas and growing at 25/3= 8.3 % per annum.

### *2.3.5 Bias: A Summary*

Aggregating the separate projections of several *noninteracting* heterogeneous populations will give rise to a total greater than one that would be obtained by projecting the aggregate population at its average rate of growth at the outset (Keyfitz 1977, p. 16). Aggregation prior to projection introduced an aggregation bias that is guaranteed to be *negative*, giving rise to an underprojection relative to the consolidation of the corresponding disaggregated projection. The aggregation of noninteracting heterogeneous populations prior to projection, then, always produces an underprojection: the aggregate population never stabilizes, the aggregate rate of growth forever increases, and the population's composition varies continuously.

Does the same guaranteed *negative* bias also arise in the aggregation introduced by the consolidation of *interacting* heterogeneous populations? The answer is no. Rogers (1985), for example, offers an illustration of a positive aggregation bias which is introduced by the consolidation of the Swedish female population across four "regions" that are marital status categories. The consolidated projection, in this instance, produces an overprojection relative to the aggregation of the deconsolidated projection. So clearly, the answer in this situation is an ambiguous one; the aggregation bias can be positive or negative. This can be readily demonstrated by carrying out a projection across two time intervals with both the deconsolidated and consolidated models and then comparing the two projections, as in our simple numerical example. The aggregation of interacting heterogeneous multiregional populations prior to projection, it can be shown, produces either under- or overprojection: the aggregate population ultimately stabilizes and both its aggregate rate of growth and its composition become fixed.

What about decomposition bias? The separation of each region from the others in a multiregional system by means of a net migration rate form of compensated decomposition will always create a bias in the projected regional totals, except in the two relatively uninteresting cases of an interregionally immobile population or one that is experiencing stable growth. Because net migration rates confound movement propensities with populations stocks, the conditions for "perfect decomposition" turn out to be even more stringent that those for "perfect aggregation" (Rogers 1969). A particularly simple, yet pervasive, form of decomposition bias is the relative overprojection of the population experiencing net gains from migration and the underprojection of the corresponding population that is the net loser of migrants.

## 2.4 Netting Out the Age Patterns of Directional Migration Rates

Crude rates are weighted averages of age-specific rates, where the rates are the proportional shares accorded to each age group to reflect that group's relative size in the total population. For example, if one of two populations has a much older age structure (say, Sweden) than the other (for instance, Costa Rica), its crude death rate is higher than the corresponding rate in the other, even though its every age-specific death rate is lower than the corresponding rate in the other population. This counterintuitive reversal is an illustration of what demographers and statisticians call Simpson's paradox–an apparent contradiction of two statements that arises as a consequence of the stratification of the populations into two or more subgroups and the resulting reversal of the rank ordering of those populations on the variable of interest. Similar counterintuitive reversals may occur in comparisons of a wide array of demographic processes, including migration. The crude outmigration rate of one population may be higher than that of another, even though its every age-specific outmigration rate is lower than the corresponding rate of the other.

Most illustrations of Simpson's paradox have focused until recently on cross-sectional comparisons. The impacts of changes in the relative weights used in the averaging process typically have been examined across several populations at one moment in time. Vaupel and Yashin (1985) and others have broadened this perspective to include the demographic dynamics of selectivity and the impacts of the changes that they bring about in the relative weights themselves, over time. In this chapter, their perspective is widened even further, by focusing on *interacting* population subgroups linked by migration, and on the dynamic impacts that this linkage generates through its contribution of *increments* as well as decrements to each of these population subgroups. Because migration, unlike mortality, say, is a repeatable event that directly affects two populations (origin and destination) the spatial population dynamics that it creates may introduce counterintuitive demographic consequences, some of which apparently have not been studied either empirically or theoretically (Rogers 1992).

Imagine a population of a million people in Country A and another of the same number in Country B. During the course of a year, ten thousand individuals die in the former and nine thousand die in the latter. A comparison of the mortality regimes prevailing in the two countries suggests that mortality is higher in Country A (1.0 % against 0.9 %).

Suppose that the population of Country A is equally divided among Young and Old people, half-a-million being in each age group. Country B, on the other hand has a younger age composition, with 70 % of its population being in the Young age group. Suppose, further, that of the thousand deaths in Country A a quarter occurred among the Young, whereas in Country B the corresponding total was 4.2 thousand. Then the age-specific death rates in Country A were 0.5 % among the Young population and 1.5 % among the Old, both lower than the corresponding percentages for Country B: 0.6 and 1.6 %, respectively. A comparison of these percentages indicates that mortality is *lower* in Country A at each age. The cause of this apparent contradiction with our earlier finding is the relatively younger age composition of Country B. Since crude rates are weighted sums of the constituent disaggregated rates, the relatively heavier weight accorded to the death rate of the Young population in Country B lowered its aggregate crude rate with respect to Country A:

$$Country\,A : 0.5(0.5\,\%) + 0.5(1.5\,\%) = 1.0\,\%.$$
$$Country\,B : 0.7(0.6\,\%) + 0.3(1.6\,\%) = 0.9\,\%$$

What is true of crude mortality rates is, of course, also true of crude outmigration rates and, therefore, of crude net migration rates. Assume that the above figures now refer to emigration from one country to the other. The aggregate flows then reveal that Country B gains a thousand net migrants from the exchange. This total results from the combination of a net loss of Young people (−1.7 thousand) and a net gain of Old people (+2.7 thousand). *Thus Country B gains net migrants, even though its rates of emigration are higher at each age than those of Country A.* This compositional artifact could possibly be a contributing factor to the counter-intuitive directional behavior of *net* interstate migrants that puzzled David Plane (1988, p. 10) who observed that in recent years something like two-thirds of all the net interstate streams of migration in the United States point in the direction of the lower average wage state.

But, of course, another contributing factor also could have been the decomposition bias introduced by a net migration perspective. Consider, for example, *identical* Young-Old age compositions of a half and a half, say, and the same directional age-specific emigration rates. But now assume that Country A has twice the population of Country B, say two million to Country B's one million. Then,

$$Country\,A : 0.5(0.5\,\%) + 0.5(1.5\,\%) = 1.0\,\%.$$
$$Country\,B : 0.5(0.6\,\%) + 0.5(1.6\,\%) = 1.1\,\%.$$

During the course of a year, then, twenty thousand individuals emigrate from Country A and only eleven thousand leave Country B. The result is that Country B shows a positive *net* migration rate of 0.9 %, while Country A exhibits a corresponding negative rate of 0.45. *And Country B gains net migrants once again, even though its rates of emigration are higher at each age than those of Country A.*

Our numerical illustration also clearly reveals how similar age profiles of gross migration rates may be hidden in the corresponding age profiles of net migration rates. For example, the age-specific immigration rates for Country A in the first illustration are

$$i_A(Y) = 0.6(\frac{7}{5}) = 0.84\,\%,$$
$$i_A(O) = 1.6(\frac{3}{5}) = 0.96\,\%,$$

and for country B they are

$$i_B(Y) = 0.5(\frac{5}{7}) = 0.36\,\%,$$
$$i_B(O) = 1.5(\frac{5}{3}) = 2.50\,\%.$$

Hence the corresponding net migration rates are

$$m_A(Y) = 0.84 - 0.5 = +0.34\,\%,$$
$$m_A(O) = 0.96 - 1.5 = -0.54\,\%,$$
$$m_B(Y) = 0.36 - 0.6 = -0.24\,\%,$$
$$m_B(O) = 2.5 - 1.6 = +0.90\,\%.$$

Figure 2.1 sets out these age-specific patterns of migration and illustrates how the netting out of similar age patterns of gross migration rates gives rise to totally different corresponding age patterns of net migration rates. Note that merely reversing the Young-Old proportional relationship between the two nations, totally reverses the corresponding age pattern of net migration rates.

Net migration rates are often viewed as crude indices that reflect differences in propensities of movement. But as we have seen, net migration rates also reflect the relative sizes of population stocks. The consequence for age patterns of migration rates is the disintegration of a well-established regularity in age profile. To see this, imagine a migration exchange between two neighboring regions of a biregional system, regions $i$ and $j$, say, that initially contain populations of equal size, $P_i = P_j$, say. Assume that the gross migraproduction rates (the areas under the migration schedules) are equal to unity in both directions, and that the age profile of both flows is that of the top age profile in Fig. 2.2. Under these conditions, the net migration rate in region $i$ is zero at all ages, as shown by the dotted line in Fig. 2.3. At each age, the number of migrants from region $j$ to region $i$ exactly equals the number in the reverse direction, and the equality also holds for the corresponding rates.

Now imagine that because of higher fertility and immigration levels, say, one population in region $j$ grows more rapidly than the other, such that it becomes

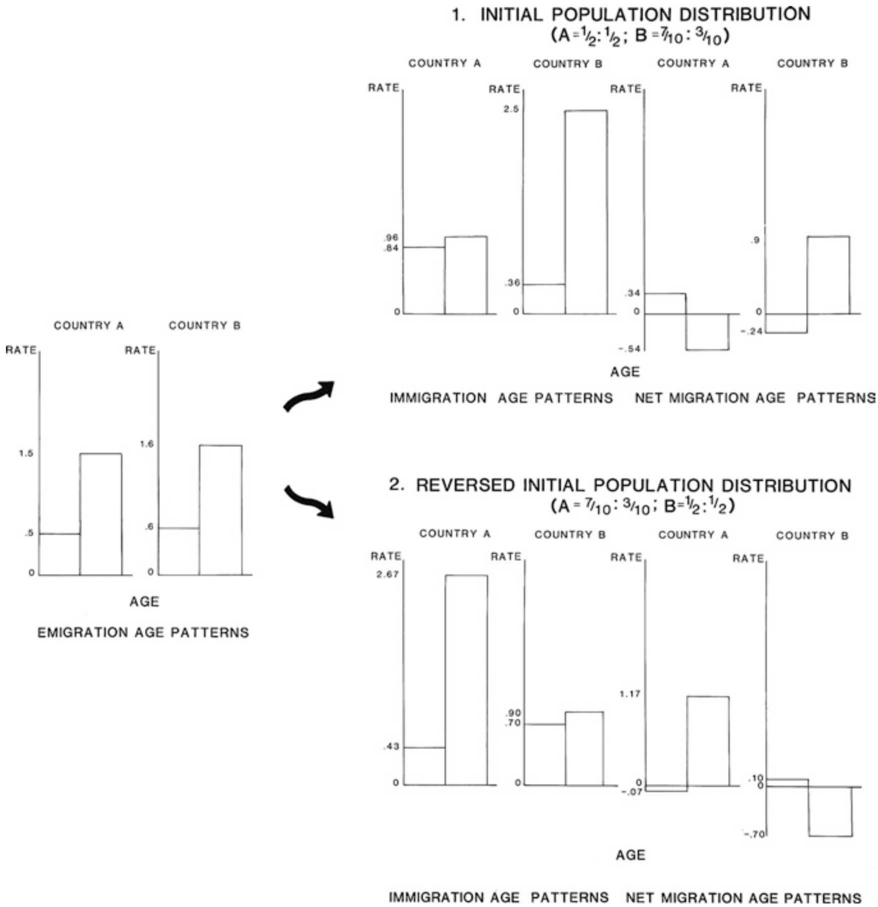


**Fig. 2.1** Netting out the age patterns of directional migration rates: I. Two age groups. *Source* Rogers (1990)

twice as large as its neighbor, that is, $P_j = 2P_i$ . Assume that the propensities to migrate in both directions and the associated age profiles remain the same as before for all ages of $x$. Then the resulting net migration rate schedule of one region becomes that of the solid line in Fig. 2.2, that is, the "standard" profile with a gross migraproduction rate of unity. We also include the corresponding net migration rate schedule when $P_j = P_i\,/2$ (the broken line in Fig. 2.2).

The three net migration schedules in Fig. 2.2 all reflect the same pair of gross migration schedules. In each instance the propensity to migrate in the two directions is the same, and so is the age profile. Yet the net migration rate for region $i$, say, varies directly with the relative sizes of the two populations, that is, with the ratio $P_j/P_i$. The net rate is zero at all ages when the ratio is unity, positive at all ages when the ratio exceeds unity, and negative at all ages when the ratio falls

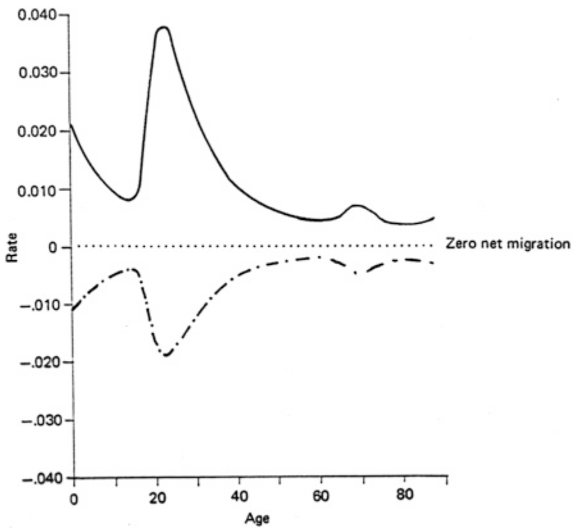


**Fig. 2.2** Netting out the age patterns of directional migration rates: II. Eighty age groups. *Source* Rogers (1990)

short of unity, in the latter two instances following the age profile of the migration schedule standard. Thus, in this illustration, net migration once again depends on relative populations sizes; the effects of flows are confounded with the effects of changes in stocks.

Because net rates confound flows with changes in stocks, they hide regularities that prevail among gross flows. Although the latter tend to always follow the conventional age profile, the former exhibit a surprisingly wide variety of shapes, a few of which appeared earlier in Fig. 1.2.

## 2.5 The Proximate Sources of India's Urban Population Growth: Mostly Migration or Mostly Natural Increase?

### *2.5.1 Introduction*

The urban population of India increased by 3.7 % a year during the late 1960s and early 1970s. The urban growth rate, $r_u$, was the outcome of a birth rate $b_u$ of 30 per 1000, a death rate $d_u$ of 10 per 1000, an inmigration rate $i_u$ of 27 per 1000, and an outmigration rate $o_u$ of 10 per 1000 (Rogers 1982, 1985). Expressing these rates on a *per capita* basis leads to the fundamental identity

$$
\begin{aligned}
r_u &= b_u - d_u + i_u - o_u \\
&= 0.030 - 0.010 + 0.027 - 0.010 \\
&= 0.037
\end{aligned}
$$

The corresponding identity for the rural population was

$$\begin{aligned} r_v &= b_v - d_v + i_v - o_v \\ &= 0.039 - 0.017 + 0.002 - 0.007 \\ &= 0.017 \end{aligned}$$

The total national population of India in 1970 was about 548 million, of which roughly 109 million (20 %) was classified as urban. Multiplying this latter total by the urban growth rate gives 109(0.037) = 4.03 million as the projected *increase* for 1971. An analogous calculation for the rural population gives 7.46 million for the corresponding projected increase in the rural population. These changes imply, for 1971, an urban population of 113 million, a rural population of 446 million, and a rate of national population increase of

$$r. = 0.20r_u + 0.80r_v = 0.021$$

What would be the immediate contributions of net migration and natural increase to urban population growth if rates either of net migration or of natural increase were suddenly to drop to zero? This reveals that urban natural increase in 1970 India contributed 0.020/0.037 = 0.54, or just over a half of the urban population growth rate. But this is a static cross-sectional view that ignores the evolution of the changing contributions of migration and natural increase to urban growth *over time*. The long-run impacts of current patterns of natural increase and migration on urban population growth and urbanization levels can only be assessed by population projection. And, according to Keyfitz (1980), the results indicate that migration is the principal contributor at first, but then is overcome by natural increase. Following Keyfitz, imagine a hypothetical population, initially entirely rural, that experiences the annual national rate of natural increase of r., say, and a net rural outmigration rate of $m_v$. Then the projected evolution of the rural populations should follow the path defined by

$$P_v(t) = (1 + r. - m_v)^t P_v(0)$$

whereas that of the national population exhibits the path set by

$$P.(t) = P_u(t) + P_v(t) = (1 + r.)^t P.(0).$$

Clearly, one can obtain $P_u(t)$ as a residual.

On the Indian data, this gives the following *uniregionally* projected totals for, say, 1980:

$$\begin{aligned} P_v(10) &= (1 + 0.021 - 0.005)^{10} 439 = 515\,\text{million} \\ P.(10) &= (1 + 0.021)^{10} 548 = 665\,\text{million} \end{aligned}$$

and

$$P_u(10) = P.(10) - P_v(10) = 665 - 515 = 150\,\text{million}$$

Once again, notice that instead of a chained multiplication of one-year at a time, one can simply raise the quantity in the parentheses to the tenth power.

Alternatively, one could project the urban and rural populations at their own rates of growth instead of the national, and then obtain the latter population by simple addition. In this event,

$$P_u(10) = (1 + 0.037)^{10} 109 = 157\,\text{million}$$
$$P_v(10) = (1 + 0.017)^{10} 439 = 520\,\text{million}$$

and

$$P.(10) = P_u(10) + P_v(10) = 157 + 520 = 677\,\text{million}$$

or 12 million more persons than in the previous projection for India as a whole. Continuing on with the latter equation to the target year 2000, say, gives an urban population of 324 million and a corresponding rural population of 728 million for the uniregional specification. Clearly, model specification matters.

A uniregional perspective must rely on the notion of *net* migration. An immediate consequence of such a perspective in this application is an ultimate and total urbanization, that is India's initial urbanization level of U(0) = 20 % in 1970, is headed toward an ultimate level of 100 %. And, correspondingly, the *absolute* contribution of urban net migration *must*, of necessity tend toward zero in the long-run… a somewhat problematic situation for an analysis that seeks to answer the question of whether it is net migration or natural increase that contributes most to urban population growth over time.

### 2.5.2 The Problematic Net Migration Rate

Recall the crude rates listed earlier to specify the corresponding biregional components-of-change model

$$P_u(t+1) = (1 + b_u - d_u - o_u)P_u(t) + o_v P_v(t) \tag{2.12}$$

$$P_v(t+1) = (1 + b_v - d_v - o_v)P_v(t) + o_u P_u(t) \tag{2.13}$$

where, for example,

$$\begin{aligned} P_u(1971) &= (1 + 0.030 - 0.010 - 0.010)109 + (0.007)439 \\ &= 113.0\,\text{million}\,\textit{persons} \end{aligned} \tag{2.14}$$

and

$$\begin{aligned} P_v(1971) &= (1 + 0.039 - 0.017 - 0.007)439 + (0.010)109 \\ &= 446.5\,\text{million}\,\textit{persons} \end{aligned} \tag{2.15}$$

Projecting to the target year 2000 with the *biregional* model defined by Eqs. (2.14) and (2.15) produces different future population totals than before: an urban population of 285 million and a corresponding rural population of 753 million, for a grand total of $P(2000) = 1.038$ billion.

### 2.5.3 A Disaggregation by Age

Having examined the sources of urban growth in India—first using the uniregional and then the biregional model, with both models ignoring age and both assuming a fixed rate of natural increase, $n_u$, and fixed outmigration rates, $o_u$ and $o_v$, we saw that, because the level of urbanization U(t) increased over time, the urban *net* migration rate, $m_u(t)$ was certain to decline over time, thereby guaranteeing that the *relative* contribution of migration to the urban growth rate would decline as well. A more realistic model is needed, one that allows natural increase to decline also. Introducing age-specific rates is a first step in that direction.

To illustrate the problematic nature of the net migration rate, consider next a *biregional* (and still closed to international migration), constant-coefficient, baseline projection to the target year 2000, say. Such a projection of India's urban and rural total population growth, using the *age-specific* rates of 1970 in Appendix B of Rogers (1985), projects a total urban population of 291 million and a corresponding rural population of 760 million.

The introduction of age favors migration as a contributor to urban growth. In the Indian illustration it increases migration's *ultimate* (stable growth) contribution threefold. In other illustrations it can reverse the ranking itself, making migration the principal source of urban growth (Rogers 1985, p. 75). What accounts for this reversal?

The disaggregation by age does not change the pattern of evolution of the aggregate urban net inmigration rate, $m_u(t)$. In the Indian illustration it declines sharply from its initial level. But now the aggregate rate of natural increase no longer remains constant, dropping from 2 to 1.5 %. The cause of this decline in the aggregate rate is, of course, the gradual aging of the population and the associated shifts in its age composition. This shift alters the relative weights with which the fixed age-specific rates are consolidated to form the aggregate crude rates. The result is an increased relative contribution of net migration as a source of urban population growth, a consequence apparently of the fact that, as with mortality (but not with fertility), the risks of migration are experienced by individuals of all ages.

In conclusion, it appears that the principal effect of introducing age composition into the fixed-rate projection model is to decrease the aggregate rate of natural increase over time, while slowing down the decline of the urban net migration rate. Because these two contributors to urban growth now can exhibit different rates of decline over time, their relative importance as sources of urban growth also can change.

## 2.6 Discussion and Conclusion

Much of literature on aggregate, cross-sectional behavioral models of internal migration continue to exhibit a curiously ambivalent position with regard to the measurement of geographical mobility. This ambivalence is particularly surprising because it stands in striking contrast to the corresponding studies of mortality and fertility, which often devote considerable attention to measurement problems.

Models that seek to explain patterns of net migration are founded on inadequate perspectives. Net migration rates confound movement propensities with relative population stock levels. They hide well-established regularities in the age pattern of geographical mobility. They can lead to misspecified explanatory models, and they make it virtually impossible to consider properly the impacts of important violations of the basic assumptions underlying many spatial demographic studies: homogeneity, stationarity, and temporal independence.

Gross migration stream (multiregional) models, on the other hand, more realistically depict the phenomenon being modeled (since there are no net migrants). The rates they use to represent directional movements are linked to the populations at risk of moving and therefore measure true propensities of migrating (a feature that net migration rates lack). Gross migration models can generate changes in migration streams that arise out of changes in the sizes of the various populations at risk of moving (something that net migration models cannot do since they only consider the size of the destination population). And, finally, gross migration models permit their users to keep track of important population attributes such as places of birth and places of former residence, a feature that for example, allows one to differentiate the migration rates of return migrants from those of nonreturn migrants.

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