Preface

It is well known that the celebrated Hille-Yosida theorem, discovered independently by Hille [1] and Yosida [1], gave the first characterization of the infinitesimal generator of a strongly continuous semigroup of contractions. This was the beginning of a systematic development of the theory of semigroups of bounded linear operators. The bounded linear operator A_λ appearing in the sufficiency part of Yosida's proof of this theorem is called the Yosida approximation of A; see Pazy [1]. The objective of this research monograph is to present a systematic study on Yosida approximations of stochastic differential equations in infinite dimensions and applications.

On the other hand, a study on stochastic differential equations (SDEs) in infinite dimensions was initiated in the mid-1960s; see, for instance, Curtain and Falb [1, 2], Chojnowska-Michalik [1], Ichikawa [1–4], and Metivier and Pistone [1] using the semigroup theoretic approach and Pardoux [1] using the variational approach of Lions [1] from the deterministic case. Note, however, that a strong foundation of SDEs, in infinite dimensions in the semilinear case was first laid by Ichikawa [1–4]. It is also worth mentioning here the earlier works of Haussman [1] and Zabczyk [1]. All these aforementioned attempts in infinite dimensions were generalizations of stochastic ordinary differential equations introduced by K. Itô in the 1940s and independently by Gikhman [1] in a different form, perhaps motivated by applications to stochastic partial differential equations in one dimension, like heat equations. Today, SDEs in the sense of Itô, in infinite dimensions are a well-established area of research; see the excellent monographs by Curtain and Pritchard [1], Itô [1], Rozovskii [1], Ahmed [1], Da Prato and Zabczyk [1], Kallianpur and Xiong [1], and Gawarecki and Mandrekar [1]. Throughout this book, we shall use mainly the semigroup theoretic approach as it is our interest to study mild solutions of SDEs in infinite dimensions. However, we shall also use the variational approach to study stochastic evolution equations with delay and multivalued stochastic partial differential equations.

To the best of our knowledge, Ichikawa [2] was the first to use Yosida approximations to study control problems for SDEs. It is a well-known fact that Itô's formula is not applicable to mild solutions; see Curtain [1]. This motivates the need to look for a way out, and Yosida approximations come in handy as these Yosida approximating SDEs have the so-called strong solutions and Itô's formula is applicable only to strong solutions. Yosida approximations, since then, have been used widely for various classes of SDEs; see Chapters 3 and 4 below, to study many diverse problems considered in Chapters 5 and 6.

The book begins in Chapter 1 with a brief introduction mentioning motivating problems like heat equations, an electric circuit, an interacting particle system, a lumped control system, and the option and stock price dynamics to study the corresponding abstract stochastic equations in infinite dimensions like stochastic evolution equations including such equations with delay, McKean-Vlasov stochastic evolution equations, neutral stochastic partial differential equations, and stochastic evolution equations with Poisson jumps. The book also deals with stochastic integrodifferential equations, multivalued stochastic differential equations, stochastic evolution equations with Markovian switchings driven by Lévy martingales, and time-varying stochastic evolution equations.

In Chapter 2, to make the book as self-contained as possible and reader friendly, some important mathematical machinery, namely, concepts and definitions, lemmas, and theorems, that will be needed later on in the book will be provided. As the book studies SDEs using mainly the semigroup theory, it is first intended to provide this theory starting with the fundamental Hille-Yosida theorem and then define precisely the Yosida approximations as well as such approximations for multivalued monotone maps. There is an interesting connection between the semigroup theory and the probability theory. Using this, we shall also delve into some recent results on asymptotic expansions and optimal convergence rate of Yosida approximations. Next, some basics from probability and analysis in Banach spaces are considered like those of the concepts of probability and random variables, Wiener process, Poisson process, and Lévy process, among others. With this preparation, stochastic calculus in infinite dimensions is dealt with next, namely, the concepts of Itô stochastic integral with respect to Q-Wiener and cylindrical Wiener processes, stochastic integral with respect to a compensated Poisson random measure, and Itô's formulas in various settings. In some parts of the book, the theory of stochastic convolution integrals is needed. So, we then state some results from this theory without proofs. This chapter coupled with Appendices dealing with multivalued maps, maximal monotone operators, duality maps, random multivalued maps, and operators on Hilbert spaces, more precisely, notions of trace class operators, nuclear and Hilbert-Schmidt operators, etc., should give a sound background. Since there are many excellent references on this subject matter like Curtain and Pritchard [1], Ahmed [1], Altman [1], Bharucha-Reid [1], Bichteler [1], Da Prato and Zabczyk [1, 2], Dunford and Schwartz [1], Ichikawa [3], Gawarecki and Mandrekar [1], Joshi and Bose [1], Pazy [1], Barbu [1, 2], Knoche [1], Peszat and Zabczyk [1], Prévôt and Röckner [1], Padgett [1], Padgett and Rao [1], Stephan [1], Tudor [1], Yosida [1], and Vilkiene [1–3], among others, the objective here is to keep this chapter brief.

Chapter 3 addresses the main results on Yosida approximations of stochastic differential equations in infinite dimensions in the sense of Itô. The chapter begins by motivating this study from linear stochastic evolution equations. After a brief discussion on linear equations, the pioneering work by Ichikawa (1982) on semilinear stochastic evolution equations is considered in detail next. We introduce Yosida approximating system as it has strong solutions so that Itô's formula can be applied. It will be interesting to show that these approximating strong solutions converge to mild solutions of the original system in mean square. This result is then generalized to stochastic evolution equations with delay. We next consider a special form of a stochastic evolution equation that is related to the so-called McKean-Vlasov measure-valued stochastic evolution equation. We introduce Yosida approximations to this class of equations, showing their existence and uniqueness of strong solutions and also the mean-square convergence of these strong solutions to the mild solutions of the original system. We next generalize this theory to McKean-Vlasov-type stochastic evolution equations with a multiplicative diffusion. In the rest of the chapter, we consider Yosida approximation problems of many more general stochastic models including neutral stochastic partial functional differential equations, stochastic integrodifferential equations, multivalued-valued stochastic differential equations, and time-varying stochastic evolution equations. The chapter concludes with some interesting Yosida approximations of controlled stochastic differential equations, notably, stochastic evolution equations driven by stochastic vector measures, McKean-Vlasov measure-valued evolution equations, and also stochastic equations with partially observed relaxed controls.

In Chapter 4, we consider Yosida approximations of stochastic differential equations with Poisson jumps. More precisely, we introduce Yosida approximations to stochastic delay evolution equations with Poisson jumps, stochastic evolution equations with Markovian switching driven by Lévy martingales, multivalued-valued stochastic differential equations driven by Poisson noise, and also such equations with a general drift term with respect to a general measure. As before, we shall also obtain mean-square convergence results of strong solutions of such Yosida approximate systems to mild solutions of the original equations.

In Chapter 5, many consequences and applications of Yosida approximations to stochastic stability theory are given. First, we consider the pioneering work of Ichikawa (1982) on exponential stability of semilinear stochastic evolution equation in detail and also the stability in distribution of mild solutions of such semilinear equations. As an interesting consequence, exponential stabilizability for mild solutions of semilinear stochastic evolution equations is considered next. Since an uncertainty is present in the system, we obtain robustness in stability of such systems with constant and general decays. This study is then generalized to stochastic equations with delay; that is, polynomial stability with a general decay is established for such delay systems. Consequently, robust exponential stabilization of such delay equations is obtained. Subsequently, stability in distribution is considered for stochastic evolution equations with delays driven by Poisson jumps. Moreover, moment exponential stability and also almost sure exponential stability of sample paths of mild solutions of stochastic evolution equations with Markovian switching with Poisson jumps are dealt with. We also study the weak convergence of induced probability measures of mild solutions of McKean-Vlasov stochastic evolution equations, neutral stochastic partial functional differential equations, and stochastic integrodifferential equations. Furthermore, the exponential stability of mild solutions of McKean-Vlasov-type stochastic evolution equations with a multiplicative diffusion, stochastic integrodifferential evolution equations, and time-varying stochastic evolution equations are considered.

Finally, in Chapter 6, it will be interesting to consider some applications of Yosida approximations to stochastic optimal control problems like optimal control over finite time horizon, a periodic control problem of stochastic evolution equations, and an optimal control problem of McKean-Vlasov measure-valued evolution equations. Moreover, we also consider some necessary conditions of optimality of relaxed controls of stochastic evolution equations. The chapter as well as the book concludes with optimal feedback control problems of stochastic evolution equations driven by stochastic vector measures.

I have tried to keep the work of various authors drawn from all over the literature as original as possible. I thank sincerely all of them whose work have been included in the book with due citations they deserve in the bibliographical notes and remarks and elsewhere. I believe to the best of my knowledge that I have covered in this monograph all the work that I have known. There may be more interesting materials, but it is impossible to include all in one book. I apologize to those authors in case I have missed out their work. This is certainly not deliberate.

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