Preface

In the landscape of stochastic analysis, Dirichlet forms are particularly endearing to those who have met their amazing properties. After having opened the finite dimensional potential theory to Hilbertian techniques, they have proved very useful in infinite dimensions for the study of Wiener space and diffusion processes as well as for an error calculus adapted to stochastic analysis, while retaining some mystery—it must be said—due to the fact that they have kept several astonishing conjectures such as the energy image density property which remains a quite fascinating enigma to this day.

The purpose of this book is to implement their properties to build new techniques concerning Poisson measures and Lévy processes that are central to the study of processes with jumps. More precisely, we develop a new approach to Malliavin calculus for Poisson measures and jump processes. The method gives rise to a new explicit calculus: if one considers a Poisson measure as a random system of particles, it consists in adding a particle and taking it back after computing the gradient i.e. "derivating" with respect to the position of the particle (the size of the jumps in the case of a Lévy process). This method permits to establish absolute continuity of Poisson functionals such as Lévy areas, solutions of SDE's driven by Poisson measure and many other examples. By iterating the gradient we also construct associated Sobolev spaces of any order and get some criteria ensuring smoothness of the law of Poisson functionals.

The work of many mathematicians have put us on the path of such a program, especially that of Paul Malliavin, Masatoshi Fukushima, Sergio Albeverio, Michael Roeckner and particularly Francis Hirsch to whom we are particularly acknowledging. Our ideas gradually took their final form during various lectures and conferences in several universities where fruitful discussions have been possible: in Japan thanks to Iroshi Sugita, Yasushi Ishikawa and Arturo Kohatsu-Higa, at Swansea Wales thanks to Niels Jacob, Eugene Lytvynov, in the Mittag-Leffler Institute Stockholm in September–October 2007 thanks to Marta Sanz, in Hong Kong in June 2009 thanks to Nicolas Privault and at Roscoff in 2012 thanks to Rainer Buckdahn. We acknowledge these colleagues gratefully. Frequent

discussions at the university of Marne-la-Vallée with Vlad Bally, Damien Lamberton, Benjamin Jourdain, at the university of Evry with Denis Feyel, Arnaud de la Pradelle, Thomas Simon, as well as with Suleyman Üstünel, Christophe Chorro and Simone Scotti, and also with Jin Ma at the University of Southern California, helped us to better define the specificity of our approach and the scope of the lent particle method which will be exposed in the book. This collective work, where we should quote many other researchers, allowed us to give a course at the Institut Henri Poincaré in Paris in 2011, from which the present book is directly derived. This series of lectures was given on the pretext of its applications in mathematical finance, which are real, but which we do not especially emphasize in the present book, preferring to focus on mathematical ideas.

Nicolas Bouleau
Laurent Denis

http://www.springer.com/978-3-319-25818-8

Dirichlet Forms Methods for Poisson Point Measures and Lévy Processes
With Emphasis on the Creation-Annihilation Techniques
Bouleau, N.; Denis, L.
2015, XVIII, 323 p. 3 illus. in color., Hardcover
ISBN: 978-3-319-25818-8