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★**Error estimates for well-balanced schemes on simple balance laws.**

One-dimensional position-dependent models.

With a foreword by François Bouchut.

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This small book is a companion to L. Gosse's prior book [*Computing qualitatively correct approximations of balance laws*, SIMAI Springer Ser., 2, Springer, Milan, 2013; [MR3053000](#)]. It introduces, through well-chosen examples, the construction and analysis of 'well-balanced' numerical schemes for hyperbolic systems of balance laws. Such systems arise, for instance, when modeling shallow water flows above a varying topography. The main purpose of the book is to present an analysis of global (in space) error bounds for well-balanced schemes with a specific emphasis on the time dependence of such bounds.

Chapter 1 is an introduction with some historical perspectives. It also gives a synthetic view of the main conclusions one can draw from the presented results. Chapter 2 discusses two possible measures of the error between exact and approximate solutions: the local truncation (or consistency) error is shown to be somehow not adapted to quasilinear hyperbolic problems for which smoothness of the solution is limited due to finite time blow-up and the regions of smoothness evolve in time due to shock propagation. The authors then discuss some more suitable error estimates on the linearized shallow water system with topography. In particular, one numerical scheme is shown to display an error estimate that grows only linearly in time.

Chapter 3 focuses on the scalar balance law

$$(1) \quad \partial_t u + \partial_x f(u) = k(x) g(u),$$

under the non-resonant assumption $f'(u) \neq 0$. This equation is rewritten as a non-conservative 2×2 hyperbolic system by introducing the supplementary equation $\partial_t a = 0$ where a denotes a primitive of k . For this augmented system, the wave front tracking algorithm is shown to display an L^1 error estimate that grows linearly in time, independently of the sign of the source term $k(x) g(u)$. An important feature of the estimate is that it only involves the function k through its L^1 norm, which allows for steep gradients in applications. Some numerical tests illustrate the results. In Chapters 4 and 5, the analysis is extended to a 2×2 semilinear hyperbolic system of relaxation type

$$(2) \quad \begin{cases} \partial_t \rho + \partial_x J = 0, \\ \partial_t J + \partial_x \rho = 2 k(x) g(\rho, J). \end{cases}$$

The authors design a well-balanced approximation for which they prove two error estimates. The first one is uniform in time but assumes smallness of the coefficient k in L^1 ; the second error estimate displays linear growth in time but does not assume smallness of k . As in Chapter 3, the assumptions on k allow for steep gradients. The authors compare the well-balanced and time splitting strategies on some numerical tests where the function k displays oscillations. Eventually, Chapter 6 addresses several possible extensions and open problems, including the coupling of the hydrodynamical model (2) with the Poisson equation, or some difficulties related to multidimensional

problems.

The book will be of interest to anyone willing to design and/or study well-balanced schemes, either from an analytical or practical point of view. Focusing on the two main examples (1) and (2) allows for a complete and accessible description of error estimates. As hoped for in the foreword, this book will surely contribute to future improvements in the field.

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One-Dimensional Position-Dependent Models

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