

# Contents

<b>1</b>	<b>Elements of Noncommutative Algebra</b>	<b>1</b>
1.1	Gauss Polynomials	2
1.2	Lyndon — Shirshov Words	3
1.2.1	Standard Words	4
1.2.2	Nonassociative Standard Words	7
1.2.3	Deg-Lex Orders	9
1.3	Gröbner–Shirshov Systems of Defining Relations	10
1.3.1	Composition Lemma	10
1.3.2	Noncommutative $G$ -Polynomials	21
1.3.3	Skew Group Rings	22
1.3.4	Poincaré–Birkhoff–Witt Theorem	23
1.4	Braid Monoid and Permutation Group	25
1.4.1	Co-sets and Shuffles	29
1.5	Hopf Algebras	31
1.5.1	Group-Like and Primitive Elements	33
1.5.2	Character Hopf Algebras	36
1.5.3	Free Character Hopf Algebra	39
1.5.4	Brackets	40
1.5.5	Defining Relations	41
1.5.6	Combinatorial Rank	43
1.5.7	Noncommutative Differential Calculi	46
1.6	Filtrations	53
1.7	Certain Concepts of P.M. Cohn’s Theory	61
1.8	Representation Theory and Crossed Products	65
1.9	Chapter Notes	67
<b>2</b>	<b>Poincaré–Birkhoff–Witt Basis</b>	<b>71</b>
2.1	PBW Bases of the Free Character Hopf Algebra	72
2.2	Coproduct on Super-Letters	77
2.3	Hard Super-Letters	79
2.4	Monomial PBW Basis	87

2.5	Serre Skew-Primitive Polynomials .....	89
2.5.1	Examples .....	92
2.6	Chapter Notes .....	95
<b>3</b>	<b>Quantizations of Kac-Moody Algebras</b> .....	99
3.1	Kac-Moody Algebras .....	100
3.2	Quantum Deformations .....	101
3.3	Defining Relations of the Main Class .....	104
3.4	Triangular Decomposition .....	111
3.5	Indecomposable Generalized Cartan Matrices .....	117
3.5.1	Regular and Exceptional Quantizations .....	117
3.5.2	Non-symmetrizable Generalized Cartan Matrices .....	119
3.5.3	Symmetrizable Generalized Cartan Matrices .....	120
3.5.4	Cartan Matrices of Finite Type .....	123
3.5.5	Isomorphism Problem .....	125
3.6	Chapter Notes .....	126
<b>4</b>	<b>Algebra of Skew-Primitive Elements</b> .....	129
4.1	Quantum Lie Operations .....	129
4.1.1	Quantum Variables .....	130
4.1.2	Linearization and Specialization .....	131
4.2	Criteria for Quantum Lie Operations .....	133
4.2.1	Left and Right Primitive Polynomials .....	133
4.2.2	Polynomial Criterion .....	135
4.2.3	Finkelstein Criterion and Specht–Wever Condition .....	136
4.3	Bilinear and Trilinear Operations .....	139
4.4	Quadrilinear Operations .....	142
4.5	Chapter Notes .....	150
<b>5</b>	<b>Multilinear Operations</b> .....	151
5.1	The Basic System of Equations .....	151
5.2	Interpretation of Operations in a Crossed Product .....	155
5.3	Co-set Decomposition .....	158
5.4	Subordinate Sequences .....	161
5.4.1	Relations in the Symmetric Group .....	167
5.5	Decreasing Modules .....	168
5.6	Second Components .....	180
5.7	Components with $s \geq 3$ .....	181
5.8	Existence Condition .....	187
5.9	Interval of Dimensions .....	190
5.10	Symmetric Operations .....	192
5.11	Chapter Notes .....	198
<b>6</b>	<b>Braided Hopf Algebras</b> .....	199
6.1	Braided Objects .....	199
6.2	Free Braided Hopf Algebra .....	203
6.3	Differential Calculi and Constants .....	216

6.4	Categorical Subspaces .....	220
6.5	Combinatorial Rank .....	223
6.6	Braided Shuffle Hopf Algebra .....	224
6.7	Nichols Algebra.....	229
6.8	Radford Biproduct .....	232
6.9	Filtrations and Subalgebras of the Free Braided Hopf Algebra .....	238
6.10	Chapter Notes .....	243
<b>7</b>	<b>Binary Structures</b> .....	245
7.1	Lie Superalgebras.....	245
7.2	Color Lie Algebras .....	247
7.3	Lie Algebras in Symmetric Categories .....	250
	7.3.1 Universal Enveloping Algebra .....	252
	7.3.2 Embedding into the Universal Enveloping Algebra .....	254
	7.3.3 PBW Isomorphism .....	259
7.4	Free Lie $\tau$ -Algebra .....	263
7.5	Chapter Notes .....	271
<b>8</b>	<b>Algebra of Primitive Nonassociative Polynomials</b> .....	275
8.1	Nonassociative Polynomials .....	276
8.2	Shestakov-Umirbaev Operations.....	279
8.3	Lie Algebra of Nonassociative Products.....	282
8.4	Chapter Notes .....	286
	<b>References</b> .....	289
	<b>Index</b> .....	299



<http://www.springer.com/978-3-319-22703-0>

Quantum Lie Theory

A Multilinear Approach

Kharchenko, V.

2015, XIII, 302 p., Softcover

ISBN: 978-3-319-22703-0