

Contents

1	Introduction	1
1.1	Branching Brownian Motion	1
1.2	Branching Random Walks	3
1.3	The Many-to-One Formula	5
1.4	Application: Velocity of the Leftmost Position	6
1.5	Examples	8
1.6	Notes	10
2	Galton–Watson Trees	11
2.1	The Extinction Probability	11
2.2	Size-Biased Galton–Watson Trees	13
2.3	Application: The Kesten–Stigum Theorem	16
2.4	Notes	17
3	Branching Random Walks and Martingales	19
3.1	Branching Random Walks: Basic Notation	19
3.2	The Additive Martingale	21
3.3	The Multiplicative Martingale	22
3.4	The Derivative Martingale	26
3.5	Notes	27
4	The Spinal Decomposition Theorem	29
4.1	Attaching a Spine to the Branching Random Walk	29
4.2	Harmonic Functions and Doob’s h -Transform	30
4.3	Change of Probabilities	31
4.4	The Spinal Decomposition Theorem	33
4.5	Proof of the Spinal Decomposition Theorem	35
4.6	Example: Size-Biased Branching Random Walks	39
4.7	Example: Above a Given Value Along the Spine	40
4.8	Application: The Biggins Martingale Convergence Theorem	42
4.9	Notes	44

5	Applications of the Spinal Decomposition Theorem	45
5.1	Assumption (H)	45
5.2	Convergence of the Derivative Martingale	47
5.3	Leftmost Position: Weak Convergence	54
5.4	Leftmost Position: Limiting Law	62
5.4.1	Step 1: The Derivative Martingale is Useful	63
5.4.2	Step 2: Proof of the Key Estimate	65
5.4.3	Step 3a: Proof of Lemma 5.18	71
5.4.4	Step 3b: Proof of Lemma 5.19	76
5.4.5	Step 4: The Role of the Non-lattice Assumption	79
5.5	Leftmost Position: Fluctuations	82
5.6	Convergence of the Additive Martingale	87
5.7	The Genealogy of the Leftmost Position	88
5.8	Proof of the Peeling Lemma	89
5.9	Notes	97
6	Branching Random Walks with Selection	99
6.1	Branching Random Walks with Absorption	99
6.2	The N -BRW	102
6.3	The L -BRW	104
6.4	Notes	105
7	Biased Random Walks on Galton–Watson Trees	107
7.1	A Simple Example	107
7.2	The Slow Movement	108
7.3	The Maximal Displacement	110
7.4	Favourite Sites	112
7.5	Notes	113
A	Sums of i.i.d. Random Variables	115
A.1	The Renewal Function	115
A.2	Random Walks to Stay Above a Barrier	116
	References	125

Branching Random Walks

École d'Été de Probabilités de Saint-Flour XLII – 2012

Shi, Z.

2015, X, 133 p. 8 illus., 6 illus. in color., Softcover

ISBN: 978-3-319-25371-8