

Contents

1	Introduction	1
2	Introduction to the Theory of Dirichlet Forms	9
2.1	Unbounded Operators, Semigroups and Closed Forms	10
2.1.1	Self-adjoint Operators	10
2.1.2	Semigroup and Resolvent Associated to a Non-negative Self-adjoint Operator	11
2.1.3	Closed Forms	12
2.2	Dirichlet Forms	14
2.2.1	Definition and Fundamental Relationships	14
2.2.2	Carré du Champ, Gradient and the (EID) Property	15
2.3	The Ornstein–Uhlenbeck Structure on the Wiener Space	19
2.4	Sufficient Conditions for (EID) Property	22
2.4.1	A Sufficient Condition on $(\mathbb{R}^r, \mathcal{B}(\mathbb{R}^r))$	23
2.4.2	The Case of a Product Structure	27
2.4.3	The Case of Structures Obtained by Injective Images	28
3	Reminders on Poisson Random Measures	31
3.1	Poisson Random Measures	31
3.2	Lévy Processes	33
3.3	Framework Adopted in the Sequel	34
3.3.1	Density Lemmas	35
3.3.2	Some Basic Formulas	36
3.4	Multiple Integrals and Chaos Decomposition (Without Dirichlet Forms)	37
4	Construction of the Dirichlet Structure on the Upper Space	41
4.1	Operators ε^+ , ε^- and Related Formulas	41
4.2	The Ornstein–Uhlenbeck Structure on the Poisson Space	43
4.2.1	Definition	43
4.2.2	The Kabanov Integral	46

4.3	Dirichlet Structure on the Upper Space	48
4.3.1	Hypotheses and Choice of a Gradient on the Bottom Space	49
4.3.2	The Extended Poisson Measure $N \odot \rho$	50
4.3.3	Upper Semigroup and Closed Form.	57
4.3.4	Definition of the Semigroup on the Upper Space Using Chaos.	57
4.3.5	Positive Closed Form and Generator Associated to (P_t)	58
4.3.6	Chaos Decomposition of \mathbb{D}	59
4.4	Positivity of P_t and Dirichlet Form	60
4.4.1	Proof Based on Friedrichs' Theorem	61
4.4.2	Proof Based on the Contractions	66
4.4.3	Proof Based on a Mehler-Type Formula.	68
4.5	Main Properties of the Dirichlet form $(\mathbb{D}, \mathcal{E})$	70
4.5.1	The Local Property and the Carré du Champ Operator	70
4.5.2	Remarks on the Scheme of Fock Spaces and Chaos Decompositions.	75
4.6	(EID) Property on the Upper Space from (EID) Property on the Bottom Space and the Domain \mathbb{D}_{loc}	76
4.6.1	The Case Where $\nu(X)$ is Finite	76
4.6.2	The General Case	78
5	The Lent Particle Formula	83
5.1	The Lent Particle Formula	85
5.1.1	Negligible Sets	87
5.1.2	The Divergence Operator δ_{\sharp}	88
5.1.3	The LPF for δ_{\sharp}	89
5.1.4	The LPF for the Generator A	90
5.1.5	Starting with LPF in the Case Where the Intensity Measure is Finite	92
5.1.6	Historical Origin of the Lent Particle Formula (LPF)	94
5.2	Various Formulae	94
5.2.1	Notational Remarks	94
5.2.2	Useful Computations	95
5.2.3	Some Computations of Functionals only Depending on the Marks.	96
5.2.4	Factorial Measures	98
5.2.5	Link with the Combinatorial Approach	101

5.3	Practical Features of the Method	102
5.3.1	Computation with the Lent Particle Formula.	102
5.3.2	A Simplified Sufficient Condition for Existence of Density	103
5.3.3	Computation of the Carré du Champ Thanks to the LPF	103
5.3.4	Interpretation in Terms of Error Calculus	104
6	Space-Time Setting and Examples	107
6.1	The Case $X = \mathbb{R}_+ \times \Xi$ and Lévy Processes.	107
6.1.1	The Framework	107
6.1.2	The Predictable Representation Property.	108
6.1.3	Operator δ_{\sharp} in the Case of Lévy Processes	110
6.1.4	A Useful Theorem of Paul Lévy	113
6.2	Applications to Some Examples in Stochastic Calculus	114
6.2.1	Application to Lévy Doléans-Dade Exponential	114
6.2.2	Upper Bound of a Process on $[0, t]$	116
6.2.3	Processes Whose Speed is a Lévy Process	117
6.2.4	Generalized Ornstein–Uhlenbeck Process in Dimension One.	117
6.2.5	Interaction Potential.	119
6.2.6	Error Calculus in Incomplete Markets	120
6.2.7	Regularizing Properties of Lévy Processes	122
6.2.8	Regularization Thanks to the Jumps When the Hörmander Conditions are not Satisfied.	124
6.2.9	Gas of Brownian Particles	127
6.2.10	Regularity Results on Multiple Poisson Integrals.	130
6.2.11	Remarks on the Range of the Method	135
7	Sobolev Spaces and Distributions on Poisson Space	137
7.1	Notation on Hilbertian Extensions of Spaces and Operators	137
7.2	Sobolev Spaces.	138
7.2.1	On the Bottom Space	138
7.2.2	Sobolev Spaces on the Upper Space	141
7.3	Identity of $\mathbb{D}^\infty(\mathbb{E})$ and $\overline{\mathbb{D}^\infty(E)}$ and Meyer Inequalities in the Cases of Classical Bottom Spaces	144
7.3.1	Khintchine’s Inequalities	145
7.3.2	An Equivalence of Norms	145
7.3.3	Meyer Inequalities in the Euclidean Case.	148
7.3.4	Other Cases of Transfer of Inequalities on the Poisson Space.	150

7.4	Criterion of Smoothness for the Law of Poisson Functionals.	150
7.4.1	The Criterion	151
7.4.2	Expression of the Density Thanks to Composition with a Schwartz Distribution.	154
7.5	Theory of Distributions on Poisson Space	157
7.5.1	Reminder on Finite Energy Distributions in Dirichlet Structures.	158
7.5.2	Finite Energy Distributions and Quadratic Sobolev Spaces.	160
7.5.3	Quadratic Meyer-Yan Distributions	161
7.5.4	Gradient and Wick Products.	162
7.6	Calculation of Chaos Decompositions	165
7.6.1	Case of Cauchy Principal Value	166
7.6.2	Approach by the Stroock Representation	167
7.6.3	Regularity of Some Functionals Defined by Their Chaos Expansions	168
8	Applications to Stochastic Differential Equations Driven by a Random Measure	171
8.1	Framework and the Equation We Consider.	171
8.1.1	The Poisson Measure and the Auxiliary Semi-martingale	171
8.1.2	Dirichlet Structure on the Upper Space	172
8.1.3	The Equation We Consider	172
8.1.4	Spaces of Processes.	173
8.2	The Solution as an Element in $\mathcal{H}_{\mathbb{D}}^d$, Expression of the Derivative.	177
8.3	Existence of Density	182
8.3.1	Hypotheses.	182
8.3.2	Obtaining the Malliavin Matrix Thanks to the Lent Particle Method	184
8.3.3	Applications.	187
8.3.4	Computation of Greeks by Internalization.	190
8.3.5	McKean-Vlasov Type Equation Driven by a Lévy Process.	193
8.3.6	Stable-Like Processes	195
8.3.7	Non-linear Subordination	197
8.3.8	Diffusive Particle Subjected to a Lévy Field of Force.	203
8.4	Smoothness of the Law	204
8.4.1	Spaces of Processes.	206
8.4.2	Functional Calculus Related to Stochastic Integrals	207
8.4.3	Existence of Smooth Density for the Solution.	211
8.4.4	Applications: The Locally Elliptic Case	216

8.5	Explicit Computation of Densities	217
8.5.1	Reducing the Bias	220
8.5.2	Comparison of Speeds of Convergence	223
8.5.3	Direct Formula for the Density	224
9	Affine Processes, Rates Models	229
9.1	Construction of Lévy Processes with Values in Diffusion Processes	229
9.1.1	Homographic Branch of the Bernstein Monoid	229
9.1.2	Link with Bessel Processes.	231
9.1.3	Lévy Measure of the Excursions	233
9.1.4	Pitman–Yor Formula	233
9.1.5	The Cox–Ingersoll–Ross Branch	234
9.2	Malliavin Calculus on Functional Lévy Processes	235
9.2.1	Increasing System of Dirichlet Structures	235
9.2.2	Study of a Functional Thanks to the LPF.	236
9.2.3	Processes with Three Indices	237
10	Non Poissonian Cases	239
10.1	Marked Random Point Measures.	239
10.2	A Particle Method for the Brownian Motion.	246
10.2.1	Second Order Stationary Process of Rotations of Normal Martingales.	247
10.2.2	The Notion of Chaotic Extension	247
10.2.3	Derivative in θ and Malliavin Gradient	251
10.2.4	Functional Calculus of Class $\mathcal{C}^1 \cap Lip$	256
10.2.5	The Unit Jump on the Interval $[0, 1]$	259
	Appendix A: Error Structures.	265
	Appendix B: The Co-Area Formula.	303
	Bibliography	307
	Index	321

Dirichlet Forms Methods for Poisson Point Measures
and Lévy Processes

With Emphasis on the Creation-Annihilation Techniques

Bouleau, N.; Denis, L.

2015, XVIII, 323 p. 3 illus. in color., Hardcover

ISBN: 978-3-319-25818-8