Joint Probability Density of the Intervals Length of Modulated Semi-synchronous Integrated Flow of Events in Conditions of a Constant Dead Time and the Flow Recurrence Conditions

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Abstract. This paper is focused on studying the modulated semi-synchronous integrated flow of events which is one of the mathematical models for incoming streams of events (claims) in computer communication networks and is related to the class of doubly stochastic Poisson processes (DSPPs). The flow is considered in conditions of its incomplete observability, when the dead time period of a constant duration T is generated after every registered event. In this paper we propose a technique for obtaining the formulas for calculation the probability density of the interval length between two neighboring flow events and the joint probability density of the length of two successive intervals. Also we find the conditions of the flow recurrence.

Keywords: Modulated semi-synchronous integrated flow of events · Doubly stochastic poisson process (DSPP) · Markovian arrival process (MAP) · Dead time · Flow parameters estimation · Probability density · Joint probability density · Flow recurrence conditions

1 Introduction

Mathematical models of the queueing theory have found wide application in describing real physical, technical and other objects and systems. It is worthwhile to note that the conditions of the real objects and systems operation are such that we can assert that the servers parameters are known and stable as time goes, but we can not tell this about the intensity processes of the input flows of events that come to the servers. Moreover, the intensities of the input flows usually vary within time and frequently their changes are accidental. As a result, it is necessary to consider the mathematical models of doubly stochastic Poisson processes (DSPPs), which are characterized by having the number of events in any given time interval as being Poisson distributed, conditionally to another positive stochastic process $\lambda(t)$ called intensity [1–5].

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There are two known classes of doubly stochastic flows of events. The first class contains the flows of events, which intensity process is a continuous random process. The second class contains flows, which intensity is a piecewise constant stationary random process with a finite number of states. The second-class flows are most typical for telecommunication networks. They were considered for the first time and independently presented in works [6,7]. Since the early 1990s to date, these flows of events are called as doubly stochastic flows of events or MAP-flows, or MC-flows [8–13].

In turn, MC-flows may be divided into three groups depending on how the intensity process changes its state from one to another: (1) synchronous flows – flows, which intensity process changes its state from one state to another at random times, which are the time moments of the flow events arrival [14–16]; (2) asynchronous flows – flows, which intensity process changes its state from one state to another at random times, which do not depend on the time moments of the flow events arrival [17–19]; (3) semi-synchronous flows – flows, for which for the one set of states the first definition is valid and for another set of states the second definition is valid [20–22]. We shall emphasize that synchronous, asynchronous and semi-synchronous flows can be presented as the mathematical models of MAP-flows of events with the constraints on the flow parameters [23].

In the recent literature, the problem of estimating the intensity process from observations of doubly stochastic Poisson processes (DSPPs) has been of a great interest, since DSPPs have found applications in many fields such as network theory, peer-to-peer streaming networks and adaptive data streaming, optical communication systems, statistical modeling, quantitative finance, spatial epidemiology, etc. [24–29]. As has been mentioned above, in the real situations the input flow parameters can be unknown or partially known or, worse, may vary in time in a random way. That is why, the central problems faced when modeling these processes are: (1) flow states estimation on monitoring the time moments of the events occurrence (the filtering of the underlying and unobservable intensity process) [30–33]; (2) flow parameters estimation on monitoring the time moments of the events occurrence [34–37].

It is worth noting, that in most of the cases researchers consider the mathematical models of the flows, where time moments of the flow events occurrence are observable. In practice, however, any recording device (server in this context) spends some finite time for event measurement and registration, during which server can not handle the next event correctly. In other words, every event registered by a server causes the period which is called the period of a dead time [38], during which no other events are observed (they are lost). We may suppose that this period has a fixed duration (constant dead time). Particularly, we may find examples of this mathematical model in the real computer networks using CSMA/CD (Carrier Sense Multiple Access with Collision Detection) protocol. At the moment of a conflict recording at the in-port of a network node, a jam signal is transmitted across the network. During the signal transmission, calls coming to a node of the network are declined and sent to a source of repeated calls. Here time, during which the network node is closed for calls serving after a conflict recording, can be interpreted as a dead time of a server, which registers the conflict in the network nodes.

In this paper we continue to study the modulated semi-synchronous integrated flow of events [31–33], which is a generalization of the semi-synchronous flow of events [20] and semi-synchronous integrated flow of events [39] and belonging to the class of Markovian arrival processes (MAPs). The rest of the paper is organized as follows. In Sect. 2 we present the modulated semi-synchronous integrated flow of events, which provides our modeling framework. In Sects. 3 and 4 we obtain the expressions for probability density of the interval length between two neighboring flow events $p_T(\tau)$, $\tau \geq 0$, and the joint probability density of the length of two successive intervals $p_T(\tau_1, \tau_2)$, $\tau_1 \geq 0$, $\tau_2 \geq 0$, explicitly. And finally, in Sect. 5 we obtain the recurrence conditions of the observable flow of events.

2 Problem Statement

In this paper we consider the modulated semi-synchronous integrated flow of events (further flow of events), which intensity process is a piecewise constant stationary random process $\lambda(t)$ with two states 1, 2 (first, second correspondingly). In the state 1 $\lambda(t) = \lambda_1$ and in the state 2 $\lambda(t) = \lambda_2$ ($\lambda_1 > \lambda_2$). The duration of the process $\lambda(t)$ staying in the first (second) state is distributed according to the exponential law with parameter β (α). If at the time moment t the process $\lambda(t)$ is found in the first (second) state, then at the interval $[t, t + \Delta t)$, where Δt (hereinafter) is sufficiently small, with probability $\beta\Delta t + o(\Delta t)$ ($\alpha\Delta t + o(\Delta t)$) the sojourn time of the process $\lambda(t)$ in the first (second) state comes to the end and process $\lambda(t)$ transits to the second (first) state. During the time interval when $\lambda(t) = \lambda_i$, a Poisson flow of events with intensity λ_i, $i = 1, 2$, arrives. Also at any moment of an event occurrence in state 1 of the process $\lambda(t)$, the process can change its state to state 2 with probability p ($0 \leq p \leq 1$) or continue to stay in state 1 with complementary probability $1 - p$. I.e., after an event occurrence the process $\lambda(t)$ can change or not change its state from state 1 to state 2. The transition of the process $\lambda(t)$ from state 2 to state 1 at the moment of an event occurrence in the second state is impossible. At the moment when the state changes from the second to the first state, an additional event is assumed to be initiated with probability δ ($0 \leq \delta \leq 1$). Such flows with additional events initiation are called integrated flows. Under the made assumptions we can assert that $\lambda(t)$ is a Markovian process. So the flow can be characterized by $\{D_0, D_1\}$, in terms of the rate matrices,

$$D_0 = \begin{Vmatrix} -(\lambda_1 + \beta) & \beta \\ (1-\delta)\alpha & -(\lambda_2 + \alpha) \end{Vmatrix}, \ D_1 = \begin{Vmatrix} (1-p)\lambda_1 & p\lambda_1 \\ \delta\alpha & \lambda_2 \end{Vmatrix}.$$

Intensities of the process $\lambda(t)$ transitions from state to state with the event occurrence fill in the matrix D_1. Nondiagonal elements of the matrix D_0 are intensities of the process $\lambda(t)$ transitions from state to state without the event occurrence. Diagonal elements of the matrix D_0 are intensities of the process $\lambda(t)$ output from its states taken with the opposite signs. Note also that if $\beta = 0$, then the integrated semi-synchronous flow of events will take place [39].

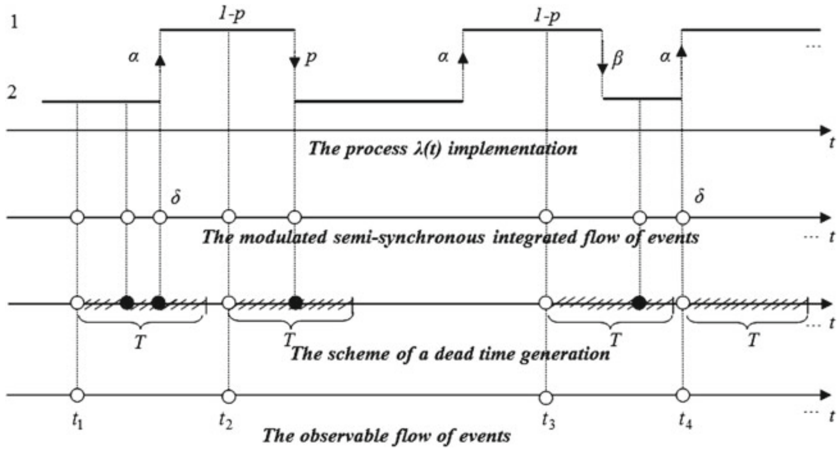

Fig. 1. The formation of an observable flow of events

The registration of the flow events is considered in condition of a constant dead time (of its incomplete observability). The dead time period of a constant duration T begins after every registered at the moment t_k, $k \geq 1$, event. During this period no other events are observed. When the dead time period is over, the first coming event causes the next interval of a dead time of duration T and so on. Figure 1 shows the possible variant of the flow operation and observation. Here 1, 2 are the states of the process $\lambda(t)$; additional events, that may occur at the moment of the process $\lambda(t)$ transition from state 2 to state 1, are marked with letter δ; dead time periods of duration T are marked with hatching; unobserved events are displayed as black circles, observed events t_1, t_2, ..., are shown as white circles.

It should be mentioned that it is not specified exactly, in which state an additional event is assumed to be initiated with probability δ, when the process $\lambda(t)$ changes its state from the second to the first one. This fact is inessential for further formulas derivation as the event occurrence and the process $\lambda(t)$ transition to the first state happens instantly. In practical situations, two variants are possible: (1) first an additional event is initiated with probability δ in state 2 and thereafter the process $\lambda(t)$ transition from state 2 to state 1 is made; (2) first the process $\lambda(t)$ transition from state 2 to state 1 is made and thereafter an additional event is initiated with probability δ in state 1. But to obtain numerical results during simulation procedure, we should take the mentioned details into account and fix, what occurs first, event or transition.

We should note that the process $\lambda(t)$ is basically unobservable. We register only time moments t_1, t_2, ... of the events occurrence in observable flow. The process $\lambda(t)$ is considered in a steady-state conditions. So under the made assumptions we can assert that the sequence of the time moments t_1, t_2, ... corresponds to an embedded Markov chain $\{\lambda(t_k)\}$, i.e. the flow has the Markov property if the evolution of the flow is considered from the time moment t_k,

$k = 1, 2, ...,$ of the event occurrence. Denote by $\tau_k = t_{k+1} - t_k$, $k = 1, 2, ...,$ the value of the k interval length between two neighboring flow events. In a steady-state conditions we may take that the probability density of the k interval length is $p_T(\tau_k) = p_T(\tau)$, $\tau \geq 0$, for any k (the index T stresses that the probability density depends on the dead time period duration). Thereby we may also take that the time moment t_k is equal to zero, i.e. the moment of the event occurrence is $\tau = 0$. Now let (t_k, t_{k+1}), (t_{k+1}, t_{k+2}) be the successive intervals with the corresponding values of interval length $\tau_k = t_{k+1} - t_k$, $\tau_{k+1} = t_{k+2} - t_{k+1}$. Due to the stationary of the flow, the arrangement of the intervals on a time axis is arbitrarily. That is why we may consider two successive intervals (t_1, t_2), (t_2, t_3) with the corresponding values of the interval length $\tau_1 = t_2 - t_1$, $\tau_2 = t_3 - t_2$, $\tau_1 \geq 0$, $\tau_2 \geq 0$, wherein $\tau_1 = 0$ corresponds to the time moment t_1 and $\tau_2 = 0$ corresponds to the time moment t_2 of the flow events arrival. The respective joint probability density is defined as $p_T(\tau_1, \tau_2)$, $\tau_1 \geq 0$, $\tau_2 \geq 0$.

In that way, the main problem is to obtain the expressions for probability density $p_T(\tau)$, $\tau \geq 0$, and the joint probability density $p_T(\tau_1, \tau_2)$, $\tau_1 \geq 0$, $\tau_2 \geq 0$, explicitly, and also to find the recurrence conditions of the observable flow of events.

3 The Expressions for Probability Density $p_T(\tau)$

Let us consider the interval $(0, \tau)$ between two neighboring events of the observable flow, which length can be written as $\tau = T + t$, where t is a duration of the interval between the end of the dead time period and the next observable event ($t \geq 0$). Let $p_{jk}(t)$ be the conditional probability that there is no observable events at the interval $(0, t)$ and $\lambda(t) = \lambda_k$ in condition that at the time moment $t = 0$ the value of the process $\lambda(t)$ is $\lambda(0) = \lambda_j$, $j, k = 1, 2$. Denote the corresponding probability density by $\widetilde{p}_{jk}(t)$, $j, k = 1, 2$. Next introduce into consideration probability $q_{ij}(T)$ – the transitional probability that the process $\lambda(\tau)$ changes its state from the state i (at the time moment $\tau = 0$) to the state j (at the time moment $\tau = T$), $i, j = 1, 2$, during the dead time period of the duration T, and probability $\pi_i(0|T)$ – the conditional probability that the process $\lambda(\tau)$ sojourns in the state i ($i = 1, 2$) at the time moment $\tau = 0$ in condition that at this time moment the event of the observable flow arrived and the dead time period of a constant duration T was generated. With the above-stated notations the desired probability density $p_T(\tau)$ can be written as

$$p_T(\tau) = \begin{cases} 0, & 0 \leq \tau < T, \\ \sum_{i=1}^{2} \pi_i(0|T) \sum_{j=1}^{2} q_{ij}(T) \sum_{k=1}^{2} \widetilde{p}_{jk}(\tau - T), & \tau \geq T. \end{cases} \quad (1)$$

Let us obtain the explicit expressions for $\widetilde{p}_{jk}(\tau - T)$, $q_{ij}(T)$, $\pi_i(0|T)$, $i, j, k = 1, 2$.

The probabilities $p_{jk}(t)$ satisfy the following systems of differential equations:

$$p'_{11}(t) = -(\lambda_1 + \beta)p_{11}(t) + \alpha(1 - \delta)p_{12}(t), \quad p'_{12}(t) = \beta p_{11}(t) - (\lambda_2 + \alpha)p_{12}(t);$$
$$p'_{21}(t) = -(\lambda_1 + \beta)p_{21}(t) + \alpha(1 - \delta)p_{22}(t), \quad p'_{22}(t) = \beta p_{21}(t) - (\lambda_2 + \alpha)p_{22}(t);$$

with the boundary conditions: $p_{11}(0)=1$, $p_{12}(0)=0$; $p_{21}(0)=0$, $p_{22}(0)=1$. Solving these systems, we find

$$\begin{aligned}
&p_{11}(t)=\tfrac{1}{z_2-z_1}\left[(\lambda_2+\alpha-z_1)e^{-z_1t}-(\lambda_2+\alpha-z_2)e^{-z_2t}\right],\\
&p_{12}(t)=\tfrac{\beta}{z_2-z_1}(e^{-z_1t}-e^{-z_2t}),\ p_{21}(t)=\tfrac{\alpha(1-\delta)}{z_2-z_1}(e^{-z_1t}-e^{-z_2t}),\\
&p_{22}(t)=\tfrac{1}{z_2-z_1}\left[(\lambda_1+\beta-z_1)e^{-z_1t}-(\lambda_1+\beta-z_2)e^{-z_2t}\right],\\
&z_1=\tfrac{1}{2}\left[\lambda_1+\lambda_2+\alpha+\beta-\sqrt{(\lambda_1-\lambda_2-\alpha+\beta)^2+4\alpha\beta(1-\delta)}\right],\\
&z_2=\tfrac{1}{2}\left[\lambda_1+\lambda_2+\alpha+\beta+\sqrt{(\lambda_1-\lambda_2-\alpha+\beta)^2+4\alpha\beta(1-\delta)}\right],\\
&0<z_1<z_2.
\end{aligned}\tag{2}$$

According to the definition of the modulated semi-synchronous integrated flow of events we introduce the probability $p_{11}(t)\,e^{-\beta\Delta t}(1-e^{-\lambda_1\Delta t})(1-p)=p_{11}(t)\lambda_1(1-p)\Delta t+o(\Delta t)$ – the joint probability that the process $\lambda(t)$ changes its state from the first state to the first one at the interval $(0,t)$ without the event occurring ($\lambda(0)=\lambda_1$, $\lambda(t)=\lambda_1$), and at the half-interval $[t,t+\Delta t)$ the duration of the first state of the process $\lambda(t)$ does not come to the end, the event of the Poisson flow with intensity λ_1 arrives and the process $\lambda(t)$ remains in the first state. The joint probabilities take the following form for different j and k ($j,\ k=1,\ 2$)

$$\begin{array}{ll}
p_{11}(t)\lambda_1(1-p)\Delta t+o(\Delta t), & p_{12}(t)\alpha\delta\Delta t+o(\Delta t),\\
p_{11}(t)\lambda_1 p\Delta t+o(\Delta t), & p_{12}(t)\lambda_2\Delta t+o(\Delta t),\\
p_{21}(t)\lambda_1(1-p)\Delta t+o(\Delta t), & p_{22}(t)\alpha\delta\Delta t+o(\Delta t),\\
p_{21}(t)\lambda_1 p\Delta t+o(\Delta t), & p_{22}(t)\lambda_2\Delta t+o(\Delta t).
\end{array}$$

The corresponding probability densities take the form

$$\begin{array}{ll}
\widetilde{p}_{11}^{(1)}(t)=p_{11}(t)\lambda_1(1-p), & \widetilde{p}_{11}^{(2)}(t)=p_{12}(t)\alpha\delta,\\
\widetilde{p}_{12}^{(1)}(t)=p_{11}(t)\lambda_1 p, & \widetilde{p}_{12}^{(2)}(t)=p_{12}(t)\lambda_2,\\
\widetilde{p}_{21}^{(1)}(t)=p_{21}(t)\lambda_1(1-p), & \widetilde{p}_{21}^{(2)}(t)=p_{22}(t)\alpha\delta,\\
\widetilde{p}_{22}^{(1)}(t)=p_{21}(t)\lambda_1 p, & \widetilde{p}_{22}^{(2)}(t)=p_{22}(t)\lambda_2.
\end{array}$$

Then the probability densities $\widetilde{p}_{jk}(t)$ that the process $\lambda(t)$ changes its state from the state j to the state k without the event occurrence at the interval $(0,t)$ and with the event occurrence at the time moment t, can be written for different j and k ($j,\ k=1,\ 2$) as

$$\begin{array}{ll}
\widetilde{p}_{11}(t)=p_{11}(t)\lambda_1(1-p)+p_{12}(t)\alpha\delta, & \widetilde{p}_{12}(t)=p_{11}(t)\lambda_1 p+p_{12}(t)\lambda_2,\\
\widetilde{p}_{21}(t)=p_{21}(t)\lambda_1(1-p)+p_{22}(t)\alpha\delta, & \widetilde{p}_{22}(t)=p_{21}(t)\lambda_1 p+p_{22}(t)\lambda_2.
\end{array}\tag{3}$$

Substituting (2) into (3), we obtain the explicit formulas for probability densities $\widetilde{p}_{jk}(t)$, $j,\ k=1,\ 2$.

The probabilities $q_{ij}(\tau)$, $0\le\tau\le T$, satisfy the following systems of differential equations:

$$\begin{array}{ll}
q_{11}'(\tau)=-(p\lambda_1+\beta)q_{11}(\tau)+\alpha q_{12}(\tau), & q_{12}'(\tau)=(p\lambda_1+\beta)q_{11}(\tau)-\alpha q_{12}(\tau);\\
q_{21}'(\tau)=-(p\lambda_1+\beta)q_{21}(\tau)+\alpha q_{22}(\tau), & q_{22}'(\tau)=(p\lambda_1+\beta)q_{21}(\tau)-\alpha q_{22}(\tau);
\end{array}$$

with the boundary conditions: $q_{11}(0) = 1$, $q_{12}(0) = 0$; $q_{21}(0) = 0$, $q_{22}(0) = 1$. Solving these systems, we obtain for $\tau = T$

$$\begin{array}{ll} q_{11}(T) = \pi_1 + \pi_2 e^{-(p\lambda_1+\beta+\alpha)T}, & q_{12}(T) = \pi_2 - \pi_2 e^{-(p\lambda_1+\beta+\alpha)T}, \\ q_{21}(T) = \pi_1 - \pi_1 e^{-(p\lambda_1+\beta+\alpha)T}, & q_{22}(T) = \pi_2 + \pi_1 e^{-(p\lambda_1+\beta+\alpha)T}, \\ \pi_1 = \frac{\alpha}{p\lambda_1+\beta+\alpha}, & \pi_2 = \frac{p\lambda_1+\beta}{p\lambda_1+\beta+\alpha}. \end{array} \tag{4}$$

Turn now to obtaining the probabilities $\pi_i(0|T)$, $i = 1, 2$. Denote by π_{ij} the transitional probability that the process $\lambda(\tau)$ changes its state from state i to state j $(i, j = 1, 2)$ during the time from the moment $\tau = 0$ till the moment of the next event arrival in observable flow. Since the sequence of the time moments of the events occurrence in observable flow corresponds to an embedded Markov chain, the following system of differential equations for $\pi_i(0|T)$ takes place:

$$\begin{array}{l} \pi_1(0|T) = \pi_1(0|T)\pi_{11} + \pi_2(0|T)\pi_{21}, \\ \pi_2(0|T) = \pi_1(0|T)\pi_{12} + \pi_2(0|T)\pi_{22};\ \pi_1(0|T) + \pi_2(0|T) = 1. \end{array} \tag{5}$$

Let us introduce into consideration probability p_{ij} – a transitional probability that the process $\lambda(t)$ changes its state from state i to state j $(i, j = 1, 2)$ during the time from the time moment $t = 0$ (the end of the dead time period) till the moment of the next observable flow event arrival. Here the probabilities p_{ij} are determined as

$$p_{ij} = \int_0^\infty \widetilde{p}_{ij}(t)\, dt, \tag{6}$$

where $\widetilde{p}_{ij}(t)$ are defined by (3), $p_{ij}(t)$ are defined by (2) $(i, j = 1, 2)$. Calculating the corresponding integrals (6) for different i and j $(i, j = 1, 2)$

$$\begin{array}{l} p_{11} = \int_0^\infty \widetilde{p}_{11}(t)\, dt = \lambda_1(1-p)\int_0^\infty p_{11}(t)\, dt + \alpha\delta\int_0^\infty p_{12}(t)\, dt, \\ p_{12} = \int_0^\infty \widetilde{p}_{12}(t)\, dt = \lambda_1 p\int_0^\infty p_{11}(t)\, dt + \lambda_2\int_0^\infty p_{12}(t)\, dt, \\ p_{21} = \int_0^\infty \widetilde{p}_{21}(t)\, dt = \lambda_1(1-p)\int_0^\infty p_{21}(t)\, dt + \alpha\delta\int_0^\infty p_{22}(t)\, dt, \\ p_{22} = \int_0^\infty \widetilde{p}_{22}(t)\, dt = \lambda_1 p\int_0^\infty p_{21}(t)\, dt + \lambda_2\int_0^\infty p_{22}(t)\, dt, \end{array}$$

we obtain

$$\begin{array}{l} p_{11} = \frac{1}{z_1 z_2}[\lambda_1(1-p)(\lambda_2+\alpha) + \alpha\delta\beta], \\ p_{12} = \frac{1}{z_1 z_2}[\lambda_1 p(\lambda_2+\alpha) + \lambda_2\beta], \\ p_{21} = \frac{1}{z_1 z_2}[\lambda_1\alpha(1-p+p\delta) + \alpha\delta\beta], \\ p_{22} = \frac{1}{z_1 z_2}[\lambda_2(\lambda_1+\beta) + p\lambda_1\alpha(1-\delta)], \end{array} \tag{7}$$

where $z_1 z_2 = \lambda_1\lambda_2 + \lambda_1\alpha + \lambda_2\beta + \alpha\delta\beta$.

Since the process $\lambda(t)$ is a Markovian process, the obtained earlier transitional probabilities $q_{ij}(T)$ and p_{ij}, $i, j = 1, 2$, allow us to write the expressions for transitional probabilities π_{ij}, $i, j = 1, 2$, in the following form

$$\begin{array}{ll} \pi_{11} = q_{11}(T)p_{11} + q_{12}(T)p_{21}, & \pi_{12} = q_{11}(T)p_{12} + q_{12}(T)p_{22}, \\ \pi_{21} = q_{21}(T)p_{11} + q_{22}(T)p_{21}, & \pi_{22} = q_{21}(T)p_{12} + q_{22}(T)p_{22}. \end{array} \tag{8}$$

Substituting first (4) into (8) and next (7) into (8), we obtain

$$\begin{array}{l} \pi_{11} = \frac{1}{z_1 z_2}\left\{\lambda_1(1-p)(\lambda_2+\alpha)+\alpha\delta\beta-\lambda_1\pi_2[\lambda_2-p(\lambda_2+\alpha\delta)]\left[1-e^{-(p\lambda_1+\beta+\alpha)T}\right]\right\}, \\ \pi_{12} = \frac{1}{z_1 z_2}\left\{p\lambda_1(\lambda_2+\alpha)+\lambda_2\beta+\lambda_1\pi_2[\lambda_2-p(\lambda_2+\alpha\delta)]\left[1-e^{-(p\lambda_1+\beta+\alpha)T}\right]\right\}, \\ \pi_{21} = \frac{1}{z_1 z_2}\left\{\alpha[\lambda_1(1-p+p\delta)+\delta\beta]+\lambda_1\pi_1[\lambda_2-p(\lambda_2+\alpha\delta)]\left[1-e^{-(p\lambda_1+\beta+\alpha)T}\right]\right\}, \\ \pi_{22} = \frac{1}{z_1 z_2}\left\{\lambda_2(\lambda_1+\beta)+p\,\lambda_1\alpha(1-\delta)-\lambda_1\pi_1[\lambda_2-p(\lambda_2+\alpha\delta)]\left[1-e^{-(p\lambda_1+\beta+\alpha)T}\right]\right\}. \end{array} \quad (9)$$

Then, substituting (9) into (5), we obtain the expressions for $\pi_i(0|T)$, $i,\ j=1,\ 2$:

$$\begin{array}{l} \pi_1(0|T) = \frac{\alpha[\lambda_1(1-p+p\delta)+\delta\beta]+\lambda_1\pi_1[\lambda_2-p(\lambda_2+\alpha\delta)]\left[1-e^{-(p\lambda_1+\beta+\alpha)T}\right]}{\lambda_1\alpha+(p\lambda_1+\beta)(\lambda_2+\alpha\delta)+\lambda_1[\lambda_2-p(\lambda_2+\alpha\delta)]\left[1-e^{-(p\lambda_1+\beta+\alpha)T}\right]}, \\ \pi_2(0|T) = \frac{p\lambda_1(\lambda_2+\alpha)+\lambda_2\beta+\lambda_1\pi_2[\lambda_2-p(\lambda_2+\alpha\delta)]\left[1-e^{-(p\lambda_1+\beta+\alpha)T}\right]}{\lambda_1\alpha+(p\lambda_1+\beta)(\lambda_2+\alpha\delta)+\lambda_1[\lambda_2-p(\lambda_2+\alpha\delta)]\left[1-e^{-(p\lambda_1+\beta+\alpha)T}\right]}, \end{array} \quad (10)$$

where π_1, π_2 are defined in (4).

Substituting first (3) into (1) and next (2), (4) and (10) into (1), carrying out laborious transformations and considering that $t=\tau-T$, we obtain

$$p_T(\tau) = \begin{cases} 0, & 0 \le \tau < T, \\ \gamma(T) z_1 e^{-z_1(\tau-T)} + (1-\gamma(T))\, z_2 e^{-z_2(\tau-T)}, & \tau \ge T, \end{cases} \quad (11)$$
$$\gamma(T) = \frac{1}{z_2-z_1}\left[z_2-\lambda_1+(\lambda_1-\lambda_2-\alpha\delta)\pi_2(T)\right],$$

$$\begin{array}{l} \pi_1(T) = \pi_1 + \left[\pi_2-\pi_2(0|T)\right] e^{-(p\lambda_1+\beta+\alpha)T}, \\ \pi_2(T) = \pi_2 - \left[\pi_2-\pi_2(0|T)\right] e^{-(p\lambda_1+\beta+\alpha)T}, \end{array} \quad (12)$$

where z_i are defined in (2); π_i – in (4); $\pi_i(0|T)$ – in (10), $i=1,\ 2$.

In particular, by setting *T=0* in (11), (12), we obtain the formulas for $p(\tau)$ that were presented in [40].

4 The Expressions for Joint Probability Density $p_T(\tau_1, \tau_2)$

Let $\tau_1 = T + t^{(1)}$, $\tau_2 = T + t^{(2)}$ be the values of the intervals length for two successive intervals between the time moments of the events arrival in observable flow of events, where $\tau_1 = 0$ is the arrival time for the first flow event, $\tau_2 = 0$ is the arrival time for the second flow event. Since the sequence of the time moments of the events arrival in observable flow corresponds to an embedded Markov chain, then with the above notation (see Sect. 3) the joint probability density $p_T(\tau_1, \tau_2)$ takes the following form

$$p_T(\tau_1,\tau_2) = \begin{cases} 0, \quad 0 \le \tau_1 < T, \quad 0 \le \tau_2 < T, \\ \sum_{i=1}^{2} \pi_i(0|T) \sum_{j=1}^{2} q_{ij}(T) \sum_{k=1}^{2} \widetilde{p}_{jk}(\tau_1 - T) \\ \times \sum_{s=1}^{2} q_{ks}(T) \sum_{n=1}^{2} \widetilde{p}_{sn}(\tau_2 - T), \ \tau_1 \ge T, \ \tau_2 \ge T, \end{cases} \quad (13)$$

where $\widetilde{p}_{jk}(\tau_1 - T) = \widetilde{p}_{jk}(t^{(1)})$, $\widetilde{p}_{sn}(\tau_2 - T) = \widetilde{p}_{sn}(t^{(2)})$ are defined by (3) and t should be replaced by $t^{(1)}$ and $t^{(2)}$ in expressions for $\widetilde{p}_{ij}(t)$, $i,\ j = 1,\ 2$. Then substituting first $\widetilde{p}_{jk}(t^{(1)})$, $\widetilde{p}_{sn}(t^{(2)})$, that are defined by (3), next $p_{jk}(t^{(1)})$, $p_{sn}(t^{(2)})$, that are defined by (2) for $t = t^{(1)}$ and $t = t^{(2)}$, next $q_{ij}(T)$, $q_{ks}(T)$, that are defined by (4), and finally $\pi_i(0|T)$, $i = 1, 2$, that are defined by (10), into (13) and carrying out laborious transformations, we obtain

$$p_T(\tau_1, \tau_2) = 0, \quad 0 \le \tau_1 < T, \quad 0 \le \tau_2 < T,$$

$$\begin{aligned} p_T(\tau_1,\tau_2) &= p_T(\tau_1)p_T(\tau_2) + e^{-(p\lambda_1+\beta+\alpha)T}\gamma(T)\left[1-\gamma(T)\right]\tfrac{\lambda_1[\lambda_2-p(\lambda_2+\alpha\delta)]}{z_1 z_2} \\ &\times \left[z_1 e^{-z_1(\tau_1-T)} - z_2 e^{-z_2(\tau_1-T)}\right]\left[z_1 e^{-z_1(\tau_2-T)} - z_2 e^{-z_2(\tau_2-T)}\right], \quad \tau_1 \ge T, \quad \tau_2 \ge T, \end{aligned} \tag{14}$$

where $z_1 z_2 = \lambda_1\lambda_2 + \lambda_1\alpha + \lambda_2\beta + \alpha\delta\beta$ and $\gamma(T)$, $p_T(\tau_k)$ are defined by (11) for $\tau = \tau_k$, $k = 1, 2$.

It follows from (14) that in general case the modulated semi-synchronous integrated flow of events is a correlated flow. By taking in (14) $T = 0$, we get the formula for the joint probability density $p(\tau_1, \tau_2)$ presented in [40].

There is no difficulty in obtaining the probabilistic characteristics of the observable flow of events, such as mathematical expectation of the interval length between the neighboring flow events, variance and covariance:

$$M\tau = T + \tfrac{\gamma(T)}{z_1} + \tfrac{1-\gamma(T)}{z_2}, \quad D\tau = 2\left[\tfrac{\gamma(T)}{z_1^2} + \tfrac{1-\gamma(T)}{z_2^2}\right] - \left[\tfrac{\gamma(T)}{z_1} + \tfrac{1-\gamma(T)}{z_2}\right]^2,$$

$$cov(\tau_1,\tau_2) = e^{-(p\lambda_1+\beta+\alpha)T}\lambda_1\gamma(T)\left[1-\gamma(T)\right]\left[\lambda_2 - p(\lambda_2+\alpha\delta)\right]\tfrac{(z_2-z_1)^2}{(z_1 z_2)^3}.$$

It is worthwhile to note that there are three types of events in the modulated semi-synchronous integrated flow of events: (1) events of a Poisson flow with intensity λ_1; (2) events of a Poisson flow with intensity λ_2; (3) additional events, which are indistinguishable. Introduce into consideration probabilities $q_1^{(i)}(T)$ – stationary probability that the event appeared is the event of a Poisson flow with intensity λ_1 (first type event) and the process $\lambda(t)$ changes its state from the state 1 to the state i ($i = 1, 2$); $q_2(T)$ – stationary probability that the event appeared is the event of a Poisson flow with intensity λ_2 (second type event); $q_3(T)$ – stationary probability that the event appeared is an additional event (third type event). Now it is not difficult to obtain the explicit expressions for the introduced probabilities on the basis of the above results:

$$q_1^{(1)}(T) = \lambda_1(1-p)\frac{\alpha + \left[(\lambda_2+\alpha\delta)\pi_1 - \alpha\delta\right]\left[1 - e^{-(p\lambda_1+\beta+\alpha)T}\right]}{z_1 z_2 - \lambda_1\left[\lambda_2 - p(\lambda_2+\alpha\delta)\right]e^{-(p\lambda_1+\beta+\alpha)T}},$$

$$q_1^{(2)}(T) = \lambda_1 p\frac{\alpha + \left[(\lambda_2+\alpha\delta)\pi_1 - \alpha\delta\right]\left[1 - e^{-(p\lambda_1+\beta+\alpha)T}\right]}{z_1 z_2 - \lambda_1\left[\lambda_2 - p(\lambda_2+\alpha\delta)\right]e^{-(p\lambda_1+\beta+\alpha)T}},$$

$$q_2(T) = \lambda_2\frac{p\lambda_1 + \beta + \lambda_1(1-p-\pi_1)\left[1 - e^{-(p\lambda_1+\beta+\alpha)T}\right]}{z_1 z_2 - \lambda_1\left[\lambda_2 - p(\lambda_2+\alpha\delta)\right]e^{-(p\lambda_1+\beta+\alpha)T}},$$

$$q_3(T) = \alpha\delta\,\frac{p\lambda_1 + \beta + \lambda_1(1-p-\pi_1)\left[1 - e^{-(p\lambda_1+\beta+\alpha)T}\right]}{z_1 z_2 - \lambda_1\left[\lambda_2 - p(\lambda_2+\alpha\delta)\right]e^{-(p\lambda_1+\beta+\alpha)T}}.$$

Then the stationary probability $q_1(T)$ that the event appeared is the event of a Poisson flow with intensity λ_1 (first type event) can be written as

$$q_1(T) = q_1^{(1)}(T) + q_1^{(2)}(T) = \lambda_1 \frac{\alpha + [(\lambda_2+\alpha\delta)\pi_1 - \alpha\delta]\left[1 - e^{-(p\lambda_1+\beta+\alpha)T}\right]}{z_1 z_2 - \lambda_1\left[\lambda_2 - p(\lambda_2+\alpha\delta)\right]e^{-(p\lambda_1+\beta+\alpha)T}}.$$

Finally, note that $\pi_1(0|T) = q_1^{(1)}(T) + q_3(T)$, $\pi_2(0|T) = q_1^{(2)}(T) + q_2(T)$.

5 The Conditions of the Observable Flow Recurrence

Let us consider the specific cases, when the modulated semi-synchronous integrated flow of events becomes the recurrent flow. It can be shown by using the expressions (11), (12) for $\gamma(T)$, $\pi_1(T)$, $\pi_2(T)$ and (10) for $\pi_1(0|T)$, $\pi_2(0|T)$ that

$$\begin{aligned}&\gamma(T)\left[1-\gamma(T)\right] = \tfrac{(\lambda_1-\lambda_2-\alpha\delta)[\lambda_1\alpha+(p\lambda_1+\beta)(\lambda_2+\alpha\delta)][(p\lambda_1+\beta)\pi_1(0)-\alpha\pi_2(0)]z_1z_2}{(z_2-z_1)^2(p\lambda_1+\beta+\alpha)^2\left[z_1z_2-\lambda_1[\lambda_2-p(\lambda_2+\alpha\delta)]e^{-(p\lambda_1+\beta+\alpha)T}\right]^2}\\ &\times\{z_1z_2 - \left[2z_1z_2 - (p\lambda_1+\beta+\alpha)(z_1+z_2)\right]e^{-(p\lambda_1+\beta+\alpha)T}\\ &+[z_1z_2 - (p\lambda_1+\beta+\alpha)(\lambda_1(1-p)+\lambda_2)]e^{-2(p\lambda_1+\beta+\alpha)T}\},\end{aligned} \tag{15}$$

where $\pi_i(0)$ is the conditional stationary probability that the process $\lambda(\tau)$ sojourns in the state i ($i = 1, 2$) at the time moment $\tau = 0$ in condition that at this time moment the flow event has arrived ($\pi_1(0) + \pi_2(0) = 1$). And $\pi_i(0)$, $i = 1, 2$, are defined as follows

$$\pi_1(0) = \alpha\tfrac{\lambda_1(1-p+p\delta)+\delta\beta}{\lambda_1\alpha+(p\lambda_1+\beta)(\lambda_2+\alpha\delta)},\ \pi_2(0) = \tfrac{p\lambda_1(\lambda_2+\alpha)+\lambda_2\beta}{\lambda_1\alpha+(p\lambda_1+\beta)(\lambda_2+\alpha\delta)}.$$

Note, that the expression enclosed in braces in formula (15), which we denote by $f(T)$, after the transformation can be written in form

$$\begin{aligned}&f(T) = z_1z_2\left[1-e^{-(p\lambda_1+\beta+\alpha)T}\right]^2 + (p\lambda_1+\beta+\alpha)e^{-(p\lambda_1+\beta+\alpha)T}[z_1+z_2\\ &-(\lambda_1(1-p)+\lambda_2)e^{-(p\lambda_1+\beta+\alpha)T}] = f_1(T)+f_2(T) = f_1(T)+\varphi_1(T)\varphi_2(T).\end{aligned}$$

It is easy to show, that for any $T \geq 0$ we have $f_1(T) \geq 0$, $\varphi_1(T) > 0$ and $\varphi_2(T) > 0$ and thus $f_2(T) > 0$. Hence, for any $T \geq 0$ we have $f(T) > 0$. It follows from (15) that:

(1) if $\lambda_1 - \lambda_2 - \alpha\delta = 0$, then the joint probability density (14) becomes factorable: $p_T(\tau_1,\tau_2) = p_T(\tau_1)p_T(\tau_2)$; and it follows from (2) that $z_1 = \lambda_1$, $z_2 = \lambda_2 + \alpha + \beta$; (11) implies $\gamma(T) = 1$, and then $p_T(\tau_k) = \lambda_1 e^{-\lambda_1(\tau_k - T)}$, $\tau_k \geq T$, $k = 1, 2$, i.e. $p_T(\tau) = \lambda_1 e^{-\lambda_1(\tau-T)}$, $\tau \geq T$.
(2) if $(p\lambda_1+\beta)\pi_1(0) - \alpha\pi_2(0) = 0$, then the joint probability density (14) becomes factorable: $p_T(\tau_1,\tau_2) = p_T(\tau_1)p_T(\tau_2)$; and it follows from (2) that $z_1 = \lambda_1(1-p+p\delta)+\delta\beta$; (11) implies $\gamma(T) = 1$, and then $p_T(\tau_k) = z_1 e^{-z_1(\tau_k-T)}$, $\tau_k \geq T$, $k = 1, 2$, i.e. $p_T(\tau) = z_1 e^{-z_1(\tau-T)}$, $\tau \geq T$.

The third condition of the joint probability density $p_T(\tau_1,\tau_2)$ factorization follows from (14): $\lambda_2 - p(\lambda_2+\alpha\delta) = 0$. In this case $p_T(\tau)$ is defined by the formula (11), where

$$\pi_2(0|T) = p;\ \pi_2(T) = \frac{p\lambda_1 + \beta}{p\lambda_1 + \beta + \alpha} + \left[p - \frac{p\lambda_1 + \beta}{p\lambda_1 + \beta + \alpha}\right] e^{-(p\lambda_1+\beta+\alpha)T};\ p \neq 1.$$

In particular, if we put $p = 1$ in the third condition, we have $\delta = 0$. Then $p_T(\tau)$ is defined by the formula (11), where

$$\pi_2(0|T) = 1; \qquad \pi_2(T) = \frac{1}{\lambda_1 + \beta + \alpha}\left[\lambda_1 + \beta + \alpha e^{-(p\lambda_1+\beta+\alpha)T}\right].$$

Since the sequence of the time moments t_1, t_2, ..., t_k, ... corresponds to an embedded Markov chain, then upon meeting one of the above-mentioned conditions or their combination we may show that the joint probability density $p_T(\tau_1, ..., \tau_k)$ becomes factorable for any k. This suggests that in this case the observable flow of events is a recurrent flow. For, let $p_T(\tau_1, ..., \tau_k, \tau_{k+1})$ be the joint probability density of $\tau_1, ..., \tau_k, \tau_{k+1}$, where $\tau_k = t_{k+1} - t_k$, $k = 1, 2, ...$. For $k = 2$ we have $p_T(\tau_1, \tau_2) = p_T(\tau_1)p_T(\tau_2)$. Now we proceed by mathematical induction. Assume that $p_T(\tau_1, ..., \tau_k) = p_T(\tau_1)...p_T(\tau_k)$. Since the sequence of the time moments t_1, t_2, ..., t_k, t_{k+1} of the flow events occuring is an embedded Markov chain, then the flow has the Markov property at the moments of the flow events arrival. Then $p_T(\tau_1, ..., \tau_k, \tau_{k+1}) = p_T(\tau_1, ..., \tau_k)p_T(\tau_{k+1}|\tau_1, ..., \tau_k) = p_T(\tau_1, ..., \tau_k)p_T(\tau_{k+1}|\tau_k)$, where $p_T(\tau_{k+1}|\tau_k) = p_T(\tau_k, \tau_{k+1})/p_T(\tau_k)$. Since for the neighboring intervals (t_k, t_{k+1}) and (t_{k+1}, t_{k+2}), $k = 1, 2, ...$, which location on the time axis is arbitraraly, we have $p_T(\tau_k, \tau_{k+1}) = p_T(\tau_k)p_T(\tau_{k+1})$, then $p_T(\tau_{k+1}|\tau_k) = p_T(\tau_{k+1})$. This proves the factorization of the joint probability density $p_T(\tau_1, ..., \tau_k, \tau_{k+1})$.

Note that the factorization conditions are identical for $T = 0$ [40] and $T \neq 0$.

In further discussion of the flow recurrence conditions we should consider results obtained in [31–33].

For the first recurrence condition a posteriori probability $w(\lambda_1|t)$ behavior at the intervals (t_k, t_{k+1}), $k = 1, 2, ...$, is determined with the explicit formulas:

$$\begin{aligned} &w(\lambda_1|t) = \pi_1 - [\pi_1 - w(\lambda_1|t_k + 0)]\ e^{-(p\lambda_1+\beta+\alpha)(t-t_k)},\ t_k < t \leq t_k + T, \\ &w(\lambda_1|t) = \frac{w_1[w_2 - w(\lambda_1|t_k+T)] - w_2[w_1 - w(\lambda_1|t_k+T)]\ e^{-b(t-t_k-T)}}{w_2 - w(\lambda_1|t_k+T) - [w_1 - w(\lambda_1|t_k+T)]\ e^{-b(t-t_k-T)}},\ t_k + T < t \leq t_{k+1}, \end{aligned} \tag{16}$$

where

$$w(\lambda_1|t_k + 0) = \frac{\alpha\delta + [\lambda_1(1-p) - \alpha\delta]\ w(\lambda_1|t_k - 0)}{\lambda_2 + \alpha\delta},$$

$$\begin{aligned} &w_1 = \frac{\lambda_1 - \lambda_2 + \alpha + \beta - 2\alpha\delta - b}{2(\lambda_1 - \lambda_2 - \alpha\delta)}, \qquad w_2 = \frac{\lambda_1 - \lambda_2 + \alpha + \beta - 2\alpha\delta + b}{2(\lambda_1 - \lambda_2 - \alpha\delta)}, \\ &b = \sqrt{(\lambda_1 - \lambda_2 - \alpha + \beta)^2 + 4\alpha\beta(1-\delta)}, \end{aligned} \tag{17}$$

and π_1 is defined by (4). In spite of the fact that the flow becomes recurrent and probability density $p_T(\tau)$ is exponential, a posteriori probability $w(\lambda_1|t)$ depends on prehistory, i.e. it depends on the time moments t_1, t_2, ..., t_k of the events occurrence in observable flow. In fact, $w(\lambda_1|t)$ depends on the initial condition at the time moment t_k – the value of $w(\lambda_1|t_k+0)$, $k = 1, 2, ...$. In turn $w(\lambda_1|t_k+0)$ depends on the value of $w(\lambda_1|t_k - 0)$, of probability $w(\lambda_1|t)$ at the moment t_k, when $w(\lambda_1|t)$ changes at the half-interval $[t_{k-1}, t_k)$ preceding the half-interval $[t_k, t_{k+1})$, $k = 1, 2, ...$ Thereby, all prehistory of the flow observation from the time moment $t_0 = 0$ to t_k is concentrated in the value of $w(\lambda_1|t_k + 0)$. And it may be stated that the flow is close to a simple stream. If to add an additional condition $\lambda_1(1-p)-\alpha\delta = 0$, then a posteriori probability $w(\lambda_1|t)$ will not depend on prehistory, it will depend on the value of $w(\lambda_1|t)$ at the moment of the event occurrence t_k, i.e. on $w(\lambda_1|t_k + 0) = \alpha\delta/(\lambda_2 + \alpha\delta)$, $k = 1, 2, ...$. In this case we may state that the flow is more close to a simple stream.

For the second recurrence condition a posteriori probability $w(\lambda_1|t)$ behavior at the intervals (t_k, t_{k+1}), $k = 1, 2, ...$, is determined with the explicit formulas (16), where

$$w(\lambda_1|t_k + 0) = \frac{\alpha\delta + [\lambda_1(1-p) - \alpha\delta]\ w(\lambda_1|t_k - 0)}{\lambda_2 + \alpha\delta + (\lambda_1 - \lambda_2 - \alpha\delta)\, w(\lambda_1|t_k - 0)},\ k = 1, 2, ...$$

In spite of the fact that the flow becomes recurrent and probability density $p_T(\tau)$ is exponential, a posteriori probability $w(\lambda_1|t)$ also depends on prehistory, i.e. it depends on the time moments t_1, t_2, ..., t_k of the events occurrence in observable flow. In this case we may state that the flow is close to a simple stream.

For the third recurrence condition probability density $p_T(\tau)$ is defined by the formula (11) and it is not exponential, so there is no closeness with a simple stream of events.

6 Conclusion

The obtained results provide the possibility to solve the problem of parameters estimation of the modulated semi-synchronous integrated flow of events in condition of a constant dead time. One of the most interesting and important problems of the flow parameters estimation is estimating the dead time period duration. This is necessary to estimate the quantity of the lost flow events (events carrying useful information). To solve this problem we can apply the following methods: (1) maximum-likelihood technique; (2) method of moments.

To estimate duration of the dead time period with maximum-likelihood technique, first of all, the likelihood function is constructed

$$L(T\,|\tau_1, ..., \tau_n) = \prod_{k=1}^{n} p_T(\tau_k),$$

where τ_k, $k = \overline{1, n}$, are the measured values of the intervals length duration $\tau_k = t_{k+1} - t_k$, $k = \overline{1, n}$. Then the following task of optimization is resolved

$$L(T\,|\tau_1, ..., \tau_n) \Longrightarrow \underset{T}{max}, \quad 0 \le T \le \tau_{min},$$

where $\tau_{min} = min\, \tau_k$, $k = \overline{1, n}$. The point of global maximum T^* of the likelihood function $L(T\,|\tau_1, ..., \tau_n)$ will be the desired estimation $\hat{T}$ of the dead time period duration.

To solve the estimation problem with the method of moments $\hat{cov}(\tau_1, \tau_2)$ statistic is constructed. $\hat{cov}(\tau_1, \tau_2)$ is the estimation of theoretical covariance

$$cov_T(\tau_1, \tau_2) = \int_T^\infty \int_T^\infty [\tau_1 - M\tau_1]\,[\tau_2 - M\tau_2]\, p_T(\tau_1, \tau_2) d\tau_1 d\tau_2,$$

where $M\tau_k$, $k = 1, 2$, are mathematical expectations of the intervals length $\tau_1 = t_2 - t_1$ and $\tau_2 = t_3 - t_2$. Then the equation of moments $cov_T(\tau_1, \tau_2) = \hat{cov}(\tau_1, \tau_2)$ is solved for the unknown T and a solution of this equation is chosen as $\hat{T}$.

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