

Preface

The ongoing progress both in theoretical computer algebra and in its expanding applications has led to a need for forums bringing together both the scientists working in the area of computer algebra methods and systems and the researchers who apply the tools of computer algebra for the solution of problems in scientific computing, in order to foster new and closer interactions. This need has led to the series of CASC (Computer Algebra in Scientific Computing) workshops, which started in 1998 and since then has been held annually.

This year the seventeenth CASC conference takes place in Aachen (Germany). Computer algebra in Aachen has a long and a fruitful history.

RWTH Aachen University is the birthplace of the system GAP: Groups, Algorithms and Programming (www.gap-system.org). GAP was initiated in 1986 by J. Neubüser and was managed by himself until his retirement in 1997. It is one of the most influential computer algebra systems oriented towards computational group theory. The big number of GAP citations from applied sciences alone confirms its widely recognized interdisciplinary importance.

It is interesting to know that several predecessors of GAP and – to some extent – also the system MAGMA, including the system CAYLEY, were developed with very active participation of people from Aachen. Moreover, developments “made in Aachen” include the popular package MEATAXE by M. Ringe for working with matrix representations over finite fields (www.math.rwth-aachen.de/~homes/MTX/), the package CHEVIE, providing symbolic calculations with generic character tables of groups of Lie type, Coxeter groups, Iwahori-Hecke algebras and other related structures, and the Modular Atlas project (www.math.rwth-aachen.de/~MOC/) by G. Hiss, F. Lübeck, and others.

Over the years, a number of packages for MAGMA (magma.maths.usyd.edu.au/magma/) have been written in Aachen as well.

In the group led by W. Plesken, both GAP and MAPLE (www.maplesoft.com/) have been used and developed further from very early times on. Numerous computer algebra packages evolved, notably the MAPLE suite OREMODULES (wwwb.math.rwth-aachen.de/OreModules/) by D. Robertz (RWTH) and A. Quadrat (INRIA), which includes several subpackages written not only by the authors. OREMODULES provides rich functionality for linear control systems over noncommutative Ore algebras.

In addition, a notable implementation of the involutive algorithms, designed by V.P. Gerdt (Dubna) and Yu.A. Blinkov (Saratov) and based on Janet’s monomial division, has been done in this group. The MAPLE package INVOLUTIVE (wwwb.math.rwth-aachen.de/Janet/involutive.html) computes Janet bases and Janet-like Gröbner bases for submodules of free modules over a polynomial ring, while JANET (wwwb.math.rwth-aachen.de/Janet/janet.html) can do these computations for linear systems of partial differential equations and LDA

(LINEAR DIFFERENCE ALGEBRA) for linear difference systems. A further MAPLE package called JANETORE (wwwb.math.rwth-aachen.de/Janet/janetore.html) can carry out these computations for submodules of free left modules over certain noncommutative Ore algebras. More recently, two algorithms for the triangular Thomas decomposition of both algebraic (ALGEBRAICTHOMAS) and differential (DIFFERENTIALTHOMAS) systems of equations and inequations, respectively, were designed and implemented in MAPLE by T. Bächler, M. Lange-Hegermann, and D. Robertz in cooperation with V.P. Gerdt (wwwb.math.rwth-aachen.de/thomasdecomposition/).

The open source project GINV (invo.jinr.ru/ginv/index.html), which implements in C++ the involutive algorithm by V.P. Gerdt and Y.A. Blinkov for polynomial systems, was also partially developed in Aachen.

M. Barakat initiated the homalg project (homalg.math.rwth-aachen.de/) for constructive homological algebra, which has evolved into a multi-author multi-package open source software project. The project exploits many capabilities of the GAP4 programming language (the object model, the method selection mechanism, and the deduction system). It implements on one hand high-level algorithms for constructive Abelian categories, e.g., spectral sequences of bi-complexes, and on the other hand concrete realizations of such constructive categories, e.g., categories of finitely presented modules over the wide class of so-called computable rings. The ring arithmetic and all matrix operations over such rings are delegated to other dedicated computer algebra systems.

The group of E. Zerz and V. Levandovskyy works with SINGULAR (www.singular.uni-kl.de) and has been implementing various algebraic tools towards, among other things, system and control theory. V. Levandovskyy has furthermore been developing the noncommutative extensions PLURAL (for a broad class of Noetherian domains) and LETTERPLACE (for free associative algebras) of SINGULAR, which provide extended functionality, based on Gröbner bases, for various noncommutative rings. In particular, several libraries for SINGULAR:PLURAL contain sophisticated implementations of fundamental algorithms for algebraic D -module theory.

The above-listed impressive activities in several wide areas of computer algebra have predetermined, to a large extent, the choice of Aachen as a venue for the CASC 2015 workshop.

This volume contains 33 full papers submitted to the workshop by the participants and accepted by the Program Committee after a thorough reviewing process. Additionally, the volume includes two invited talks.

Polynomial algebra, which is at the core of computer algebra, is represented by contributions devoted to the computation of resolutions and Betti numbers with the aid of a combination of Janet bases and algebraic discrete Morse theory, automatic reasoning in reduction rings using the THEOREMA software system, estimation of the complexity of a new algorithm for recognizing a tropical linear variety, conversion of a zero-dimensional standard basis into a standard basis with respect to any other local ordering, simplification of cylindrical algebraic decomposition formulas with the aid of a new multi-level heuristic algorithm,

solving polynomial systems with polynomial homotopy continuation, implementation of solvable polynomial rings in the object oriented computer algebra system JAS (Java Algebra System), explicit construction of the tangent cone of a variety using the theory of regular chains, computation of the limit points of the regular chain quasi-component by using linear changes of coordinates, obtaining new bounds for the largest positive root of a univariate polynomial with real coefficients, real root isolation by means of root radii approximation, and application of the algebra of resultants for distance evaluation between an ellipse and an ellipsoid.

Among the many existing algorithms for solving polynomial systems, perhaps the most successful numerical ones are the homotopy methods. The number of operations that these algorithms perform depends on the condition number of the roots of the polynomial system. Roughly speaking the condition number expresses the sensitivity of the roots with respect to small perturbations of the input coefficients. The invited talk by E. Tsigaridas deals with the problem of obtaining effective bounds for the condition number of polynomial systems with integer coefficients. The provided bounds depend on the number of variables, the degree, and the maximum coefficient bitsize of the input polynomials. Such bounds allow one to estimate the bit complexity of algorithms like the homotopy algorithms that depend on the condition number for solving polynomial systems.

Two papers deal with the solution of difference systems: hypergeometric solutions of first-order linear difference systems with rational-function coefficients; computation of regular solutions of linear difference systems with the aid of factorial series.

The topics of two further papers are related to problems in linear algebra and the theory of matrices: in one of them, it is proposed to perform a randomized preprocessing of Gaussian elimination without pivoting with the aid of circulants; the other paper proposes a new form of triangular decomposition of a matrix, which is termed the LDU-decomposition.

Several papers are devoted to using computer algebra for the investigation of various mathematical and applied topics related to ordinary differential equations (ODEs): obtaining the algebraic general solutions of first order algebraic ODEs, obtaining the analytic-form solutions of two-point boundary problems with one mild singularity with the aid of the Green's function and the *Theorema* software system, investigation of quasi-steady state phenomena arising in the solutions of parameter-dependent ODE systems, in particular, for reaction networks, symbolic computation of polynomial first integrals of systems of ODEs, and homotopy analysis of stochastic differential equations with maxima.

Two papers deal with applications of symbolic and symbolic-numeric computations for investigating and solving partial differential equations (PDEs) in mathematical physics: partial analytical solution of a PDE system from Kirchhoff's rod theory, symbolic-numeric solution of boundary-value problems for the Schrödinger equation.

Among the numerical methods to approximately solve partial differential equations on complicated domains, finite element methods are often the preferred

tool. The invited talk by V. Pillwein shows how symbolic computations can be used in the construction of higher-order finite element methods. The analysis and construction methods range from Gröbner basis computations and cylindrical algebraic decomposition to algorithms for symbolic summation and integration.

Applications of symbolic and symbolic-numeric algorithms in mechanics and physics are represented by the following themes: investigation of the stability of relative equilibria of oblate axisymmetric gyrostat by means of symbolic-numerical modeling, investigation of the influence of constant torque on stationary motions of satellite, investigation of invariant manifolds and their stability in the problem of motion of a rigid body under the influence of two force fields, development of a symbolic algorithm for generating irreducible bases of point groups in the space of $SO(3)$ group for its application in molecular and nuclear physics, analysis of reaction network systems using tropical geometry, and approximate quantum Fourier transform and quantum algorithm for phase estimation.

The remaining topics include the use of resultants and cylindrical algebraic decomposition for the symbolic determination of the topology of a plane algebraic curve, the solution of the problem of interpolating the reduced data in cases when the interpolation knots are unknown, and safety verification of hybrid systems using certified multiple Lyapunov-like functions.

The CASC 2015 workshop was supported financially by a generous grant from Deutsche Forschungsgemeinschaft (DFG). Further financial support was obtained from our sponsors Additive, Dr. Hornecker Software-Entwicklung and IT-Dienstleistungen, and Maplesoft.

Our particular thanks are due to the members of the CASC 2015 local organizing committee at RWTH Aachen University, i.e., Eva Zerz and Viktor Levandovskyy, who provided us with the history of the computer algebra activities at RWTH and handled all the local arrangements in Aachen. Furthermore, we would like to thank all the members of the Program Committee for their thorough work. We are grateful to Matthias Seiß (Kassel University) for his technical help in the preparation of the camera-ready manuscript for this volume. Finally, we are grateful to the CASC publicity chair Andreas Weber (Rheinische Friedrich-Wilhelms-Universität Bonn) together with Hassan Errami for the design of the conference poster and the management of the conference web page.

July 2015

Vladimir P. Gerdt
Wolfram Koepf
Werner M. Seiler
Evgenii V. Vorozhtsov

<http://www.springer.com/978-3-319-24020-6>

Computer Algebra in Scientific Computing
17th International Workshop, CASC 2015, Aachen,
Germany, September 14-18, 2015, Proceedings
Gerdt, V.P.; Koepf, W.; Seiler, W.M.; Vorozhtsov, E.V.
(Eds.)
2015, XIII, 494 p. 75 illus. in color., Softcover
ISBN: 978-3-319-24020-6