

Contents

1	Introduction	1
2	Function Spaces	7
2.1	$L^p, C^\alpha, \text{BMO}, L^{p,\lambda}, L^{p,\lambda}_w$	7
2.2	The Campanato scale $\mathcal{L}^{p,\lambda}, \mathcal{L}^{p,\lambda}(Q_0)$	8
2.3	Sobolev Spaces $W^{m,p}(\Omega), G_\alpha(L^p), I_\alpha(L^p)$	10
2.4	Morrey-Sobolev Spaces $I_\alpha(L^{p,\lambda})$	10
2.5	Dense/non-dense subspaces, Zorko Spaces, $VL^{p,\lambda}, VMO$	10
2.6	Note	12
3	Hausdorff Capacity	13
3.1	Set functions $\Lambda^d, \mathcal{H}^d, \mathcal{L}^n, 0 < d \leq n$	13
3.2	Dyadic versions: $\tilde{\Lambda}^d, \tilde{\Lambda}_0^d$	14
3.3	Frostman's Theorem	15
3.4	Strong subadditivity of $\tilde{\Lambda}_0^d$ and $\Lambda^d \sim \tilde{\Lambda}_0^d$	16
3.5	The operator M_α and Hausdorff capacity	16
3.6	Notes	17
3.6.1	Netrusov's capacity $\Lambda^{d;\theta}$ and a Netrusov-Frostman Theorem	17
3.6.2	A strong type estimate for M_α	18
4	Choquet Integrals	21
4.1	Definition and basic properties: sublinear vs. strong subadditivity	21
4.2	Adams-Orobitg-Verdera Theorem	24
4.3	Notes	26
4.3.1	Further estimates for $M_\alpha f$	26
4.3.2	Speculations on weighted Hausdorff Capacity	27
5	Duality for Morrey Spaces	29
5.1	Dual of $L^1(\Lambda^d)$	29
5.2	Three equivalent predual spaces $X^{p,\lambda}, K^{p,\lambda}, Z^{p,\lambda}$	31
5.3	The predual $H^{p',\lambda}$	33

5.4	The space $Z_0^{p,\lambda}$ and $Z^{p,\lambda}$	35
5.5	Notes	36
6	Maximal Operators and Morrey Spaces	37
6.1	M_0 on $L^{p,\lambda}$ - two proofs	37
6.2	$\ I_\alpha \mu\ _{L^{p,\lambda}} \sim \ M_\alpha \mu\ _{L^{p,\lambda}}, \ I_\alpha \mu\ _{H^{p,\lambda}} \sim \ M_\alpha \mu\ _{H^{p,\lambda}}$	39
6.3	Proof of (6.6)	40
6.4	Notes	42
7	Potential Operators on Morrey Spaces	43
7.1	$I_\alpha : L^{p,\lambda} \longrightarrow L^{\tilde{p},\lambda} \cap L^{p,\lambda-\alpha p}, \tilde{p} = \frac{\lambda p}{\lambda-\alpha p}, \alpha p < \lambda$ $I_\alpha : H^{p,\lambda} \longrightarrow H^{\tilde{p},\lambda} \cap H^{p,\lambda+\alpha p'}$	43
7.2	Wolff potentials associated with $\ I_\alpha \mu\ _{L^{p'}}$	45
7.3	Wolff potentials associated with $\ I_\alpha \mu\ _{L^{p',\lambda}}, \ I_\alpha \mu\ _{H^{p',\lambda}}$	46
7.4	A “Morrey bridge” to C^α	48
7.5	Notes	49
7.5.1	Proof of Lemma 7.4	49
7.5.2	$I_\alpha : L^{1,\lambda} \longrightarrow L_w^{\tilde{1},\lambda}, \tilde{1} = \lambda/(\lambda - \alpha), 0 < \alpha < \lambda$	50
8	Singular Integrals on Morrey Spaces	51
8.1	$T : L^{p,\lambda} \longrightarrow L^{p,\lambda}$ and $T : H^{p,\lambda} \longrightarrow H^{p,\lambda}; 1 < p < \infty, 0 < \lambda < n$	51
9	Morrey-Sobolev capacities	53
9.1	Definitions and simple properties for $C_{\alpha,p}(\cdot)$	53
9.2	Definitions and simple properties for $C_\alpha(\cdot; X)$, $X = L^{p,\lambda}$ or $H^{p,\lambda}$	54
9.3	$C_\alpha(B(x, r); L^{p,\lambda})$	56
9.4	$C_\alpha(B(x, r); H^{p,\lambda})$ and failure of CSI for $C_\alpha(\cdot; L^{p,\lambda})$	59
9.5	Notes	60
9.5.1	Weighted capacity vs. Choquet Integrals	60
9.5.2	Relations between $C_{\alpha,p}$ and $C_{\beta,q}$ via Morrey Theory	60
9.5.3	Speculations and parabolic capacities	61
10	Traces of Morrey Potentials	63
10.1	$\ I_\alpha f\ _{L^q(\mu)} \leq c \ f\ _{L^p}$	63
10.2	$\ I_\alpha f\ _{L^q(\mu)} \leq c_0 \ f\ _{L^{p,\lambda}}$	65
10.3	An Improved Trace Result	67
10.4	Notes	68
11	Interpolation of Morrey Spaces	71
11.1	Stampacchia-Peetre interpolation; Interpolation via the new duality	71
11.2	Counterexamples to interpolation with Morrey Spaces in the domain of the operator	75
11.3	Integrability of Morrey Potentials	76

12 Commutators of Morrey Potentials	77
12.1 Some history for the operators $[b, T]$ and $[b, I_\alpha]$	77
12.2 Commutators: $b \in \text{BMO}$	78
12.3 Traces of Morrey commutators, $ \nabla b \in L^n$	80
13 Mock Morrey Spaces	85
13.1 Marcinkiewicz Spaces	85
13.2 Conti's Theorem	86
13.3 Notes	88
13.3.1 $Q^{p,\lambda}$ vs $L^{p,\lambda}$	88
14 Morrey-Besov Spaces and Besov Capacity	89
14.1 Adams-Lewis inequality (Sobolev inequality for Morrey-Besov)	89
14.2 Besov capacity and the Netrusov capacity	92
14.3 Notes: CSI for Besov capacities	93
15 Morrey Potentials and PDE I	95
15.1 $-\Delta u = u^p$, $u \geq 0$	95
15.2 Notes	101
15.2.1 The Yamabe Case $p = \frac{n+2}{n-2}$	101
15.2.2 Stationary Navier-Stokes ($n = 5$)	101
15.2.3 A Comment	102
16 Morrey Potentials and PDE II	103
16.1 Examples of singular sets for elliptic systems	103
16.2 Meyers-Elcrat system	104
16.3 Notes	108
16.3.1 Harmonic Maps	108
16.3.2 Lane-Emden systems	109
17 Morrey Spaces On Complete Riemannian Manifolds	111
17.1 A counterexample	112
17.2 A Morrey-Sobolev inequality on M^n with balls having maximal growth and $\text{Ric} \geq 0$	112
17.3 Further embedding and speculations	114
Bibliography	115
Index of Symbols	121

Morrey Spaces

Adams, D.R.

2015, XVII, 124 p. 1 illus. in color., Softcover

ISBN: 978-3-319-26679-4

A product of Birkhäuser Basel