Komplexe Zahlen

Polardarstellung komplexer Zahlen:

$$z = x + \mathrm{i}\, y = |z|\, \mathrm{e}^{\mathrm{i}\varphi}, \quad |z| = \sqrt{x^2 + y^2}, \quad \tan\varphi = \frac{y}{x}$$

Euler'sche Formel:

$$\mathrm{e}^{\mathrm{i}\varphi} = \cos\varphi + \mathrm{i}\, \sin\varphi$$

Vektorrechnung

Dreidimensional

$$\begin{aligned} \boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) &= \boldsymbol{b} \cdot (\boldsymbol{c} \times \boldsymbol{a}) = \boldsymbol{c} \cdot (\boldsymbol{a} \times \boldsymbol{b}) \\ \boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) &= (\boldsymbol{a} \cdot \boldsymbol{c})\, \boldsymbol{b} - (\boldsymbol{a} \cdot \boldsymbol{b})\, \boldsymbol{c} \\ (\boldsymbol{a} \times \boldsymbol{b}) \cdot (\boldsymbol{c} \times \boldsymbol{d}) &= (\boldsymbol{a} \cdot \boldsymbol{c})\, (\boldsymbol{b} \cdot \boldsymbol{d}) - (\boldsymbol{a} \cdot \boldsymbol{d})\, (\boldsymbol{b} \cdot \boldsymbol{c}) \\ (\boldsymbol{a} \times \boldsymbol{b})^2 &= a^2\, b^2 - (\boldsymbol{a} \cdot \boldsymbol{b})^2 \end{aligned}$$

Kronecker-Symbol:

$$\delta_{ij} = \begin{cases} 1 & \text{wenn } i = j \\ 0 & \text{wenn } i \neq j \end{cases}$$

Levi-Civita-Symbol:

$$\varepsilon_{ijk} = \begin{cases} +1 & \text{wenn } ijk \text{ gerade Permutation von 123} \\ -1 & \text{wenn } ijk \text{ ungerade Permutation von 123} \\ 0 & \text{sonst} \end{cases}$$

$$\varepsilon_{ijk}\varepsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}, \qquad \varepsilon_{ijk}\varepsilon_{ljk} = 2\delta_{il}$$

Einstein'sche Summenkonvention (kartesische Koordinaten):

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_i\, b_i \equiv \sum_{i=1}^{3} a_i\, b_i, \quad (\boldsymbol{a} \times \boldsymbol{b})_i = \varepsilon_{ijk}\, a_j\, b_k$$

Ableitungen von skalaren Feldern $\phi(\boldsymbol{r})$, Vektorfeldern $\boldsymbol{X}(\boldsymbol{r})$:

$$\begin{aligned} &\text{Gradient:} && \mathbf{grad}\,\phi = \nabla\phi \quad \text{bzw.} \quad (\nabla\phi)_i = \partial_i\,\phi \\ &\text{Divergenz:} && \operatorname{div}\boldsymbol{X} = \nabla \cdot \boldsymbol{X} = \partial_i\, X_i \\ &\text{Rotation:} && \mathbf{rot}\,\boldsymbol{X} = \nabla \times \boldsymbol{X} \quad \text{bzw.} \quad (\mathbf{rot}\,\boldsymbol{X})_i = \varepsilon_{ijk}\,\partial_j\, X_k \end{aligned}$$

Zweite Ableitungen:

$$\begin{aligned} \operatorname{div}\mathbf{grad}\,\phi &= \nabla \cdot \nabla\phi = \Delta\phi \\ \mathbf{grad}\operatorname{div}\boldsymbol{X} &= \nabla(\nabla \cdot \boldsymbol{X}) \\ \operatorname{div}\mathbf{rot}\,\boldsymbol{X} &= \nabla \cdot (\nabla \times \boldsymbol{X}) = 0 \\ \mathbf{rot}\,\mathbf{grad}\,\phi &= \nabla \times (\nabla\phi) = \mathbf{0} \\ \mathbf{rot}\,\mathbf{rot}\,\boldsymbol{X} &= \nabla \times (\nabla \times \boldsymbol{X}) = \nabla(\nabla \cdot \boldsymbol{X}) - \Delta\boldsymbol{X} \end{aligned}$$

Vierdimensional

Kronecker-Symbol:

$$\delta^{\mu}_{\nu} = \begin{cases} 1 & \text{wenn } \mu = \nu \\ 0 & \text{wenn } \mu \neq \nu \end{cases}$$

Minkowski-Metrik:

$$(\eta_{\mu\nu}) = (\eta^{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad \eta^{\mu\lambda}\eta_{\lambda\nu} = \delta^{\mu}_{\nu}$$

Lorentz-invariantes Skalarprodukt (mit Summenkonvention):

$$a_\mu b^\mu = \eta_{\mu\nu} a^\nu b^\mu = a_0 b^0 + a_i b^i = a^0 b^0 - a^i b^i$$

Viererortsvektor:

$$(x^\mu) = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} \equiv \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}, \qquad \mu \in \{0, 1, 2, 3\}$$

Vierergradient:

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu}, \quad (\partial_\mu) = \begin{pmatrix} \frac{1}{c}\partial_t \\ \nabla \end{pmatrix}, \quad (\partial^\mu) = (\eta^{\mu\nu}\partial_\nu) = \begin{pmatrix} \frac{1}{c}\partial_t \\ -\nabla \end{pmatrix}$$

Wellen-, d'Alembert- oder Quabla-Operator:

$$\partial_\mu \partial^\mu \equiv \partial^2 \equiv \Box = \frac{1}{c^2}\partial_t^2 - \nabla^2 \equiv \frac{1}{c^2}\partial_t^2 - \Delta$$

Vierdimensionales Levi-Civita-Symbol:

$$\varepsilon^{\mu\nu\sigma\tau} := \begin{cases} +1 & \text{wenn } \mu\nu\sigma\tau \text{ gerade Permutation von 0123} \\ -1 & \text{wenn } \mu\nu\sigma\tau \text{ ungerade Permutation von 0123} \\ 0 & \text{sonst.} \end{cases}$$

$$\varepsilon^{0123} = +1, \quad \varepsilon_{0123} = \eta_{00}\eta_{11}\eta_{22}\eta_{33}\varepsilon^{0123} = -1$$

Integralsätze

Linienintegral:

$$\int_{\boldsymbol{x}_a}^{\boldsymbol{x}_b} \nabla\phi \cdot \mathbf{d}\boldsymbol{r} = \phi(\boldsymbol{x}_b) - \phi(\boldsymbol{x}_a)$$

$$\int_{\boldsymbol{x}_a}^{\boldsymbol{x}_b} \mathrm{d}x_i\, \partial_i\, (\ldots) = (\ldots)\big|_{\boldsymbol{x}_b} - (\ldots)\big|_{\boldsymbol{x}_a}$$

Satz von Stokes:

$$\int_F \mathbf{d}\boldsymbol{f} \cdot \mathbf{rot}\,\boldsymbol{X} = \oint_{\partial F} \mathbf{d}\boldsymbol{r} \cdot \boldsymbol{X}$$

$$\int_F \mathrm{d}f_i\, \varepsilon_{ijk}\, \partial_j\, (\ldots) = \oint_{\partial F} \mathrm{d}x_k\, (\ldots)$$

Satz von Gauß:

$$\int_V \mathrm{d}V \operatorname{div}\boldsymbol{X} = \oint_{\partial V} \mathbf{d}\boldsymbol{f} \cdot \boldsymbol{X}$$

$$\int_V \mathrm{d}V\, \partial_i\, (\ldots) = \oint_{\partial V} \mathrm{d}f_i\, (\ldots)$$

Green'sche Integralsätze:

$$\int_V \mathrm{d}V\ [(\nabla\phi)\cdot(\nabla\chi) + \phi\ \Delta\chi] = \oint_{\partial V} \mathbf{d}\boldsymbol{f}\cdot\phi\ \nabla\chi$$

$$\int_V \mathrm{d}V\ (\phi\ \Delta\chi - \chi\ \Delta\phi) = \oint_{\partial V} \mathbf{d}\boldsymbol{f}\cdot(\phi\ \nabla\chi - \chi\ \nabla\phi)$$

Zylinderkoordinaten

$$x_1 \equiv x = \varrho\cos\varphi \qquad \hat{\boldsymbol{e}}_\varrho = \cos\varphi\,\hat{\boldsymbol{e}}_1 + \sin\varphi\,\hat{\boldsymbol{e}}_2$$

$$x_2 \equiv y = \varrho\sin\varphi \qquad \hat{\boldsymbol{e}}_\varphi = \cos\varphi\,\hat{\boldsymbol{e}}_2 - \sin\varphi\,\hat{\boldsymbol{e}}_1$$

$$x_3 \equiv z \qquad \hat{\boldsymbol{e}}_z = \hat{\boldsymbol{e}}_3$$

$$\mathrm{d}V = \mathrm{d}^3x = \varrho\,\mathrm{d}\varrho\,\mathrm{d}\varphi\,\mathrm{d}z$$

$$\nabla f = \hat{\boldsymbol{e}}_\varrho\frac{\partial f}{\partial\varrho} + \hat{\boldsymbol{e}}_\varphi\frac{1}{\varrho}\frac{\partial f}{\partial\varphi} + \hat{\boldsymbol{e}}_z\frac{\partial f}{\partial z}$$

$$\Delta f = \underbrace{\frac{1}{\varrho}\frac{\partial}{\partial\varrho}\left(\varrho\frac{\partial f}{\partial\varrho}\right)}_{\frac{\partial^2 f}{\partial\varrho^2} + \frac{1}{\varrho}\frac{\partial f}{\partial\varrho}} + \frac{1}{\varrho^2}\frac{\partial^2 f}{\partial\varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

Kugelkoordinaten (sphärische Polarkoordinaten)

$$x_1 \equiv x = r\sin\vartheta\ \cos\varphi$$

$$x_2 \equiv y = r\sin\vartheta\ \sin\varphi$$

$$x_3 \equiv z = r\cos\vartheta$$

$$\mathrm{d}V = \mathrm{d}^3x = r^2\,\mathrm{d}r\,\mathrm{d}\Omega = r^2\,\mathrm{d}r\,\sin\vartheta\,\mathrm{d}\vartheta\,\mathrm{d}\varphi$$

$$\hat{\boldsymbol{e}}_r = \sin\vartheta\ \cos\varphi\,\hat{\boldsymbol{e}}_1 + \sin\vartheta\ \sin\varphi\,\hat{\boldsymbol{e}}_2 + \cos\vartheta\,\hat{\boldsymbol{e}}_3$$

$$\hat{\boldsymbol{e}}_\vartheta = \cos\vartheta\ \cos\varphi\,\hat{\boldsymbol{e}}_1 + \cos\vartheta\ \sin\varphi\,\hat{\boldsymbol{e}}_2 - \sin\vartheta\,\hat{\boldsymbol{e}}_3$$

$$\hat{\boldsymbol{e}}_\varphi = \cos\varphi\,\hat{\boldsymbol{e}}_2 - \sin\varphi\,\hat{\boldsymbol{e}}_1$$

$$\nabla f = \hat{\boldsymbol{e}}_r\frac{\partial f}{\partial r} + \hat{\boldsymbol{e}}_\vartheta\frac{1}{r}\frac{\partial f}{\partial\vartheta} + \hat{\boldsymbol{e}}_\varphi\frac{1}{r\sin\vartheta}\frac{\partial f}{\partial\varphi}$$

$$\Delta f = \left(\Delta_r + \frac{1}{r^2}\Delta_\Omega\right)f$$

$$\Delta_r := \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} \equiv \frac{1}{r}\frac{\partial^2}{\partial r^2}r \equiv \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}$$

$$\Delta_\Omega := \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta} + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}$$

Kugelflächenfunktionen

$$\Delta_\Omega Y_{lm}(\vartheta,\varphi) = -l(l+1)Y_{lm}(\vartheta,\varphi), \quad l\in\mathbb{N}_0, \quad m = -l,\ldots,l$$

$$Y_{lm}(\vartheta,\varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\vartheta)\mathrm{e}^{im\varphi}$$

$$P_l^m(u) = \frac{(-1)^{l+m}}{2^l l!}\left(1-u^2\right)^{m/2}\frac{\mathrm{d}^{l+m}}{\mathrm{d}u^{l+m}}\left(1-u^2\right)^l, \quad P_l \equiv P_l^0$$

$Y_{lm}(\vartheta,\varphi)$	$l=0$	$l=1$	$l=2$
$m=0$	$\sqrt{\frac{1}{4\pi}}$	$\sqrt{\frac{3}{4\pi}}\cos\vartheta$	$\sqrt{\frac{5}{16\pi}}(3\cos^2\vartheta-1)$
$m=1$		$-\sqrt{\frac{3}{8\pi}}\sin\vartheta\ \mathrm{e}^{\mathrm{i}\varphi}$	$-\sqrt{\frac{15}{8\pi}}\sin\vartheta\ \cos\vartheta\ \mathrm{e}^{\mathrm{i}\varphi}$
$m=2$			$\sqrt{\frac{15}{32\pi}}\sin^2\vartheta\ \mathrm{e}^{2\mathrm{i}\varphi}$

$$Y_{l,-m} = (-1)^m Y_{lm}^*, \qquad \int \mathrm{d}\Omega\ Y_{lm}^*(\vartheta,\varphi)Y_{l'm'}(\vartheta,\varphi) = \delta_{ll'}\delta_{mm'}$$

$$\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|} = \sum_{l=0}^\infty\sum_{m=-l}^{l}\frac{4\pi}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{lm}(\vartheta,\varphi)Y_{lm}^*(\vartheta',\varphi')$$

$$= \sum_{l=0}^\infty\frac{r_<^l}{r_>^{l+1}}P_l(\cos\alpha), \quad \cos\alpha \equiv \frac{\boldsymbol{r}\cdot\boldsymbol{r}'}{r\,r'},$$

$$r_< = \min(r,r'), \quad r_> = \max(r,r')$$

Fourier-Transformation

$$f(t,\boldsymbol{x}) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{\sqrt{2\pi}}\int\frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\tilde{f}(\omega,\boldsymbol{k})\,\mathrm{e}^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}$$

$$\tilde{f}(\omega,\boldsymbol{k}) = \int_{-\infty}^{\infty}\frac{\mathrm{d}t}{\sqrt{2\pi}}\int\frac{\mathrm{d}^3x}{(2\pi)^{3/2}}f(t,\boldsymbol{x})\,\mathrm{e}^{-\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}$$

Entwicklung nach einem vollständigen orthonormalen Funktionensystem f_j

$$f(x) = \sum_j a_j f_j(x), \quad a_j = \langle f_j, f\rangle, \quad \langle f,g\rangle = \int_I f^*(x)\,g(x)\,\mathrm{d}x$$

$$\langle f_j, f_k\rangle = \delta_{jk}, \quad \sum_j f_j^*(x')f_j(x) = \delta(x'-x)$$

Funktionentheorie

Cauchy-Riemann'sche Differenzialgleichungen:

$$f(z) = u(x+\mathrm{i}y) + \mathrm{i}v(x+\mathrm{i}y): \quad u_x = v_y, \quad u_y = -v_x$$

Residuensatz:

$$\oint f(z)\,\mathrm{d}z = 2\pi\mathrm{i}\sum_{j=1}^n \mathrm{Res}_{z_j}f(z)$$

Residuum bei einer einfachen Polstelle:

$$\mathrm{Res}_{z_0}f(z) = \lim_{z\to z_0}(z-z_0)f(z)$$

Mechanik

Lagrange-Gleichungen erster Art:

$$m_i \ddot{\boldsymbol{x}}_i = \boldsymbol{F}_i + \sum_{a=1}^{r} \lambda_a \nabla_i f_a, \quad f_a(t, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_N) = 0$$

Lagrange-Gleichungen zweiter Art:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} = Q_j, \quad \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = 0, \quad L = T - V$$

Hamilton'sche kanonische Gleichungen:

$$\frac{\partial H}{\partial p_i} = \dot{q}_i, \quad \frac{\partial H}{\partial q_i} = -\dot{p}_i, \quad H = \sum_i \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L$$

Poisson-Klammern:

$$\{F, G\} = \sum_i \left(\frac{\partial F}{\partial q_i}\frac{\partial G}{\partial p_i} - \frac{\partial F}{\partial p_i}\frac{\partial G}{\partial q_i} \right)$$

Elektrodynamik

Maxwell-Gleichungen (Gauß'sches System):

$$\begin{aligned}
\operatorname{div} \boldsymbol{D} &= 4\pi\rho_f, \qquad \boldsymbol{D} \equiv \boldsymbol{E} + 4\pi\boldsymbol{P} \\
\operatorname{div} \boldsymbol{B} &= 0 \\
\mathbf{rot}\, \boldsymbol{E} &= -\frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{B} \\
\mathbf{rot}\, \boldsymbol{H} &= \frac{4\pi}{c}\boldsymbol{j}_f + \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{D}, \qquad \boldsymbol{H} \equiv \boldsymbol{B} - 4\pi\boldsymbol{M}
\end{aligned}$$

Potenziale:

$$\boldsymbol{E} = -\nabla\phi - \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{A}, \qquad \boldsymbol{B} = \nabla \times \boldsymbol{A}$$

Lineare Medien:

$$\begin{aligned}
\boldsymbol{P} &= \chi_e \boldsymbol{E}, \quad \boldsymbol{D} = (1 + 4\pi\,\chi_e)\boldsymbol{E} \equiv \epsilon\boldsymbol{E}, \\
\boldsymbol{M} &= \chi_m \boldsymbol{H}, \quad \boldsymbol{B} = (1 + 4\pi\chi_m)\boldsymbol{H} \equiv \mu\boldsymbol{H}
\end{aligned}$$

Vakuum: $\boldsymbol{D} \to \boldsymbol{E}, \boldsymbol{H} \to \boldsymbol{B}, \rho_f \to \rho, \boldsymbol{j}_f \to \boldsymbol{j}$

Maxwell-Gleichungen (SI-System):

$$\begin{aligned}
\operatorname{div} \boldsymbol{D}^{[\mathrm{SI}]} &= \rho_f^{[\mathrm{SI}]}, \qquad \boldsymbol{D}^{[\mathrm{SI}]} \equiv \epsilon_0 \boldsymbol{E}^{[\mathrm{SI}]} + \boldsymbol{P}^{[\mathrm{SI}]} \\
\operatorname{div} \boldsymbol{B}^{[\mathrm{SI}]} &= 0 \\
\mathbf{rot}\, \boldsymbol{E}^{[\mathrm{SI}]} &= -\frac{\partial}{\partial t}\boldsymbol{B}^{[\mathrm{SI}]} \\
\mathbf{rot}\, \boldsymbol{H}^{[\mathrm{SI}]} &= \boldsymbol{j}_f^{[\mathrm{SI}]} + \frac{\partial}{\partial t}\boldsymbol{D}^{[\mathrm{SI}]}, \qquad \boldsymbol{H}^{[\mathrm{SI}]} \equiv \frac{1}{\mu_0}\boldsymbol{B}^{[\mathrm{SI}]} - \boldsymbol{M}^{[\mathrm{SI}]}
\end{aligned}$$

$$\mu_0 = 4\pi \cdot 10^{-7}\mathrm{N/A^2}, \quad \epsilon_0 \equiv \frac{1}{c^2\mu_0}$$

Potenziale:

$$\boldsymbol{E}^{[\mathrm{SI}]} = -\nabla\phi^{[\mathrm{SI}]} - \frac{\partial}{\partial t}\boldsymbol{A}^{[\mathrm{SI}]}, \qquad \boldsymbol{B}^{[\mathrm{SI}]} = \nabla \times \boldsymbol{A}^{[\mathrm{SI}]}$$

Lineare Medien:

$$\begin{aligned}
\boldsymbol{P}^{[\mathrm{SI}]} &= \chi_e^{[\mathrm{SI}]}\,\epsilon_0 \boldsymbol{E}^{[\mathrm{SI}]}, \quad \boldsymbol{D}^{[\mathrm{SI}]} = (1 + \chi_e^{[\mathrm{SI}]})\epsilon_0 \boldsymbol{E}^{[\mathrm{SI}]} \equiv \epsilon^{[\mathrm{SI}]}\boldsymbol{E}^{[\mathrm{SI}]}, \\
\boldsymbol{M}^{[\mathrm{SI}]} &= \chi_m^{[\mathrm{SI}]} \boldsymbol{H}^{[\mathrm{SI}]}, \quad \boldsymbol{B}^{[\mathrm{SI}]} = (1 + \chi_m^{[\mathrm{SI}]})\mu_0 \boldsymbol{H}^{[\mathrm{SI}]} \equiv \mu^{[\mathrm{SI}]}\boldsymbol{H}^{[\mathrm{SI}]}
\end{aligned}$$

Umrechnung Gauß'sches und SI-System:

$$\begin{aligned}
\boldsymbol{E} &= \sqrt{4\pi\epsilon_0}\,\boldsymbol{E}^{[\mathrm{SI}]}, \quad \boldsymbol{B} = \sqrt{\frac{4\pi}{\mu_0}}\,\boldsymbol{B}^{[\mathrm{SI}]}, \\
\phi &= \sqrt{4\pi\epsilon_0}\,\phi^{[\mathrm{SI}]}, \quad \boldsymbol{A} = \sqrt{\frac{4\pi}{\mu_0}}\,\boldsymbol{A}^{[\mathrm{SI}]}, \\
\rho &= \frac{1}{\sqrt{4\pi\epsilon_0}}\,\rho^{[\mathrm{SI}]}, \quad \boldsymbol{j} = \frac{1}{\sqrt{4\pi\epsilon_0}}\boldsymbol{j}^{[\mathrm{SI}]}
\end{aligned}$$

Kontinuitätsgleichung:

$$\nabla \cdot \boldsymbol{j} + \frac{\partial}{\partial t}\rho = 0$$

Lorentz-Kraft auf Punktladung $q = q^{[\mathrm{SI}]}/\sqrt{4\pi\epsilon_0}$:

$$\boldsymbol{F} = q\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v} \times \boldsymbol{B}\right) = q^{[\mathrm{SI}]}\left(\boldsymbol{E}^{[\mathrm{SI}]} + \boldsymbol{v} \times \boldsymbol{B}^{[\mathrm{SI}]}\right)$$

Quantenmechanik

Korrespondenzregeln im Ortsraum:

$$\boldsymbol{x} \mapsto \hat{\boldsymbol{x}} = \boldsymbol{x}\cdot, \quad \boldsymbol{p} \mapsto \hat{\boldsymbol{p}} = \frac{\hbar}{\mathrm{i}}\nabla_x$$

Zeitabhängige Schrödinger-Gleichung:

$$\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}\left|\psi(t)\right\rangle = \hat{H}\left|\psi(t)\right\rangle$$

Formale Lösung für konservative Systeme:

$$\left|\psi(t)\right\rangle = \hat{U}(t)\left|\psi(0)\right\rangle, \quad \hat{U}(t) = \mathrm{e}^{-\mathrm{i}t\hat{H}/\hbar}$$

Stationäre Schrödinger-Gleichung für konservative Systeme:

$$\hat{H}\left|\psi\right\rangle = E\left|\psi\right\rangle, \quad \left|\psi(t)\right\rangle = \mathrm{e}^{-\mathrm{i}Et/\hbar}\left|\psi\right\rangle$$

Hamilton-Operator im Ortsraum für Teilchen ohne Spin:

$$\hat{H} = \frac{1}{2m}\left(\hat{\boldsymbol{p}} - \frac{q}{c}\boldsymbol{A}(t, \hat{\boldsymbol{x}})\right)^2 + V(\hat{\boldsymbol{x}})$$

Entwicklung nach Eigenvektoren einer Observablen:

$$\hat{A}\left|a_n\right\rangle = a_n\left|a_n\right\rangle, \quad \left\langle a_m \middle| a_n\right\rangle = \delta_{mn} \Rightarrow \left|\psi\right\rangle = \sum_n \left\langle a_n \middle| \psi\right\rangle \left|a_n\right\rangle$$

Erwartungswert einer Observablen in einem Zustand:

$$\langle \hat{A} \rangle_\psi = \langle \psi | \hat{A} | \psi \rangle, \quad \langle \psi | \psi \rangle = 1$$

Unbestimmtheitsrelation für zwei Observablen:

$$\left\langle (\Delta \hat{A})^2 \right\rangle_\psi \left\langle (\Delta \hat{B})^2 \right\rangle_\psi \geq -\frac{1}{4} \left\langle [\hat{A}, \hat{B}] \right\rangle_\psi^2, \quad \Delta \hat{A} = \hat{A} - \langle \hat{A} \rangle_\psi$$

Übergang vom Schrödinger- zum Heisenberg-Bild:

$$|\psi_\mathrm{H}\rangle = \hat{U}^{-1}(t) |\psi(t)\rangle, \quad \hat{A}_\mathrm{H}(t) = \hat{U}^{-1}(t) \hat{A} \hat{U}(t)$$

Heisenberg-Gleichung für Observablen:

$$\mathrm{i}\hbar \frac{\mathrm{d}\hat{A}_\mathrm{H}}{\mathrm{d}t} = \left[\hat{A}_\mathrm{H}, \hat{H}_\mathrm{H} \right]$$

Harmonischer Oszillator:

$$\hat{H} = \hbar\omega \left(\hat{a}^\dagger \hat{a} + 1/2 \right), \quad \left[\hat{a}, \hat{a}^\dagger \right] = 1, \quad E_n = \hbar\omega(n + 1/2)$$

Hermitesche Drehimpulsoperatoren:

$$\left[\hat{J}_i, \hat{J}_j \right] = \mathrm{i}\hbar \varepsilon_{ijk} \hat{J}_k, \quad \left[\hat{\boldsymbol{J}}^2, \hat{J}_i \right] = 0$$

Eigenzustände des Drehimpulses:

$$\hat{\boldsymbol{J}}^2 |jm\rangle = \hbar^2 j(j+1) |jm\rangle, \quad \hat{J}_3 |jm\rangle = \hbar m |jm\rangle$$

Energien des Wasserstoffatoms:

$$E_n = -\frac{Z^2}{n^2} \mathrm{Ry}, \quad \mathrm{Ry} = \frac{m_\mathrm{e} c^2}{2} \alpha^2 \approx 13{,}606\,\mathrm{eV}$$

Änderung der Energie und des Zustandsvektors in erster Ordnung Störungstheorie:

$$E^{(1)} = \langle \psi_n^{(0)} | \hat{H}^{(1)} | \psi_n^{(0)} \rangle,$$
$$|\psi_n^{(1)}\rangle = \sum_{k \neq n} \frac{\langle \psi_k^{(0)} | \hat{H}^{(1)} | \psi_n^{(0)} \rangle}{E_n^{(0)} - E_k^{(0)}} |\psi_k^{(0)}\rangle$$

Thermodynamik

Einige wichtige thermodynamische Potenziale:

$U(S, V)$	innere Energie
$F(T, V) = U(S, V) - TS$	freie Energie
$H(S, P) = U(S, V) + PV$	Enthalpie
$G(T, P) = F(T, V) + PV$ $= U(S, V) - TS + PV$	freie Enthalpie

Vollständige Differenziale davon:

$$\begin{aligned} \mathrm{d}U(S, V) &= T\,\mathrm{d}S - P\,\mathrm{d}V \\ \mathrm{d}F(T, V) &= -S\,\mathrm{d}T - P\,\mathrm{d}V \\ \mathrm{d}H(S, P) &= T\,\mathrm{d}S + V\,\mathrm{d}P \\ \mathrm{d}G(T, P) &= -S\,\mathrm{d}T + V\,\mathrm{d}P \end{aligned}$$

Daraus abgeleitete Maxwell-Relationen:

$$\begin{aligned} \left(\frac{\partial T}{\partial V} \right)_S &= -\left(\frac{\partial P}{\partial S} \right)_V \\ \left(\frac{\partial S}{\partial V} \right)_T &= \left(\frac{\partial P}{\partial T} \right)_V \\ \left(\frac{\partial T}{\partial P} \right)_S &= \left(\frac{\partial V}{\partial S} \right)_P \\ \left(\frac{\partial S}{\partial P} \right)_T &= -\left(\frac{\partial V}{\partial T} \right)_P \end{aligned}$$

Großkanonische Zustandssummen für Maxwell-Boltzmann-, Fermi-Dirac- und Bose-Einstein-Systeme:

$$\begin{aligned} Z_\mathrm{gc}^\mathrm{MB} &= \sum_{N=0}^\infty \int \mathrm{d}\Gamma(x)\, \mathrm{e}^{-\beta(H(x) - \mu N)} \\ Z_\mathrm{gc}^\mathrm{FD} &= \prod_k \left(1 + \mathrm{e}^{-\beta(\epsilon_k - \mu)} \right) \\ Z_\mathrm{gc}^\mathrm{BE} &= \prod_k \left(1 - \mathrm{e}^{-\beta(\epsilon_k - \mu)} \right)^{-1} \end{aligned}$$

Großkanonische Verteilungsfunktionen dieser Systeme:

$$\begin{aligned} \rho_\mathrm{gc}^\mathrm{MB} &= \left(Z_\mathrm{gc}^\mathrm{MB} \right)^{-1} \sum_{N=0}^\infty \mathrm{e}^{-\beta(H(x) - \mu N)} \\ \left\langle n_k^\mathrm{FD} \right\rangle &= \frac{1}{\mathrm{e}^{\beta(\epsilon_k - \mu)} + 1} \\ \left\langle n_k^\mathrm{BE} \right\rangle &= \frac{1}{\mathrm{e}^{\beta(\epsilon_k - \mu)} - 1} \end{aligned}$$

Beziehungen zwischen Zustandssummen und thermodynamischen Potenzialen:

$$\begin{aligned} S &= k_\mathrm{B} \ln \Omega \\ F &= -k_\mathrm{B} T \ln Z_\mathrm{c} \\ J &= -PV = -k_\mathrm{B} T \ln Z_\mathrm{gc} \end{aligned}$$

Gibbs-Duhem-Beziehung:

$$U - TS + PV = G = \sum_i \mu^{(i)} N^{(i)}$$