

# Preface

These notes attempt to provide an elementary introduction to the one-dimensional discrete-time branching random walk and to exploit its spinal structure.

They begin with the case of the Galton–Watson tree for which the spinal structure, formulated in the form of the size-biased tree, is simple and intuitive.

Chapter 3 is devoted to a few fundamental martingales associated with the branching random walk.

The spinal decomposition is introduced in Chap. 4, first in its more general form, followed by two important examples. This chapter gives the most important mathematical tool of the notes.

Chapter 5 forms, together with Chap. 4, the main part of the text. Exploiting the spinal decomposition theorem, we study various asymptotic properties of the extremal positions in the branching random walk and of the fundamental martingales.

The last part of the notes presents a brief account of results for a few related and more complicated models.

The lecture notes by Berestycki [43] and Zeitouni [235] give a general and excellent account of, respectively, branching Brownian motion and the F-KPP equation and branching random walks with applications to Gaussian free fields.

I would like to deeply thank Yueyun Hu; together we wrote about 20 papers in the last 20 years, some of them strongly related to the material presented here. I am grateful to Élie Aïdékon, Julien Berestycki, Éric Brunet, Xinxin Chen, Bernard Derrida, Gabriel Faraud, Nina Gantert, and Jean-Baptiste Gouéré for stimulating discussions, to Bastien Mallein and Michel Pain for great assistance in the preparation of the present notes, and to Christian Houdré for correcting my English with patience.

I wish to thank Laurent Serlet and the Scientific Board of the École d'été de probabilités de Saint-Flour for the invitation to deliver these lectures.

Paris, France  
August 2015

Zhan Shi

Branching Random Walks

École d'Été de Probabilités de Saint-Flour XLII – 2012

Shi, Z.

2015, X, 133 p. 8 illus., 6 illus. in color., Softcover

ISBN: 978-3-319-25371-8