A Multiscale Adaptive Mesh Refinement Approach to Architectured Steel Specification in the Design of a Frameless Stressed Skin Structure

Paul Nicholas, David Stasiuk, Esben Clausen Nørgaard, Christopher Hutchinson and Mette Ramsgaard Thomsen

Abstract

This paper describes the development of a modelling approach for the design and fabrication of an incrementally formed, stressed skin metal structure. The term incremental forming refers to a progression of localised plastic deformation to impart 3D form onto a 2D metal sheet, directly from 3D design data. A brief introduction presents this fabrication concept, as well as the context of structures whose skin plays a significant structural role. Existing research into ISF privileges either the control of forming parameters to minimise geometric deviation, or the more accurate measurement of the impact of the forming process at the scale of the grain. But to enhance structural performance for architectural applications requires that both aspects are considered synthetically. We demonstrate a mesh-based approach that incorporates critical parameters at the scales of structure, element and material. Adaptive mesh refinement is used to support localised variance in resolution and information flow across these scales. The adaptation of mesh resolution is linked to structural analysis, panelisation, local geometric formation, connectivity, and the calculation of forming strains and material thinning.

Introduction

The research structure *StressedSkins* investigates the highly integrated material and formal specification of an inexpensive, long-standing architectural material best known for its homogeneity-steel. In this paper, we describe the process of asymmetric incremental sheet forming (ISF). We link ISF process parameters to variable

P. Nicholas (✉) · D. Stasiuk · E.C. Nørgaard
M.R. Thomsen
Centre for IT and Architecture, Royal Danish Academy of Fine Art, School of Architecture, Copenhagen, Denmark
e-mail: paul.nicholas@kadk.dk

C. Hutchinson
Department of Materials Science and Engineering, Monash University, Melbourne, Australia

M.R. Thomsen
Centre for Information Technology and Architecture (CITA), The Royal Danish Academy of Fine Art, School of Architecture, Design and Conservation, Copenhagen, Denmark

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specification at three architectural scales: within the material, cold working increases the strength; within the panel, forming out of plane increases the local stiffness of the sheet; within the structure, overall rigidity is obtained and increased in relation to panel locations, and where geometric formations on one side of the stressed skin connects with geometric formations on the other. We introduce a computational modelling approach for operationalising these relations, based on the use of an adaptive, unstructured mesh that instrumentalises interscalar feedback during the simulation process.

ISF Process

Incremental sheet forming (ISF) is an innovative fabrication method for imparting 3D form on a 2D metal sheet, directly informed by a 3D CAD model. In the ISF process, a simple tool moves over the surface of a thin (0.5–1 mm) metal sheet so as to cause localised plastic deformation (Fig. 1) (Jeswiet et al. 2005). ISF is of interest for three major reasons: it avoids the need for a costly, die (negative forming), by instead directly machining semi-finished pieces of metal. Secondly, because forming is highly localized, the force required does not increase with scale, meaning that there is theoretically no limit to the size of the sheet that is formed (Tisza 2012). Lastly, ISF has been shown to extend the formability of metals beyond what is achievable via conventional forming via stamping or deep drawing (Bagudanch et al. 2013). New research in this field examines scaling up the process: Ford and Boeing aim towards a system with an effective work area of 2 × 1.5 m (Energy 2015).

Transferred into architecture, ISF moves from a prototyping technology to a production technology. Within the context of mass customisation, it provides an alternate technology through which to incorporate, exploit and vary material capacities within the elements that make up a building system. Potential architectural applications have been identified in folded plate thin metal sheet structures (Trautz and Herkrath 2009) and customised load-adapted architectural designs (Kalo and Newsum 2014; Brüninghaus et al. 2013). Recent research has established ISF as structurally feasible at this scale (Bailly et al. 2015), and explored the utilisation of forming cone geometries as means to reach from one skin to another (Kalo and Newsum 2014; Bailly et al. 2015).

Stressed Skin Structures

In this research, the ISF process is used to fabricate a stressed skin structure. Stressed skin structures are thin sheet structures in which the skin is structurally active, bearing a considerable part of the load and providing significant rigidity. They are an intermediate between monocoque

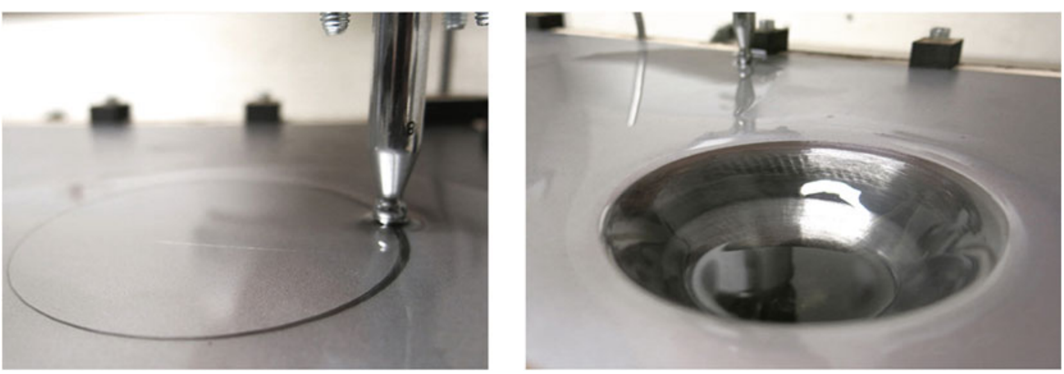

Fig. 1 Single point incremental forming

and rigid frame approaches, and have been particularly associated with light weight structures. In their design, rigidity is a central concern. One of the main problems is to ensure rigidity at multiple scales: against the instability of the whole structure and also the local buckling of the parts which have to carry compressive load. *StressedSkins* develops a structural approach in which the skin carries planar and shear forces, without an additional framing system, at the scale of a pavilion. Local corrugation avoids buckling through geometric stiffening of the skin, while shear connectors transfer loads between upper and lower skins to rigidize the entire structure (Fig. 2). These features, as well as all in-plane connections, and shear connections between the upper and lower skin, are achieved through the deformation of the skin and are outcomes of the computational modelling process.

Research Objective: A Mesh-Based Approach to Communication Across Scales

The object of this study is to explore how a mesh can work as a substrate for enacting and communicating various types of analysis across multiple scales to support the geometric specification, structural simulation and fabrication of a stressed skin structure. There are two established mesh-based methods for adapting resolution where required to capture complex dynamics, small scale geometry and scale sensitive calculations: the nesting of structured grids (multiple contiguous domains) and the adaptation of a non structured grid (a single continuous domain). The research begins with the aim to deploy a single, continuous-domain multi-scale mesh as an exclusive design medium for negotiating the form-finding and

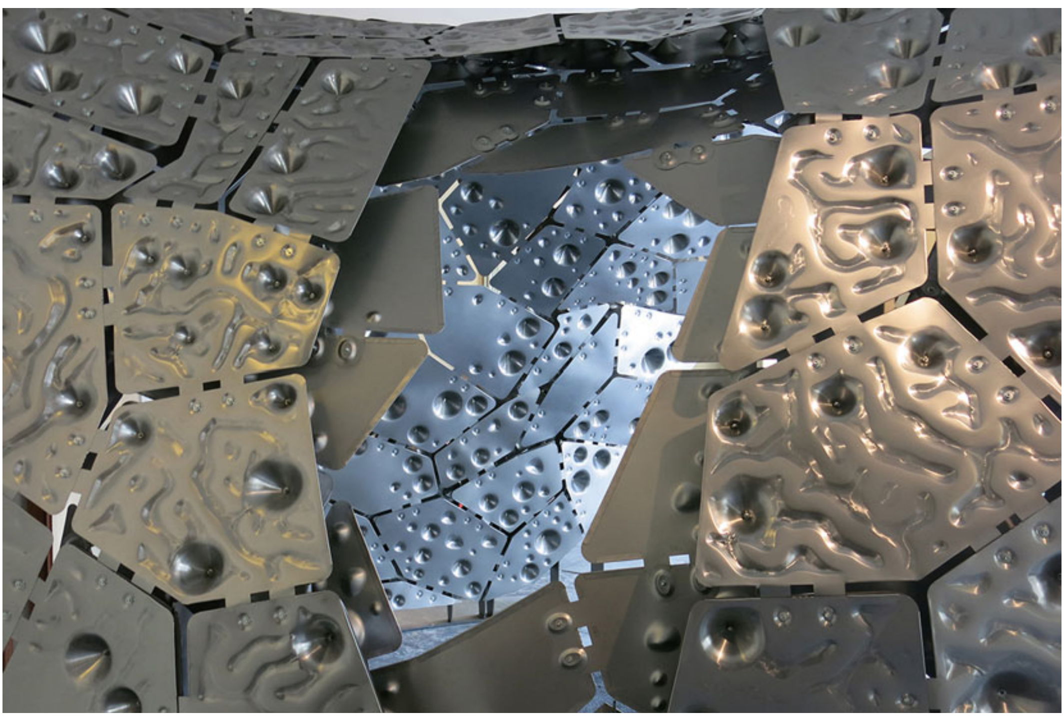

Fig. 2 The *StressedSkins* installation at the Danish Design Museum

analysis, and producing all relevant outputs for fabrication and representation. This is ultimately achieved, however, via a hybrid approach that implements both contiguous and continuous approaches.

Modelling Framework

The modelling framework for *StressedSkins* considers macro, meso and micro scales as markers along a continuum describing variable, interdependent functionalities within the design system (Fig. 3).

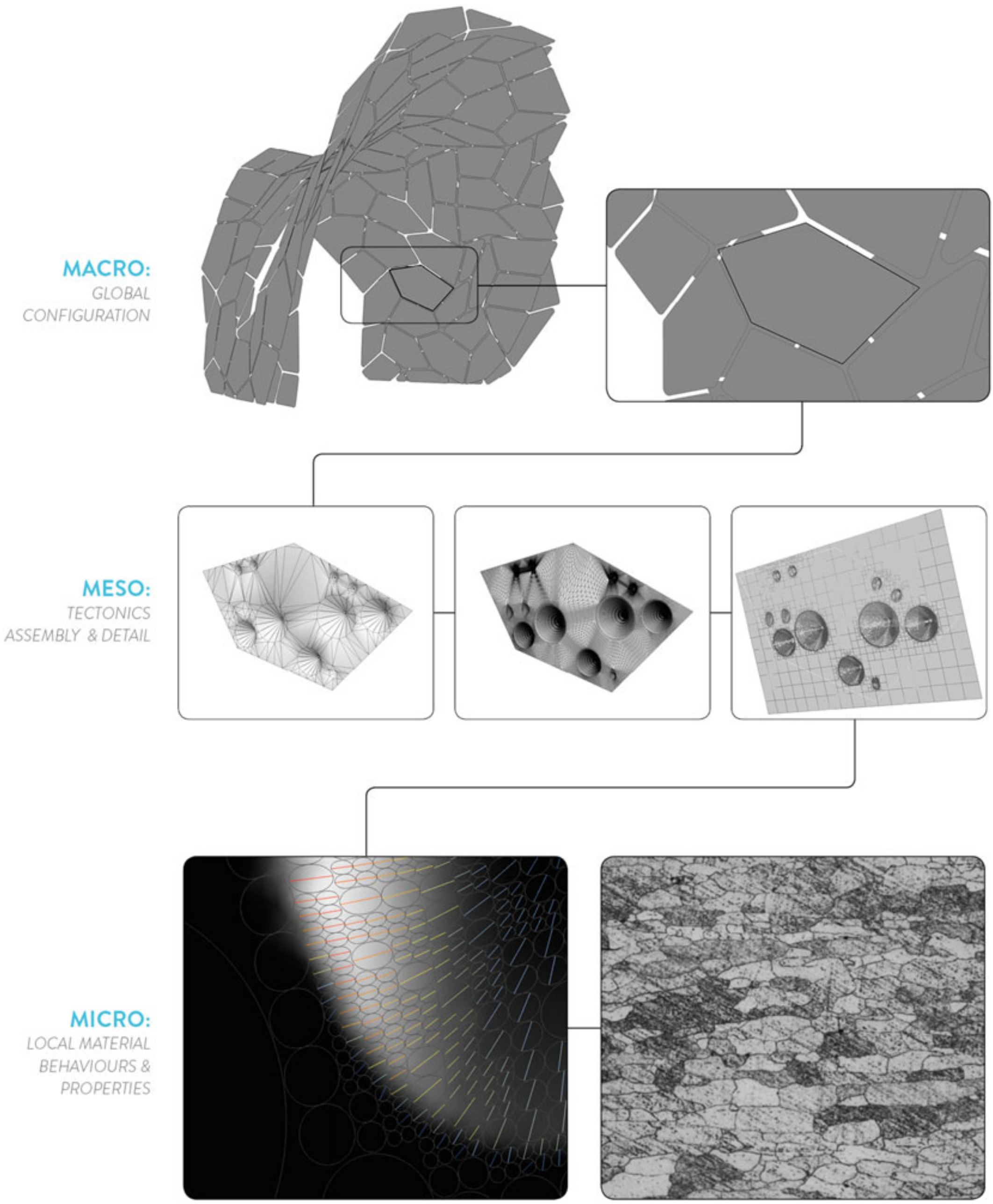

Fig. 3 Multiscale considerations in *StressedSkins*

In general, the macro scale encompasses the resolution of global design goals, overall geometric configurations, and a full-scale understanding of structural performance. The meso scale considers the project at an assembly and sub-assembly level, and is concerned with material behaviours tied to geometric transformation, detailing and component-level tectonic expression. The micro scale is concerned with relevant material characteristics at the most discretised level. The multi-scale modelling approach used here is then comprised of those techniques, which enable the information generated at each of these markers to flow both up and down the continuum.

Strategy and Computational Tooling

These modelling parameters are organised using a half-edge (or directed-edge) mesh data structure (Campagna et al. 1998). Half-edge meshes enable the deployment of n-gon faces (rather than more standard triangulated or quadrilateral faces). This opens up the possibility for designing with more complex topologies. In this case, a pentagonal tiling algorithm was used, resulting in a base mesh comprised of five-sided faces. The features of this mesh—such as its vertices, half-edges and faces—are coupled with a series of lists, dictionaries and Grasshopper data trees that effectively bundle within mesh elements critical design data related to: topology, form-finding and geometry; structural behaviour; material characteristics; connection detailing; and patterning and tectonic expression (Fig. 4).

The primary digital design instruments used are Rhino and Grasshopper as a base modelling environment, the Plankton library for scripting half-edge meshes, a beta library of the Kangaroo2 physics engine for form finding, and Karamba for FE analysis. A series of bespoke tools and implementations are created for managing and modifying the design meshes. These operate such that the meshes are transformed to operate at scales appropriate to particular function—or, when necessary, handshake with other meshes—while retaining the integrity of these key relationships and informing the design with new relevant data.

Macro Scale

Preconfiguration

The configuration of the overall form is developed through a multi-stage modelling process. The first stage entails the top-down drawing of a single target design surface in response to constraints introduced by the site and in pursuit of specific design ambitions. This small site—the foyer of the Danish Design Museum—requires the maintenance of adequate open circulation along its two primary axes and through two additional doors. The design ambitions lie in producing an asymmetrical, cantilevered form that will help test the forming limits of the steel, its capacity for geometric adaptation, and its structural performance (Fig. 5).

With this surface form established, a shell FE analysis is performed, loaded under self-weight. Utilisation of shell elements resulting from this analysis are used to drive locally varying offsets from this single surface into two discrete target surfaces, one for the upper and one for the lower skin. Higher utilisations result in greater target depth between the two skins.

Generative Model

The two skins are then grown on each of these offset target surfaces by sequentially locating pentagonal panels, spiralling outward from a seed tile on each surface. The Kangaroo2 physics engine is a constraint (or goal) based form-finding and simulation design system. In addition to pre-configured goals embedded in the library, the scripting interface enables the coding of custom goals. The recursive form-finding employed here includes edge length and angle goals, which seek to maintain the ideal geometry prescribed by the planar tiling rule set. Target mesh pulling goals draw the mesh vertices out of plane and to the target surfaces. As each new panel is located in the assembly, the solver reconciles these geometrically competing interests

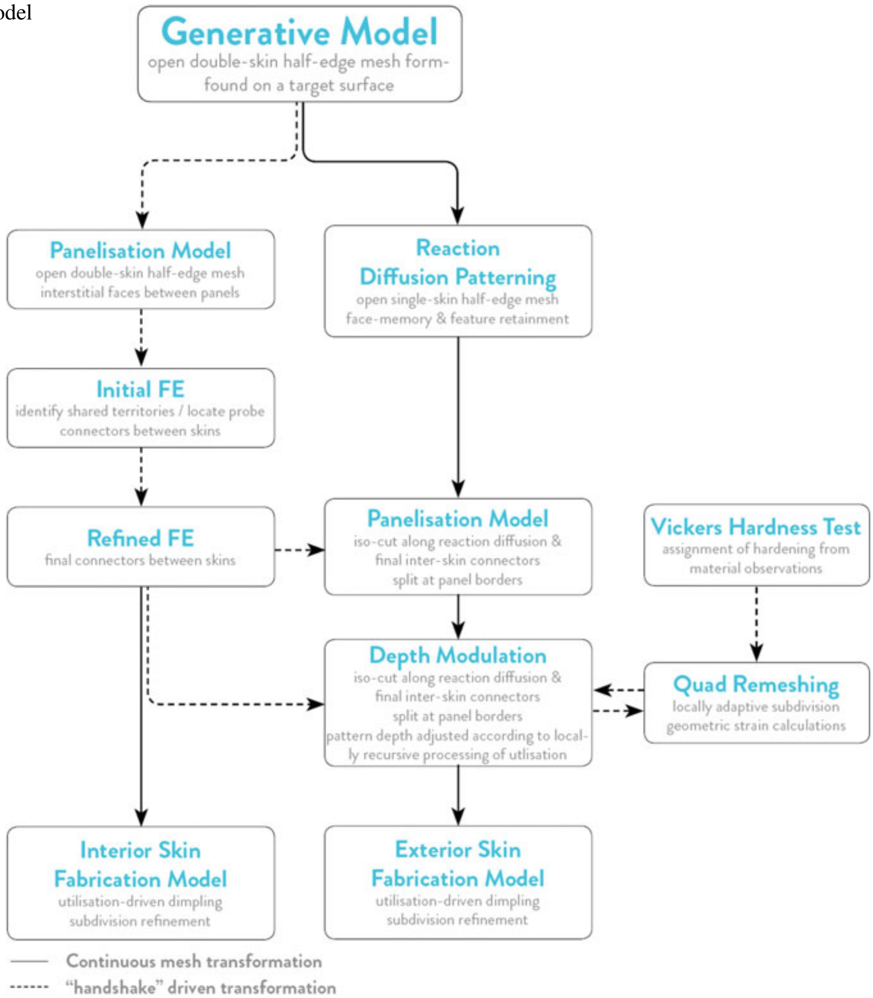

Fig. 4 Networked model dependencies

into a configuration that retains the topology of the tiling strategy but minimally adapts its emerging form to approximate the target surface. Additional goals include: vertex repelling goals between the upper and lower skins that increase diversity in connectivity between them; goals that neither skin collide with the other; and finally, after the target surfaces are fully tiled, a planarising goal is introduced to the panels. This last goal is essential to make fabrication viable, and further deforms each panel from its ideal planar geometry. Throughout this generative process, all relevant data between the form-finding solver and the mesh topology are coordinated as a unified data structure.

Panelisation Model

This pentagonal mesh then directly folds into a new, hybridised mesh that incorporates panel offsets for the geometric definition of boundaries, and specifies the connection detail between panels on the same skin. This consists of a male element on one panel and a female element on its adjacent panel. This male/female relationship is determined for each edge according to the generative sequence of each panel, such that older panels reach out with the male connection. This base model provides the specifications necessary for laser cutting the profiles—or dies—for each panel. A simple etched labelling strategy allows topological information to directly activate a

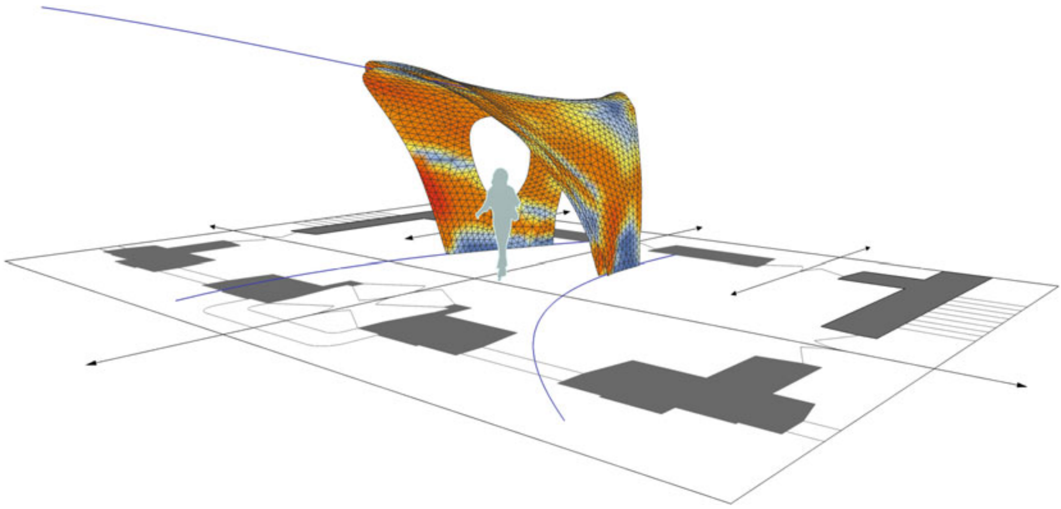

Fig. 5 Target design meshes for upper and lower skin, deployed in response to circulation constraints and with the ambition to produce an asymmetrical, cantilevered form. Variable thickness between the skins is determined by a simple FE analysis on the base mesh

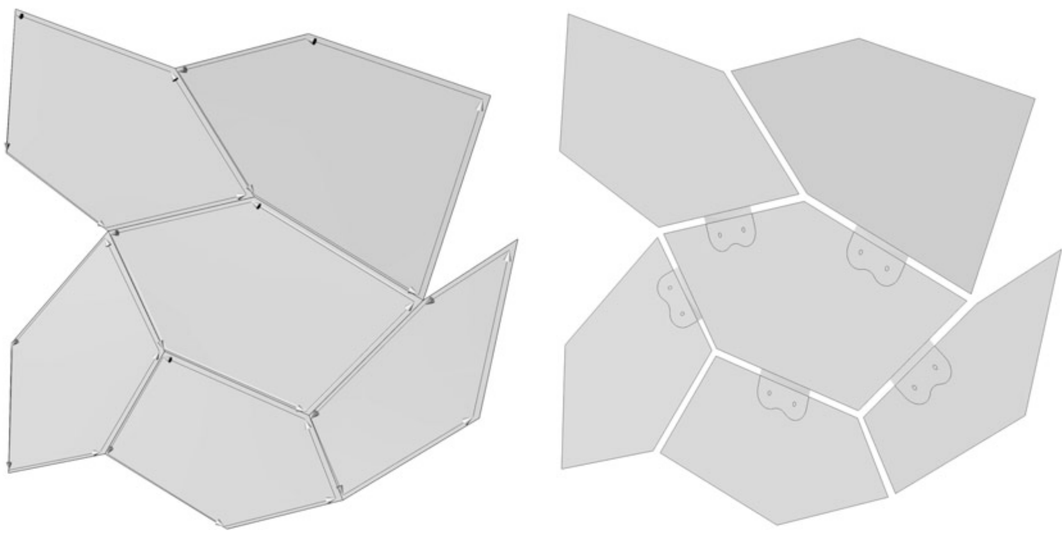

Fig. 6 Panelisation based on pentagonal tiling realised through a halfedge mesh. Planar geometry defined from generative form finding enables precise *offsets*, and order of formation and embedded topology simply organises male/female connection details

self-jigging assembly approach during installation (Fig. 6).

Simultaneously, connective faces within the design mesh are produced to bridge the gap between the newly offset panels. This supported topological consistency in the mesh and is subsequently key to the production of a finite element analysis mesh. Fabrication requires a minimum offset from the panel edges, as well as from the connection details. Based on the knowledge of these geometrical constraints, formable regions called "search boundaries" are then identified within each offset panel.

From these search boundaries, "shared territories" are identified between proximate panels on the lower and upper skins. This secondary form-finding procedure is comprised of a custom, iterative process whereby lines drawn normal to planes from opposite panels self-orient such that they each intersects at a point, with the requirement that the base of the cones defined by each potential connection fall within the search boundaries on both panels. These are derived from a large number of initial samples "feelers" from each panel which identify proximate feelers from panels on the opposite skin, and which dynamically relax within each panel's shared territory and search boundary. As a result many such many potential connection points are identified within each shared territory.

Initial FE Model

Once these territories and their constituent connection points are identified, the most central, or average, connection point is established for a single "probe connection" (Fig. 7). The conical geometries for connections between skins are integrated with the panels and connective faces—again along with inherited data structures from previous efforts—into a coarse triangulated mesh on which a finite element analysis is performed. The resulting modelling method relies on a hybrid of beam and shell elements, which in addition to data on nodal translations and rotations produces readings of shear forces at connection points between the skins, and utilisations and bending forces within the panels (Fig. 8).

Meso Scale

Refined FE Model

The results from this initial FE and the previously solved connection points within each shared territory synthesise into a refined FE model. For this it is asserted that each territory have at least one connection, and each panel—where possible—have at least three connections. Based on both of these interests, panels range in connection count from 0 to 5. The goal here is to maximise the diversity of panels connecting with each other across the skins in an effort to prevent possible hinging and maximise tri-angulation across skins each panel. When multiple connection points are allocated within a shared territory, they are aligned to configure perpendicular to shear forces in the initial models corresponding probe connection (Fig. 9).

Strategic Dimpling and Lower Skin Fabrication Model

This refined FE model is subjected to another analysis which directly drives the tectonic patterning of the lower skin. For this, utilisation forces within each panel are used to drive tectonics and improve performance through the

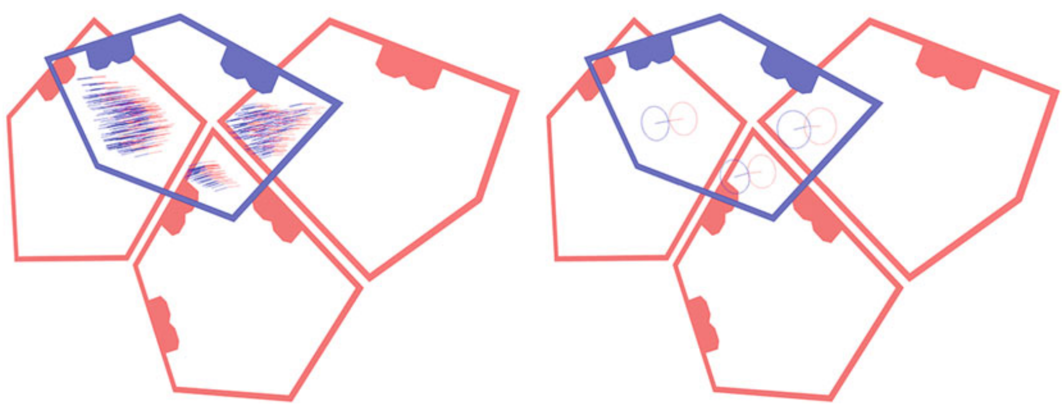

Fig. 7 Identification of "shared territories" between panels on the upper and lower skins, and the establishment of a single "probe connection" within each

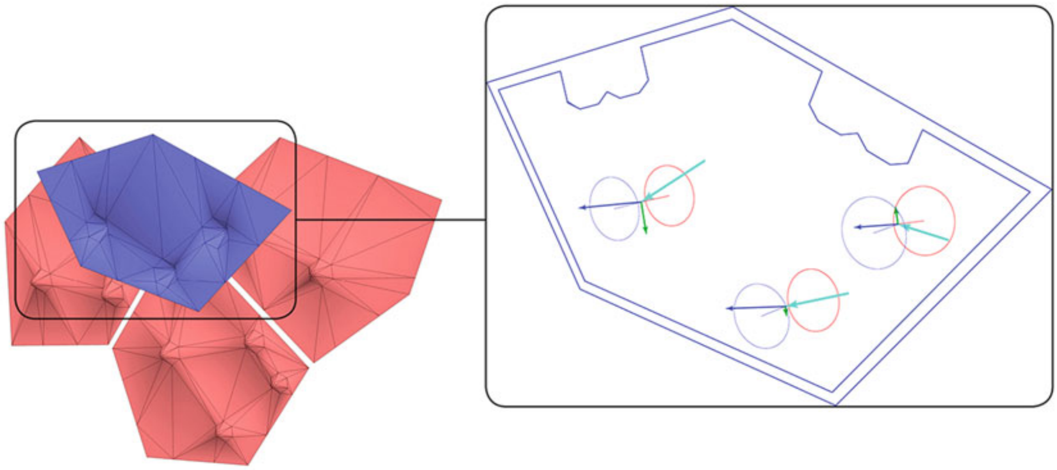

Fig. 8 Geometric definition of each probe connection, and extraction of shear forces at each connection point through an initial FE analysis

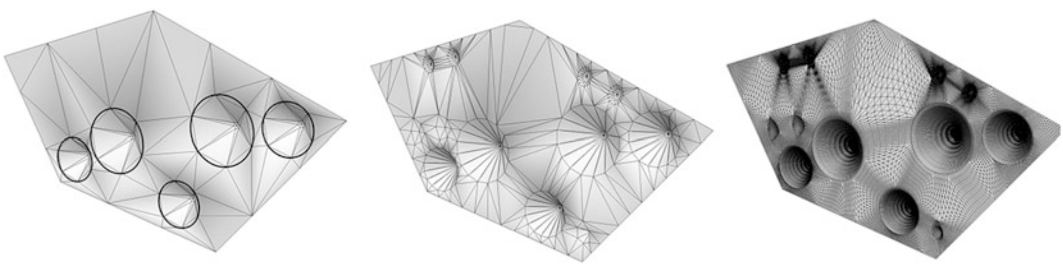

Fig. 9 (*Left* to *right*) **a** Redistribution of connections between skins based on improving resistance to shears identified in the initial FE analysis. **b** Strategic dimpling of panel as a tectonic response to in-panel utilisations. **c** Direct subdivision refinement for production of fabrication model

forming of a responsive pattern of oriented dimples within the structure. This pattern is located in areas of high utilisation and bending energy, the former of which is locally reduced through the strain hardening of the material and the latter better managed through the geometric stiffening resulting from the dimples. Shear forces within nearby connections orient the dimples, and enhance the structural expression of the pattern.

A final lower skin fabrication model is then synthesised from the refined FE model and the resulting dimple pattern, and subdivided to a higher resolution for fabrication, each panel systematically arrayed for extracting toolpaths.

Macro Scale

Reaction Diffusion

The foundation for the patterning strategy on the upper skin is the implementation of a Gray-Scott reaction-diffusion algorithm (comprised of the virtual ingredients U and V). This is executed on a higher-resolution, topologically persistent tri-angulated subdivision of the original generative pentagonal tile mesh, and informed with fixed vertex locations for connection elements that later enable the instantiation of precise connection geometries. The goal here is to produce a pattern on the upper surface whose isotropic nature assists in stiffening panels without favouring any particular directionality.

The subdivision technique used here allows for newly introduced features to inherit key data elements during both decimation and subdivision. It is an extension of a re-meshing script developed by Daniel Piker, which itself allows for the specification of fixed geometric features during these operations as well (Fig. 10). The mesh was then further discretised according to a surface-level iso cut, which split faces along edges according to the reaction-diffusion U and V ingredient parameters at each vertex. The areas inside the iso surface were considered formable (Fig. 11).

Meso to Micro Scale

Variable Resolution Quad Mesh

The meshing technique deployed here is an adaptive, variable resolution square mesh. This quad mesh is mapped onto an existing plane at a coarse resolution, starting from the panel plane. The vertices for each quad face are projected to the target geometry, and the face is tested for planarity. If the face is within a set tolerance, it remains at its current resolution. If it is not, it subdivides into four faces. This test is recursively applied to the mesh such that it locally adapts its resolution to relevant geometric features. After several iterations, each quad can be understood both in its initial unformed square state on the starting plane, and in its strained quad state resulting from geometric forming. A circle inscribed in the initial square is then projected onto the deformed quad, resulting in an ellipse whose primary and secondary axes produce both direction and lengths of strains resulting from the forming process (Fig. 12) (Emmens and Boogaard 2007). Strains are calculated as true strains (Eq. 1)

$$\varepsilon \text{ True } = \ln(\text{L0} \times \text{L1}) \qquad (1)$$

The resulting thickness strain (ε3) is determined by volume constancy (Eq. 2)

$$\varepsilon 1 + \varepsilon 2 + \varepsilon 3 = 0 \qquad (2)$$

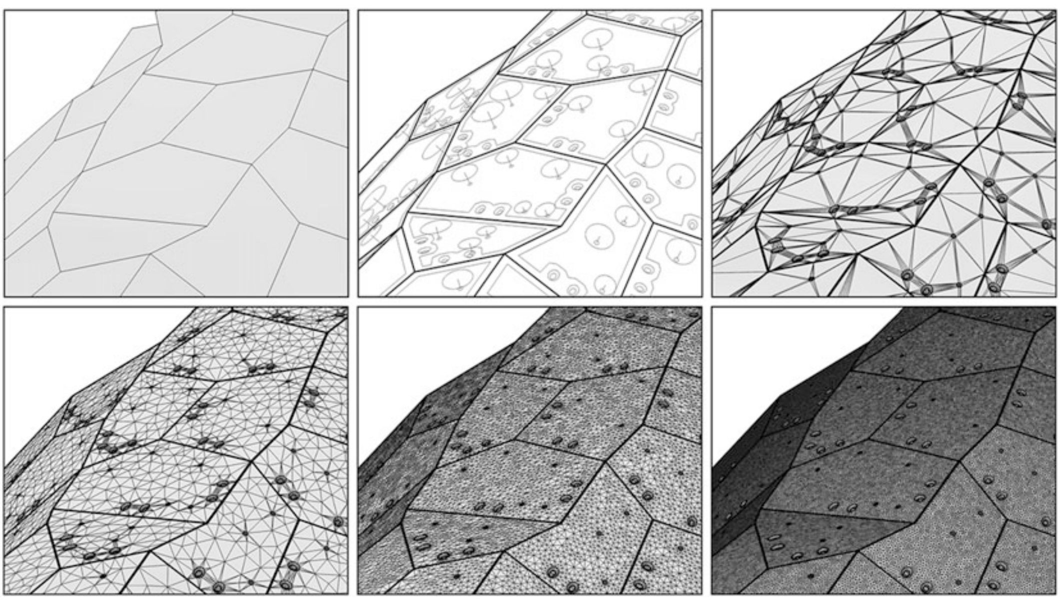

Fig. 10 (*Left* to *right*, *top* to *bottom*) **a** Unrefined upper skin pentagonal mesh. **b** Location of features for connections between panels within upper skin, as well as between upper and lower skin. **c** Introduction of features into pentagonal mesh as triangulated elements. **d** Through. **f** Subdivision of mesh to resolution required for executing reaction diffusion patterning scheme at desired resolution, while retaining key features both geometrically and topologically

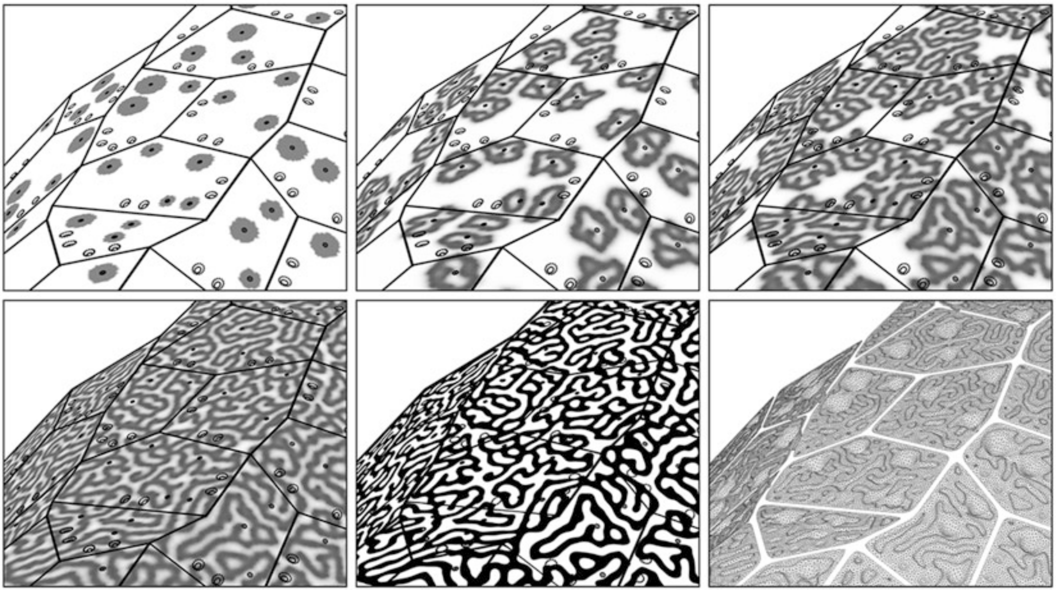

Fig. 11 (*Left* to *right*, *top* to *bottom*) **a** Initialisation of reaction diffusion algorithm on sub-divided mesh. **b** Through. **d** Pattern generation on the mesh, with values represented as colour gradient **e** Iso-splitting of pattern for sharp discretisation of formable areas. **f** Integration of connections between skins with pattern array

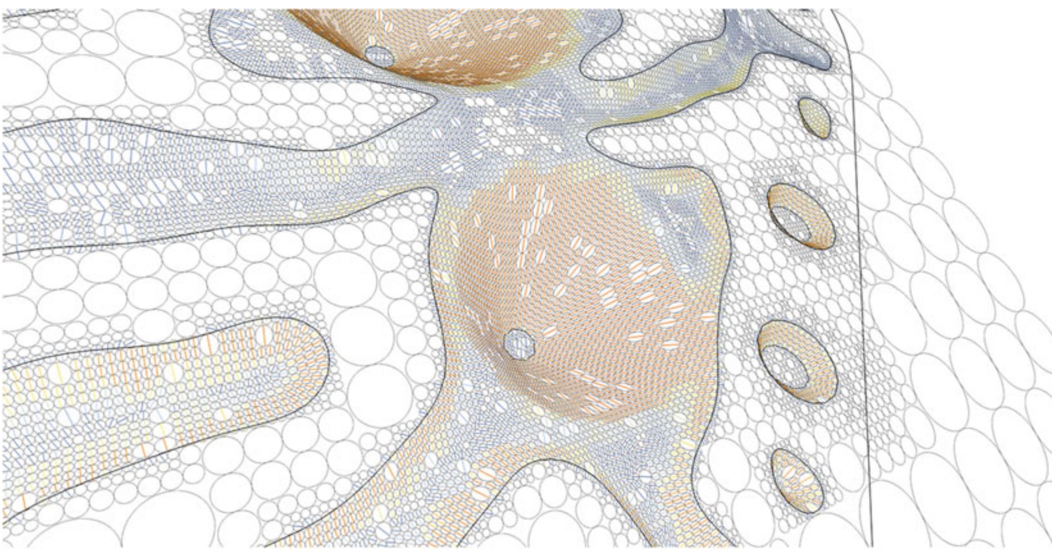

Fig. 12 Quad recursively remeshed to adapt through subdivision in order to more precisely describe geometric features. *Circles* inscribed in each quad are projected from the plane onto the mesh. Here deformation is read in the long axis of *ellipses* that have been deformed through projection. Flat areas on the mesh reflect unformed areas

Calibrating Micro Scale Calculations using Vickers Hardness and Optical Microscopy

To calibrate and verify the prediction of local strains, and to relate this to increase in yield strength, a series of empirical tests are made on material samples that vary processing parameters in a systematic fashion. To best incorporate all processing variables, all samples are produced using the same rig as used for final production. The local increase in strength is monitored using Vickers hardness tests with a 5 kg load measured on the cross sectional thickness of the sheet. Measurements are made along the cross-sectional length of a formed component and the resulting hardnesses are converted to estimated flow stresses and correlated with the local strains. Figure 13 shows a graph charting the observed relationship between local strains induced by forming and the measured flow stress calculated from the Vickers hardness measurements.

Equation 3 is derived from this curve, and is used to calculate, on individual mesh faces, local yield strength that was fed back into a Karamba finite element model.

$$\sigma Y = 43.407 \ln(\varepsilon) + 431.23 \quad (3)$$

Micro to Meso Scale

Depth Modulation and Upper Skin Fabrication Model

The modelling strategy for modulating the pattern depth on the upper skin results from the synthesis of several of these modelling systems. It relies on knowledge of connection geometries and formable areas for activating the reaction diffusion pattern within each panel; nodal translations and rotations at connection points both within and between skins from the refined, global FE analysis; and the capacity to locally calculate granular material properties through the adaptive quad mesh and the measured flow stress.

The process begins by discretising each panel from the reaction diffusion model, applying the geometry of the connection elements, and leaving the rest of the panel "un-formed". This minimum forming baseline geometry is then further subdivided into individual, triangulated elements, each of which is capable of having unique material properties assigned to it in the Karamba finite element modelling environment. An analysis is performed on each face, with extracted local strains from the adaptive quad remeshing technique described above. Resulting

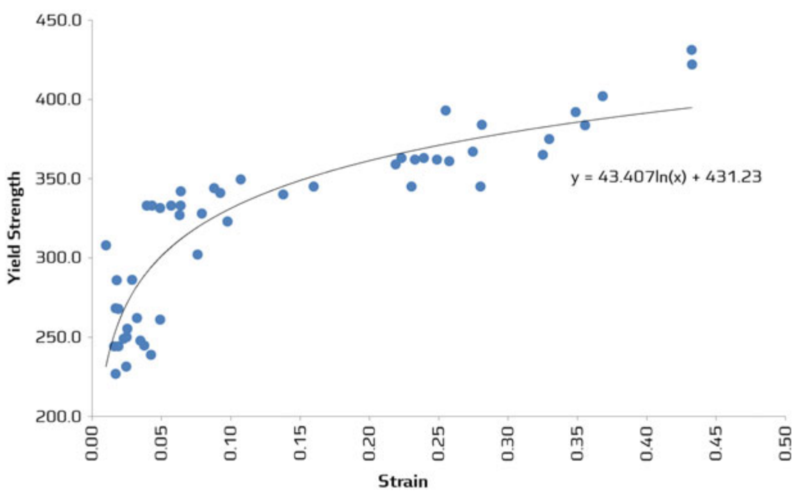

Fig. 13 Observed relationship between local strains induced by forming and measured flow stress calculated from the Vickers hardness measurements

yield strengths for each face are specified by the measured relationship between strain and yield strength.

Using Karamba's prescribed displacements enables the process of subjecting this locally informed mesh to corresponding nodal rotations and translations along its connection points, as extracted from the refined FE model. By virtually working the panel in this way, the depth of the each panel is incrementally and locally modulated. Here, vertices in the prescribed patterned area respond to proximity to faces that are heavily utilised when subjected to the prescribed displacement. Through an iterative accumulation of change in depth (Fig. 14)—and iterative analysis and application of resulting local changes in yield strength due to strain hardening (Figs. 15 and 16)—the patterning emerges as a

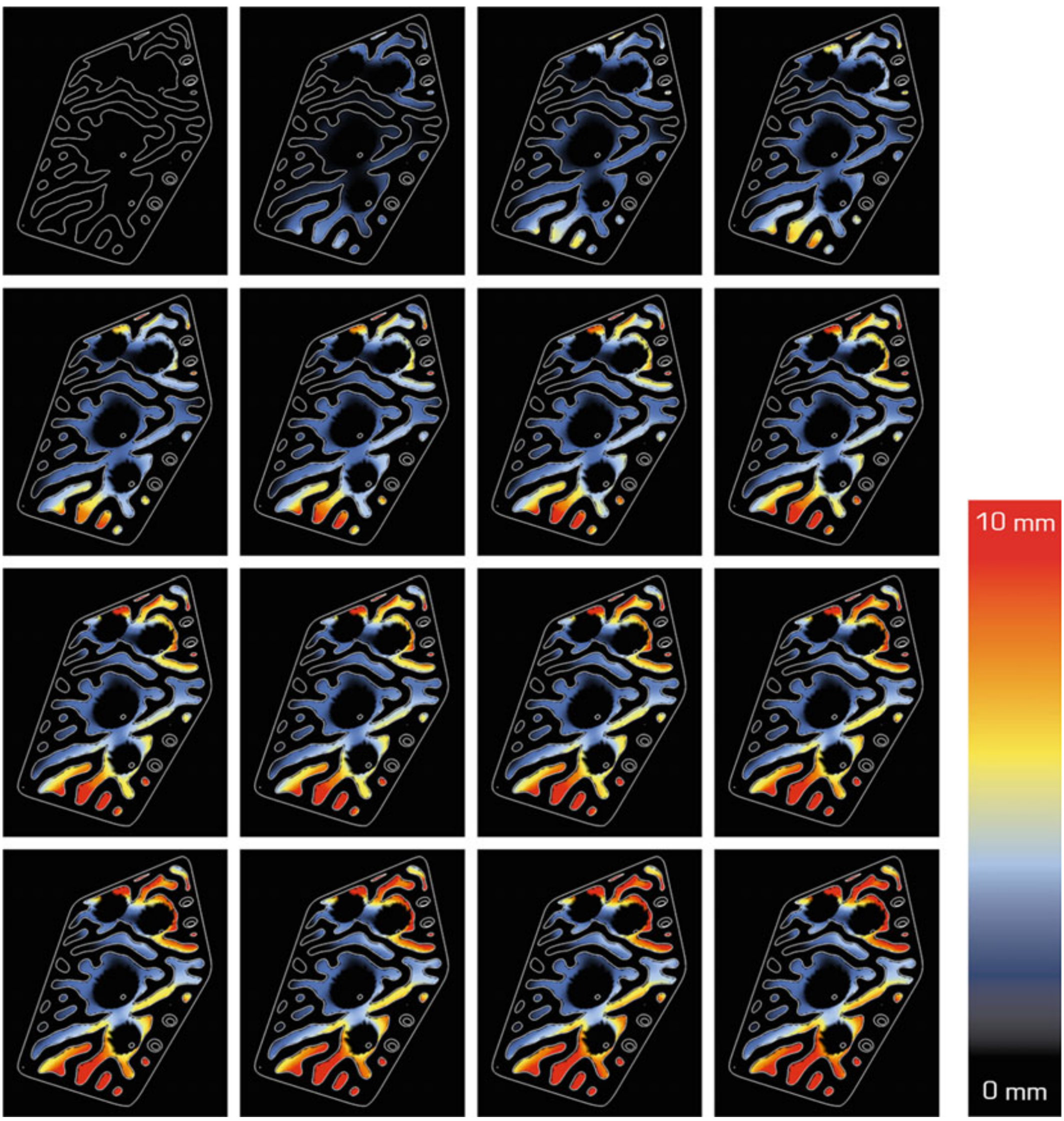

Fig. 14 Depth modulation of patterning introduced iteratively. The incremental increase in depth from a minimum *offset*. Here *black* represents areas that remain planar or are fixed at a depth required for connections, and *coloured areas* reflect formable areas iteratively deepening in response to local utilisations

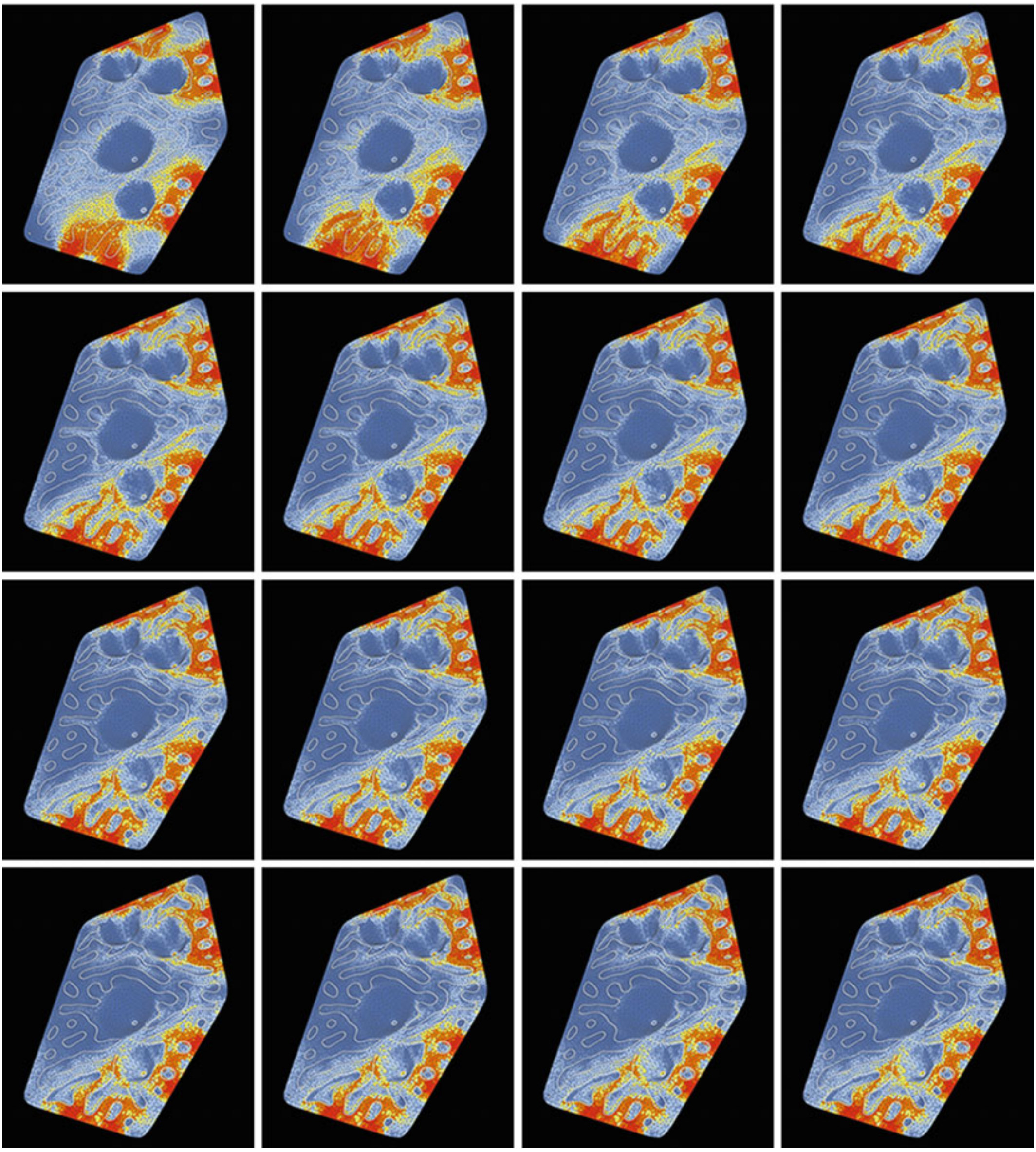

Fig. 15 Transformation in utilisation through local deepening of features and resultant strain hardening, exercised iteratively

tectonic response to utilisation and bending energies introduced by the global structural conditions (Fig. 17).

Following this iterative introduction of locally adaptive depth within channels defined by the reaction diffusion patterning, the mesh is subdivided to a higher resolution for fabrication, arrayed by panel toolpath extraction.

Fabrication and Toolpathing

An ABB IRB140 multi-purpose industrial robot is used to fabricate *StressedSkins*. During prototyping, panels up to a scale of 150 × 50 cm are produced; the working area for the final panels is approximately 50 × 100 cm. Conventional

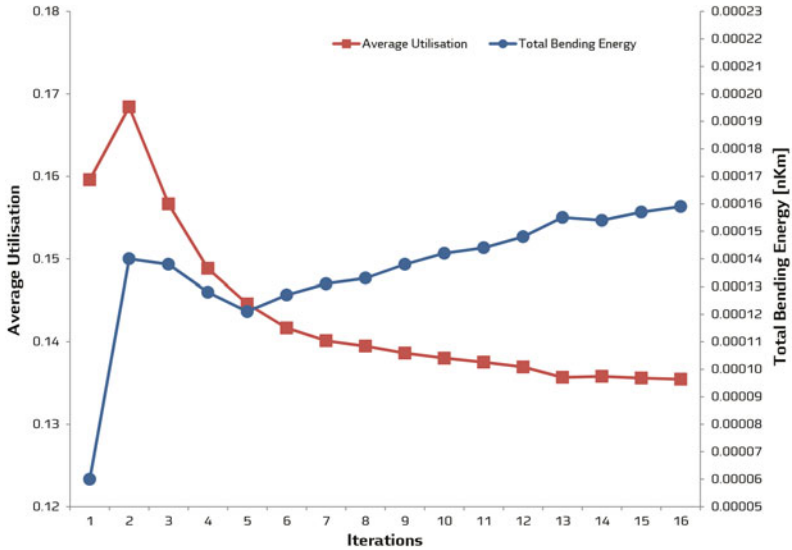

Fig. 16 Tracking of change in average utilisation for each mesh element, along with total bending energy captured within each panel as depth modulation is introduced and iterated through patterning and forming

Fig. 17 Variable depth patterning on an upper skin panel

toolpath generation algorithms generate simple sliced contours in the horizontal plane. However this strategy does not produce optimal results when applied to more complex geometries formed with ISF. To improve control over tooling time and surface quality, we develop a

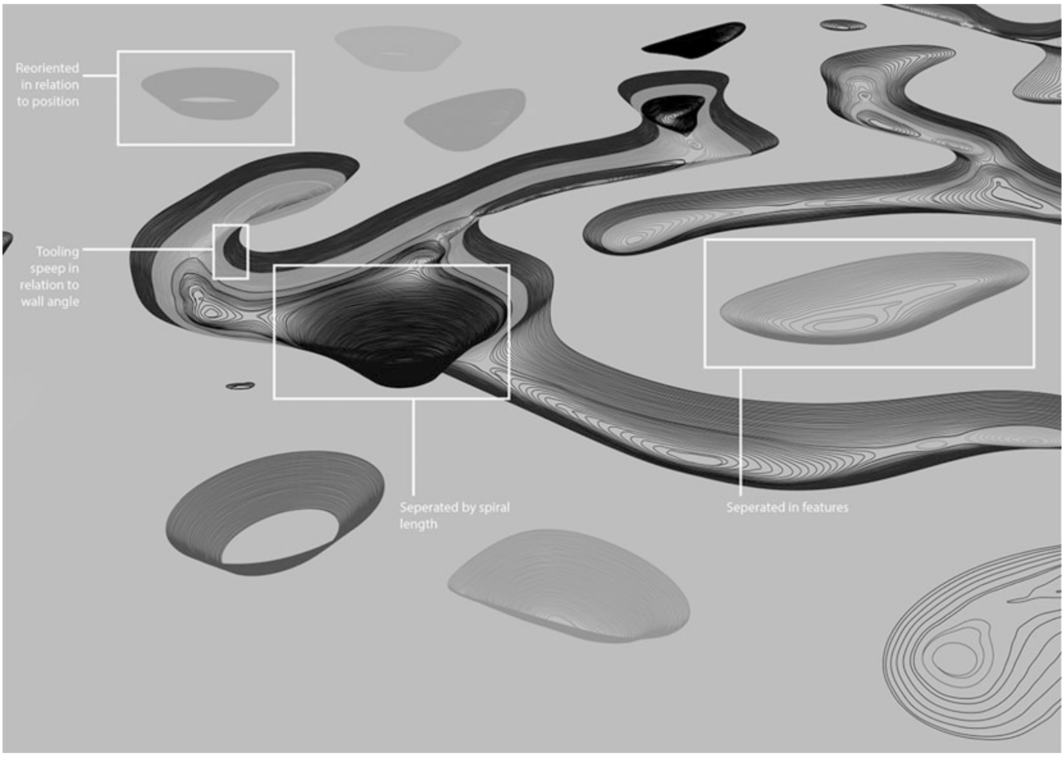

Fig. 18 Diagram of key drivers for toolpath extraction and organisation

toolpathing algorithm based on the established method of a spiral descent (Jeswiet et al. 2005). This algorithm integrates the grouping of features, the position of features, toolpath length and tooling speed in relation to wall angle (Fig. 18). Additionally, the algorithm is informed by knowledge gained directly through prototyping: this includes the observation of forming limits, optimal working areas, and tooling speed.

Discussion

This research demonstrates a mesh-based modelling method that synthesises structural performance interests with locally varying material properties, in the production of an architectural scale installation. The model is based around geometry, properties and mechanics at three scales. With the exception of two "handshakes", the model varies a single mesh topology to manage the complexity of bridging scales and functions while maintaining speed, flexibility and continuity of information flows up and down scales. Further work aims to incorporate the measure of strain into this continuous mesh topology by performing measures on triangular rather than quad faces (Fig. 19).

The model inputs include direct user inputs, geometric parameters, material and processing parameters, and feedback from finite element analysis. The micro scale calculations are calibrated through experimental testing of Vickers hardness, and are shown to be an accurate predictor. While at this stage the calculation of local yield strength is limited to informing finite element analyses, a next step will be to understand how parameters at this scale might also activate optimisation processes (Fig. 20).

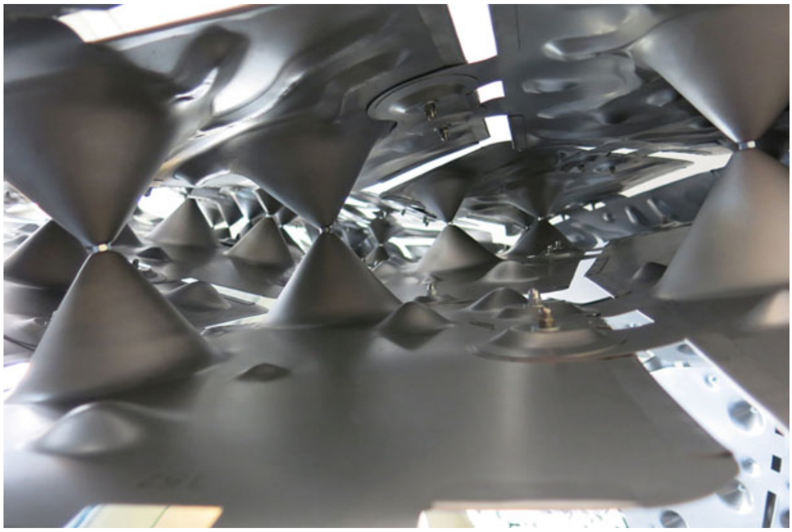

Fig. 19 Connectivity between the upper and lower skins

Fig. 20 The *StressedSkins* installation

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