

Preface

On behalf of both the Organizing and the Scientific Committees, it is our great pleasure to welcome you to the proceedings of the Second International SEE Conference on “Geometric Science of Information” (GSI 2015), hosted by École Polytechnique (Palaiseau, France), during October 28–30, 2015 (<http://www.gsi2015.org/>).

GSI 2015 benefited from the scientific sponsorship of Société de Mathématique Appliquées et Industrielles (SMAI, smai.emath.fr/) and the financial sponsorship of:

- CNRS
- École Polytechnique
- Institut des Systèmes Complexes
- Inria (<http://www.inria.fr/en/>)
- Telecom ParisTech
- THALES (www.thalesgroup.com)

GSI 2015 was also supported by CNRS Federative Networks MIA and ISIS.

The 3-day conference was organized in the framework of the relations set up between SEE (<http://www.see.asso.fr/>) and the following scientific institutions or academic laboratories: École Polytechnique, École des Mines de Paris, INRIA, Supélec, Université Paris-Sud, Institut Mathématique de Bordeaux, Sony Computer Science Laboratories, Telecom SudParis, and Telecom ParisTech.

We would like to express our thanks to the Computer Science Department LIX of École Polytechnique for hosting this second scientific event at the interface between geometry, probability, and information geometry. In particular, we warmly thank Evelynne Rayssac of LIX, École Polytechnique, for her kind administrative support that helped us book the auditorium and various resources at École Polytechnique, and Olivier Bournez (LIX Director) for providing financial support.

The GSI conference cycle was initiated by the Brillouin Seminar Team (<http://repmus.ircam.fr/brillouin/home>). The 2015 event was motivated in continuing the first initiatives launched in 2013 (see LNCS proceedings 8085, <http://www.springer.com/us/book/9783642400193>). We mention that in 2011, we organized an Indo-French workshop on “Matrix Information Geometry” that yielded an edited book in 2013 (<http://www.springer.com/us/book/9783642302312>).

The technical program of GSI 2015 covered all the main topics and highlights in the domain of “geometric science of information” including information geometry manifolds of structured data/information and their advanced applications. These proceedings consist solely of *original research papers* that were carefully *peer-reviewed* by two or three experts and revised before acceptance.

The program included the renown invited speaker Professor Charles-Michel Marle (UPMC, Université Pierre et Marie Curie, Paris, France), who gave a talk on “Actions of Lie Groups and Lie Algebras on Symplectic and Poisson Manifolds,” and three distinguished keynote speakers:

- Professor Marc Arnaudon (Bordeaux University, France): “Stochastic Euler-Poincaré Reduction”
- Professor Tudor Ratiu (EPFL, Switzerland): “Symmetry Methods in Geometric Mechanics”
- Professor Matilde Marcolli (Caltech, USA): “From Geometry and Physics to Computational Linguistics”

A short course was given by Professor Dominique Spehner (Grenoble University, France) on the “Geometry on the Set of Quantum States and Quantum Correlations” chaired by Roger Balian (CEA, France).

The collection of papers have been arranged into the following 17 thematic sessions, illustrating the richness and versatility of the field:

- Dimension Reduction on Riemannian Manifolds
- Optimal Transport
- Optimal Transport and Applications in Imagery/Statistics
- Shape Space and Diffeomorphic Mappings
- Random Geometry and Homology
- Hessian Information Geometry
- Topological Forms and Information
- Information Geometry Optimization
- Information Geometry in Image Analysis
- Divergence Geometry
- Optimization on Manifold
- Lie Groups and Geometric Mechanics/Thermodynamics
- Computational Information Geometry
- Lie Groups: Novel Statistical and Computational Frontiers
- Geometry of Time Series and Linear Dynamical Systems
- Bayesian and Information Geometry for Inverse Problems
- Probability Density Estimation

Historical Background

As for the first edition of GSI (2013) and in past publications (<https://www.see.asso.fr/node/11950>), GSI 2015 addressed inter-relations between different mathematical domains such as shape spaces (geometric statistics on manifolds and Lie groups, deformations in shape space), probability/optimization and algorithms on manifolds (structured matrix manifold, structured data/information), relational and discrete metric spaces (graph metrics, distance geometry, relational analysis), computational and Hessian information geometry, algebraic/infinite dimensional/Banach information manifolds, divergence geometry, tensor-valued morphology, optimal transport theory, and manifold and topology learning, as well as applications such as geometries of audio-processing, inverse problems, and signal processing.

At the turn of the century, new and fruitful interactions were discovered between several branches of science: information science (information theory, digital communications, statistical signal processing), mathematics (group theory, geometry and

topology, probability, statistics), and physics (geometric mechanics, thermodynamics, statistical physics, quantum mechanics).

From Statistics to Geometry

In the middle of the last century, a new branch in the geometric approach of statistical problems was initiated independently by Harold Hotelling and Calyampudi Radhakrishna Rao, who introduced a metric space in the parameter space of probability densities. The metric tensor was proved to be equal to the Fisher information matrix. This result was axiomatized by Nikolai Nikolaevich Chentsov in the framework of category theory. This idea was also latent in the work of Maurice Fréchet, who had noticed that the “distinguished densities” that reach lower bounds of statistical estimators are defined by a function that is given by a solution of the Legendre–Clairaut equation (cornerstone equation of “information geometry”), and in the works of Jean-Louis Koszul with a generalized notion of characteristic function.

From Probability to Geometry

Probability is again the subject of a new foundation to apprehend new structures and generalize the theory to more abstract spaces (metric spaces, shape space, homogeneous manifolds, graphs). An initial attempt to probability generalization in metric spaces was made by Maurice Fréchet in the middle of the last century, in the framework of abstract spaces topologically affine and “distance space” (“espace distancié”). More recently, Misha Gromov, at IHES (Institute of Advanced Scientific Studies), indicated the possibilities for (non-)homological linearization of basic notions of probability theory and also the replacement of real numbers as values of probabilities by objects of suitable combinatorial categories. In parallel, Daniel Bennequin, from Institut mathématique de Jussieu, observed that entropy is a universal co-homological class in a theory associated with a family of observable quantities and a family of probability distributions.

From Groups Theory to Geometry

As observed by Gaston Bachelard, “The group provides evidence of a mathematic closed on itself. Its discovery closes the era of conventions, more or less independent, more or less coherent.” About Elie Cartan’s work on group theory, Henri Poincaré said that “The problems addressed by Elie Cartan are among the most important, most abstract, and most general dealing with mathematics; group theory is, so to speak, the whole mathematics, stripped of its material and reduced to pure form. This extreme level of abstraction has probably made my presentation a little dry. To assess each of the results, I would have had to virtually render it the material of which it had been stripped; but this refund can be made in a thousand different ways; and this is the only

form that can be found as well as a host of various garments, which is the common link between mathematical theories whose proximity is often surprising.”

From Mechanics to Geometry

The last elaboration of geometric structure on information is emerging at the inter-relations between “geometric mechanics” and “information theory” that was largely debated at the GSI 2015 conference with invited speakers including C.M. Marle, T. Ratiu, and M. Arnaudon. Elie Cartan, the master of geometry during the last century, said: “distinguished service that has rendered and will make even the absolute differential calculus of Ricci and Levi-Civita should not prevent us from avoiding too exclusively formal calculations, where debauchery indices often mask a very simple geometric fact. It is this reality that I have sought to put in evidence everywhere.” Elie Cartan was the son of Joseph Cartan, who was the village blacksmith, and Elie recalled that his childhood had passed under “blows of the anvil, which started every morning from dawn.” One can imagine that the hammer blows made by Joseph on the anvil, giving shape and curvature to the metal, influenced Elie’s mind with germinal intuition of fundamental geometric concepts. The alliance between geometry and mechanics is beautifully illustrated by the image of Forge, in the painting of Velasquez about the Vulcan God (see Figure 1). This concordance of meaning is also confirmed by the etymology of the word “forge,” which comes from late fourteenth century, “a smithy,” from the Old French *forge* “forge, smithy” (twelfth century), earlier *faverge*, from the Latin *fabrica* “workshop, smith’s shop,” from *faber* (genitive *fabri*) “workman in hard materials, smith.”

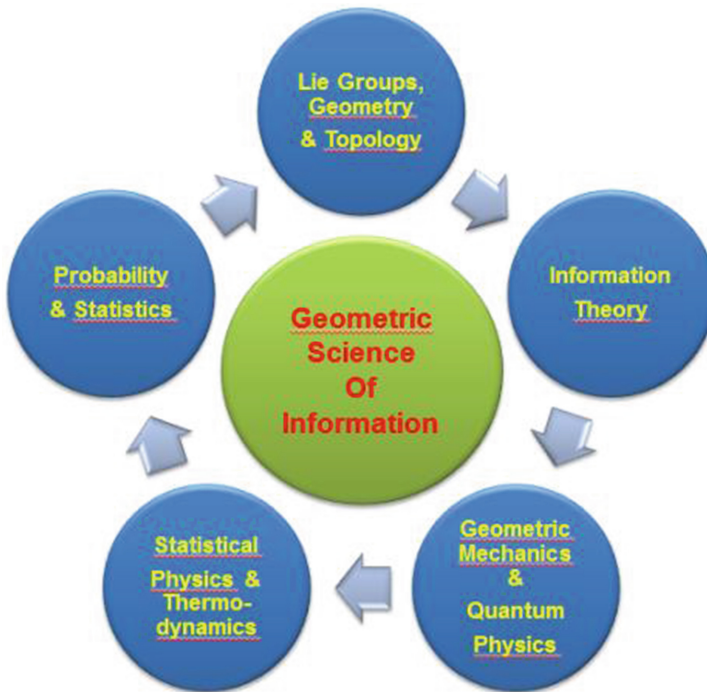


Fig. 1. Into the Flaming Forge of Vulcan, into the Ninth Sphere, Mars descends in order to retemper his flaming sword and conquer the heart of Venus (Diego Velázquez, Museo Nacional del Prado). Public domain image, courtesy of https://en.wikipedia.org/wiki/Apollo_in_the_Forge_of_Vulcan

As Henri Bergson said in his book *The Creative Evolution* in 1907: “As regards human intelligence, there is not enough [acknowledgment] that mechanical invention was first its essential approach ... we should say perhaps not *Homo sapiens*, but *Homo faber*. In short, intelligence, considered in what seems to be its original feature, is the faculty of manufacturing artificial objects, especially tools to make tools, and of indefinitely varying the manufacture.”

Geometric Science of Information: A new Grammar of Sciences

Henri Poincaré said that “mathematics is the art of giving the same name to different things” (“La mathématique est l’art de donner le même nom à des choses différentes” in *Science et méthode*, 1908). By paraphrasing Henri Poincaré, we could claim that the “geometric science of information” is the art of giving the same name to different sciences. The rules and the structures developed at the GSI 2015 conference comprise a kind of new grammar for these sciences.



We give our thanks to all the authors and co-authors for their tremendous effort and scientific contribution. We would also like to acknowledge all the Organizing and Scientific Committee members for their hard work in evaluating the submissions. We warmly thank Jean Vieille, Valerie Alidor, and Flore Manier from the SEE for their kind support.

As with GSI 2013, a selected number of contributions focusing on a core topic were invited to contribute a chapter without page restriction to the edited book *Geometric Theory of Information* (<http://www.springer.com/us/book/9783319053165>) in 2014. Similarly, for GSI 2015, we invite prospective authors to submit their original work to a special issue on “advances in differential geometrical theory of statistics” of the MDPI *Entropy* journal (http://www.mdpi.com/journal/entropy/special_issues/entropy-statistics).

It is our hope that the fine collection of peer-reviewed papers presented in these LNCS proceedings will be a valuable resource for researchers working in the field of information geometry and for graduate students.

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