

# Preface

The purpose of this book is to provide a bridge between new research results motivated by the financial crisis and classical literature on interest rate modeling.

**Motivation.** The traditional textbooks on interest rate modeling are no longer adequate in a modern context as overviews of the techniques needed for the valuation of interest rate derivatives. In the years following the crisis, the problem of developing new models for interest rate derivatives has attracted significant attention, both from researchers working in financial institutions, as well as researchers working in academia. Various models are continually being proposed. The aim of this book is therefore two-fold. On the one hand, it aims at providing an overview of the state-of-the art techniques in modern interest rate modeling and, on the other hand, it attempts to clarify the link between these models and the classical literature. From the practical point of view, the importance of up-to-date interest rate models can be best illustrated when viewing the fixed-income market as a part of the global derivatives market. According to the yearly statistics provided by the Bank for International Settlements, the notional amounts outstanding each year for over-the-counter (OTC) interest rate derivatives sum up to 80 % of the total trade volume in OTC derivatives (\$505 trillion out of the total volume of \$630 trillion corresponded to interest rate derivatives in 2014).

**Audience.** The book is intended to serve as a guide for graduate students, researchers, and practitioners interested in the paradigm change that affected all fixed-income markets due to the financial crisis. More generally, we intend to address people who are already quite knowledgeable of mathematical finance, who have some familiarity with classical interest rate theory, but who have little or no familiarity with issues on multi-curve modeling. As mathematical prerequisites we expect the reader to have a basic knowledge of probability theory together with notions from stochastic processes and stochastic calculus that are commonly used in the mathematical finance literature.

**Approach.** Our approach to multiple curves consists mainly in modeling for the purpose of pricing interest derivatives, rather than in view of hedging and/or risk management. It corresponds to what in the classical single-curve interest rate theory is called “martingale modeling” in the sense that the models are defined under a martingale measure  $Q$  that has as numéraire the money market account and has to be calibrated to each specific basket of products, given that the market as such is incomplete. From this measure  $Q$ , and using the discount curve, one can then derive the various forward measures used for pricing of interest rate derivatives. After calibration, this will lead to unique prices. We can and shall follow the same procedure also in the multi-curve setup after justifying (see Sect. 1.3.1 below) the choice of a single specific curve for discounting future cash flows and with it the money market account and thus also a reference martingale measure  $Q$ .

For simplicity of exposition we limit ourselves to Wiener-driven models, but extensions to jump-diffusions can in most cases be obtained in a relatively straightforward manner. In each of the chapters, when it comes to pricing, we consider only what according to some of the literature is called “clean valuation” and in Sect. 1.2.3 of the introductory Chap. 1 we justify this choice. Even limiting ourselves to clean valuation, there are various possible approaches that one can find in the literature over the past years. Multi-curve modeling is very recent as a research topic and it is still early to evaluate the advantages of one multi-curve approach with respect to others; we therefore opted to limit ourselves to an overview.

**Structure.** In the classical pre-crisis interest rate theory one considers various models and model classes and there does not exist a single model that is uniformly better than the others. Different model classes are in fact suited for different situations and products. Consequently, also when passing to multi-curve modeling, one considers various model classes as well. The following major classical interest rate model classes have so far found an extension to the multi-curve setting and form the basis for this monograph: (i) short-rate and rational pricing kernel models; (ii) forward rate models (Heath–Jarrow–Morton setup); (iii) Libor market models (more generally, market forward rate models). In the pre-crisis setting there are also other interest rate models that are briefly cited in Chap. 1. The above ordering of the three classes reflects a “bottom-up” point of view and in our exposition we follow this ordering. In fact, starting from the introductory Chap. 1 where we explain the main notions and concepts related to the post-crisis fixed-income markets, we then proceed with three chapters as follows:

Chapter 2: This chapter concerns mainly the classical, strict-sense short-rate models, but also some wider sense short-rate models represented by the rational pricing kernel models. For the strict-sense short-rate models, we consider for each tenor a short-rate spread to be added to the short rate from the outset. For the dynamics of the short rate and the spreads we consider factor models that belong to the exponentially affine or the exponentially quadratic model classes. We develop in detail the results for the exponentially affine class, for

which we are able to obtain closed or semi-closed formulas for the prices of linear and optional interest rate derivatives. For linear derivatives we are able to compare directly single curve (pre-crisis) and multiple curve (post-crisis) derivative values by means of an adjustment factor. Finally, we summarize recent multiple curve extensions of rational pricing kernel models.

**Chapter 3:** This chapter concerns forward rate models in a Heath–Jarrow–Morton (HJM) setup. Similarly to the short rate and its additive spreads in Chap. 2, here we consider the reference forward OIS rate and the corresponding spreads. Major emphasis is put on obtaining arbitrage-free models by deriving for them no-arbitrage conditions in the form of a “drift condition” analogous to the classical HJM condition. Since the HJM framework is situated in between the short-rate models and the Libor market models (LMMs), we proceed essentially along two ways: (i) mimicking the LMMs by a hybrid LMM–HJM approach, where we consider a linear transform of the Libor rate that is modeled directly under the standard martingale measure, but by its definition has to be a martingale under the forward measure; this then leads to no-arbitrage conditions. We shall call these “real no-arbitrage conditions” in the sense that they represent intrinsic no-arbitrage conditions in relation to the basic traded assets that are FRA contracts; (ii) the other alternative consists in reproducing the pre-crisis relationship between discretely compounding forward rates and bond prices also for the forward Libor rates in the post-crisis setup, but replacing the standard zero coupon bonds by fictitious ones that are supposed to be affected by the same risk factors as the Libor rates. In this latter context we obtain no-arbitrage conditions analogously to point (i) by imposing that the ratio of fictitious bond prices in this relationship has to be a martingale under the forward measure. In addition to this, we also discuss “pseudo no-arbitrage conditions” by assigning different interpretations to the fictitious bond prices, in particular via a credit risk and a foreign exchange analogy. The last part of the chapter concerns interest rate derivative pricing in this HJM context.

**Chapter 4:** This chapter presents basically an overview of two major existing approaches to obtain multi-curve models on a discrete tenor in the spirit of the Libor market models. The first approach concerns a series of papers by Mercurio and co-authors, as well as by authors related to them, in which multiple curve extensions of the classical LMMs are developed. The other approach is concerned with an affine Libor model for multiple curves. The spreads in the above approaches are additive; we conclude the chapter by mentioning approaches based on modeling multiplicative spreads.

The material presented in these chapters corresponds to a selection that we had to make among the possible material to fit into the allowable size of the monograph within the “SpringerBriefs” series. Among the topics that we were not able to include we would like to mention the issues of numerical implementation and calibration of the presented models, for which we shall always refer to corresponding articles dealing with this, as well as the questions of hedging and risk management in the multi-curve environment, which have currently been less well

studied in a systematic way in the available literature, although they are of utmost practical importance. In particular, in view of hedging, one might also raise the issue of defining the price of a derivative as a cost of its hedging portfolio and a related issue of possible non-uniqueness of prices. As mentioned above, here we opt rather for the approach based on martingale modeling, where an existence of a martingale measure is assumed, the models are developed under this measure, and the prices defined as corresponding conditional expectations. This approach has the advantage of allowing to compute various post-crisis valuation adjustments such as CVA, which have to be computed for the whole aggregate portfolio of derivatives between two counterparties, and thus require a unified pricing method for all derivatives.

During the preparation of the manuscript, in the European markets we have witnessed a continuous important decrease in the level of all interest rates, as well as the appearance of negative interest rates, firstly only for the Swiss Franc, but more recently also for the Euro. This phenomenon has been observed for several months already and it has by now occurred not only at the shortest end of the interest rate curve, but also in the midterm rates. Negative rates arise because of frictions not addressed by the models, such as the “cost of carry” associated with keeping large amounts of cash. Due to this current market situation we are prompted to slowly readjust one of the long-standing modeling axioms that the interest rates should be positive. This is an interesting modeling situation, where the models in which the interest rates can become negative suddenly seem to be perfectly suited for the task at hand. One should still be cautious when addressing this issue because, even though negative rates have been observed, the multiple curve spreads still remain positive, so ideally one would need a model which combines both of these features. In this book we present some models that ensure positive interest rates and some models that do not have this property and, as mentioned above, we do not evaluate the approaches based on this quality. However, we do mention when discussing certain models providing positive interest rates that they can be modified without increasing the level of their complexity to allow for the rates to fall below zero. It is still left to be seen if the negative rate phenomenon will persist in the future as well and will become a standing modeling requirement, such as it was the case with the multiple interest rate curves.

**Literature.** The literature on interest rates is too vast to mention it all. Our monograph concerns multiple curves. While we made an effort to cite the relevant literature for interest rates in general, we found it most natural to concentrate mainly on the literature that concerns models that so far have found an extension to the multi-curve setting. We have tried to be as complete as possible and we apologize for having possibly overlooked some relevant literature. It is also not an easy task to keep track of all work, especially the more recent, since the subject is currently in rapid evolution and some of the key work after the crisis has been made inside the investment banks, and cannot be accessed as long as it remains inside the companies. Furthermore, it would bring us too far if for each concept we would trace back its evolution over time in the literature and thus we limited ourselves to

references that in some sense summarize previous achievements. On the other hand, given that one of our purposes is to provide an overview of the state-of-the-art of multi-curve modeling, instead of taking the approach of addressing “who did what,” it was natural to make repeated references since each reference does not treat only a single topic, but touches upon various arguments.

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