Preface

When dealing with real-world problems, one can rarely avoid uncertainty. Very often in practice, it is not possible to obtain precise information about the values of the parameters of a modelled system. For a long time, probability approaches were dominant in the literature devoted to problems involving uncertainty. However, the most common situation in practice is when some model parameters are subject to *aleatory uncertainty*, whereas others are subject to *epistemic uncertainty*. This situation is especially common in economic calculus, which stems from the fact that economic data usually comes from various sources, but also concerns areas such as human decision making, risk analysis or engineering applications.

Aleatory uncertainty, also called *variability* or *statistical uncertainty*, is a description of naturally random behaviour in a physical process or property. Probability theory is a natural model for this type of uncertainty, and the quantification for the aleatory uncertainty is usually performed using the Monte Carlo techniques. On the other hand, epistemic uncertainty refers to limited knowledge about the system (modelled or real) or lack of information. This type of uncertainty can be reduced by, e.g. taking more measurements, conducting more tests, "buying" more information, etc. Because of that, the epistemic uncertainty is also called *reducible uncertainty*, *incertitude* or *subjective uncertainty*. Often, uncertainty quantification intends to work toward reducing epistemic uncertainties to aleatory uncertainties. However, epistemic uncertainty is not well characterised by probabilistic approaches. To evaluate epistemic uncertainties, methods such as *interval analysis*, *fuzzy logic* or *evidence theory* (Dempster–Shafer theory) are more suitable.

Measurement errors are one of the possible sources of epistemic uncertainty. It is well known that at the empirical level, uncertainty is an inseparable companion of almost any measurement. The difference between the measured and the actual values is called a *measurement error*. Since the absolute value of the measurement error can usually be bounded, it is therefore guaranteed that the actual (unknown) value of the desired quantity belongs to the *interval* (*number*) with the midpoint being the measured value, and the radius being the upper bound for the (absolute value) of the possible errors. Therefore imprecision, approximation, or uncertainty in the knowledge of the exact values of physical, technical or economic parameters can be modelled conveniently by intervals. By its nature, interval arithmetic yields rigorous enclosures for the range of operations and functions. The results are intervals in which the exact results must lie. In order that this inclusion remains true in numerical computations, the problem of round-off errors must be taken into account during the implementation of the interval computations. Proper handling of outward rounding in numerical computations forces the result to be the interval approximation of the correct real interval (that could be hypothetically obtained assuming that infinite precision is available).

A finite number of increasingly precise measurements give a finite family of usually increasingly narrower intervals (interval numbers). By assigning a respective *level of possibility* to each interval number from this family, a discrete collection of the so-called α–cuts is obtained, which can be viewed as a finite representation of a *fuzzy number*. The strict relation between interval and fuzzy numbers is often emphasised in the literature devoted to fuzzy theory. Fuzzy numbers enhance the expressive power of intervals, and therefore they are often referred to as generalised intervals.

This book presents some advances in fuzzy decision making. It is organised in eight chapters. Chapter 1 introduces some basic concepts from the fuzzy numbers theory. The main part of this chapter concerns the problem of performing arithmetic operations on fuzzy numbers. Linear operations such as addition and subtraction are rather obvious, whereas nonlinear operations, such as multiplication or division, pose a problem. Nonlinear operations usually result in fuzzy numbers of different types than operands. Therefore, various, the so-called "shape preserving", approaches to multiplication and division of fuzzy numbers are proposed in the literature. On the other hand, many researchers recommend to use the α–cuts based approach to performing operations on fuzzy numbers, because this approach allows fuzzy and interval techniques to be combine and used to effectively solve problems involving both types of uncertainty. The remaining part of this chapter is devoted to the problem of performing arithmetic operations on interactive fuzzy numbers. The reason is that all of the above-mentioned approaches implicitly assume that there is no dependency between fuzzy numbers involved in a computation. In practice this assumption is rarely satisfied, especially when dealing with economic problems. To cope with the dependency problem, stochastic simulation of fuzzy systems and nonlinear programming approaches are proposed.

The problem of comparing and ordering fuzzy numbers is described in Chap. 2. Theoretically, fuzzy numbers can only be partially ordered, and hence cannot be compared. However, in practical applications, such as decision making, scheduling, market analysis or optimisation with fuzzy uncertainties, the comparison of fuzzy numbers becomes crucial. That is why several methods for comparing and ordering fuzzy numbers have been proposed in the literature. They can be generally divided into two groups. The first group consists of methods which enable two fuzzy numbers to be compared. One can mention the probabilistic approach, centroid point approach or radius of gyration approach. To order a set of fuzzy numbers using these methods, some dedicated procedures are required. The second group consists of methods, which assign to a fuzzy number a real value. These are, for example Yager ranking index based approach, defuzzification approach or weighted average. The methods from the second group can be directly used to order a set of fuzzy numbers, by employing one of the several methods for ordering (sorting) real numbers. All the described methods are compared using a simple example.

Chapter 3 presents the concept of a fuzzy random variable and the Dempster–Shafer theory of evidence. Fuzzy random variable extends the classical definition of a random variable and is one of the possible ways to jointly consider randomness and imprecision. The simultaneous occurrence of randomness and imprecision is often the case in real-world decision problems, because data in such problems usually comes from various sources, such as historical datasets or experts opinions. The theory of evidence (also called the theory of belief functions), on the other hand, provides mathematical tools to process information, which is, at the same time, of random and imprecise nature. It allows imprecision and variability to be treated separately within a single framework. The evidence theory encompasses both possibility and probability theories.

Chapter 4 discusses selected issues of the fuzzy multi-criteria decision making (FMCDM). Generally, multi-attribute decision making (MADM) is concerned with ranking alternatives with respect to multiple criteria. Two basic techniques of multi-criteria decision making are analytical hierarchy process (AHP) and technique for order of preference by similarity to ideal solution (TOPSIS). Both AHP and TOPSIS were initially designed to deal only with crisp numbers. Later, fuzzy variants of those methods were developed, because in the real world available data are often imprecise and vague. The integrated fuzzy approach to solve multi-attribute decision problems is proposed in this chapter. Its use is illustrated using a real case from a steel industry.

Chapter 5 is devoted to a method which is able to process hybrid data, i.e. to jointly handle both randomness and imprecision. Random variables are described by probability distributions and imprecise values are modelled using possibility distributions. The main advantage of the proposed method is that it takes into account the dependencies between economic parameters. The correlation matrix is used to model dependencies between stochastic parameters, whereas interval regression is used to model both dependencies between fuzzy parameters and between fuzzy and stochastic parameters. The proposed method combines tools such as Monte Carlo simulation, interval regression and nonlinear programming. As the result of hybrid data processing, a random fuzzy set is received. Assessment of risk is obtained by computing the standard deviation and also by estimating the upper and lower cumulative distribution functions of the analysed variable. The method is verified through computing the operating profit for a metallurgical industry enterprise.

Chapter 6 describes the application of fuzzy sets to planning and scheduling of production in steel industry. Primarily the problem of steel grade assignment to customers' orders is analysed, which is the first stage of steel production planning.

Fuzzy sets are used to reduce the variety of potential steel grades and to describe the characteristic of materials by decision makers. Next, the use of fuzzy logic systems for steel production scheduling is examined. Parameters like timeliness, amount, priority and the sequence length on casters are expressed using linguistic variables. Whereas fuzzy rules are used to determine an initial schedule and to perform a quick rescheduling. More advanced systems use a multi-agent approach. Each agent may use its own fuzzy logic in order to satisfy the constraints related to the certain level of steel production. Finally, an example of a fuzzy scheduling agent is provided. It uses a genetic algorithm to generate feasible and economically efficient schedules for a continuous caster.

Chapter 7 presents a new method for forecasting the level and structure of market demand for industrial goods. The method employs two data mining methods: k-means clustering and fuzzy decision trees. The k-means method serves to separate groups with items of a similar consumption level and structure of the analysed products (consumption patterns). Whereas fuzzy decision trees are used to determine the dependencies between consumption patterns and predictors (parameters determining the level and structure of consumption). The proposed method is verified using the extensive statistical material on the level and structure of steel products consumption in selected countries during 1960–2010.

Chapter 8 discusses various techniques of visualisation of fuzzy numbers in one-, two- and more-dimensional spaces. As canonical box-and-whiskers representation is not suitable for visualisation of uncertainty in three-dimensional spaces, an approach based on ScPovPlot3D templates for POVRay, which is a powerful photorealistic renderer equipped with domain-specific programming language, dubbed scene description language (SDL), is proposed. In order to show the usefulness of the proposed technique some examples of visualisation of fuzzy objects are included, in one, two and three dimensions. For three-dimensional approach two examples of application of fuzzy visualisation are presented. The first one shows a surface defined by a function which assigns a fuzzy number to a point on the real plane. In the second example, a lesion deposited in a segment of cardiac vessel is depicted using "thick surface" approach.

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