Preface

This book is about how symmetric functions can be used in enumeration. The development is entirely self-contained, including an extensive introduction to the ring of symmetric functions. Many of the proofs are combinatorial and involve bijections or sign-reversing involutions. There are numerous exercises with full solutions, many of which highlight interesting mathematical gems.

The intended audience is graduate students and researchers in mathematics or related subjects who are interested in counting methods, generating functions, or symmetric functions. The mathematical prerequisites are relatively low; we assume the readers possess a knowledge of elementary combinatorics and linear algebra. We use the basic ideas of group theory and ring theory sparingly in the book, using them mostly in Chapter 6.

Chapter 1 introduces fundamental combinatorial objects such as permutations and integer partitions. Statistics on permutations and rearrangements are defined and relationships between q-analogues of n, $n!$, and $\binom{n}{k}$ are given, as these are used in later chapters. We also provide an introduction to generating functions. Much of the material in this introductory chapter is classic.

Symmetric functions are introduced in Chapter 2. Our development emphasizes the combinatorics of the transition matrices between bases of symmetric functions in a way that cannot be found elsewhere. Readers may find this approach more accessible than those in other books that discuss symmetric functions. This material is essential to understanding the later chapters in the book; after all, this book is all about how to use the relationships between symmetric functions to solve counting problems.

One of the major ideas this book highlights is that ring homomorphisms applied to the ring of symmetric functions can be used to find interesting generating functions. This is first applied in Chapter 3, where we use the background material introduced in Chapters 1 and 2 to find an assortment of generating functions for permutation statistics. We are able to count and refine permutations according to restricted appearances of descents and prove a number of results about words.

In Chapter 4, the techniques introduced in Chapter 3 are extended to find generating functions for a variety of objects. The exponential formula and the generating functions derived from linear recurrence equations can be found with the methods introduced in Chapter 4.

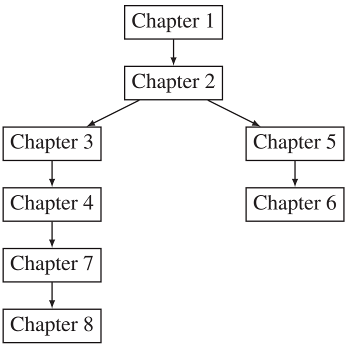
The Robinson-Schensted-Knuth algorithm is presented in Chapter 5, an important algorithm which needs to be included in any book on symmetric functions and enumeration. Connections are made to increasing subsequences in permutations and words and the Schur symmetric functions. A q-analogue of the celebrated hook length formula is proved.

Symmetric functions are used to prove Pólya's enumeration theorem in Chapter 6, allowing us to count objects modulo symmetries. This is a standard topic in many courses on combinatorics, but too often it is not made clear that Pólya's enumeration theorem can be properly phrased using the language of symmetric functions. We also give a new combinatorial proof of the Murnaghan-Nakayama rule from the Pieri rules.

Chapters 7 and 8 are more specialized chapters than the others, and may appeal to researchers in this area. In Chapter 7 we study consecutive pattern matches in permutations, words, cycles, and alternating permutations. Chapter 8 introduces the reciprocity method, an approach which can provide a way to define ring homomorphisms with desirable properties.

Most of the results and exercises found in Chapters 3, 4, 7, and 8 are appearing in book form for the first time.

The chapter dependency chart for the text is as follows:



Anthony Mendes thanks Jeff Remmel for introducing him to some wonderful mathematics and for working with him over the years. He thanks all students who took Math 435 or Math 530 in the fall of 2014 at Cal Poly San Luis Obispo for carefully reading a preliminary copy of this text. Thanks also go to the following people who pointed out at least one typographical error or suggested a specific improvement to the text: Shelby Burnett, Maggie Conley, Saba Gerami, Mike LaMartina, Amanda Lombard, Thomas Stienke, and Thomas J. Taylor. Most importantly, Anthony Mendes thanks his wife Amy and daughters Ava, Tabitha, and Ruby for their support.

Jeff Remmel would like to thank Adriano Garsia, Domique Foata, and Ian Macdonald who introduced him to the theory of symmetric functions and enumerative combinatorics. He also thanks the following Ph.D. students who helped him over the years to develop parts of the theory presented in this book: Tamsen Whitehead, Diseree Beck, Tom Langley, Jennifer Wagner, Tony Mendes, Amanda Riehl, Jeff Liese, Evan Fuller, Andy Niedermaier, Andre Harmse, Miles Jones, Adrian Duane and Quang Bach. Finally, he thanks his family members, especially his wife Paula, for their continuing support which made his research career possible.

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July 2015

http://www.springer.com/978-3-319-23617-9

Counting with Symmetric Functions
Mendes, A.; Remmel, J.
2015, X, 292 p. 209 illus., Hardcover
ISBN: 978-3-319-23617-9