
2.1 Introduction

A quant revolution started on Wall Street in 1952, when Harry M. Markowitz established the *modern portfolio theory* (MPT) which applies mathematical concepts to finance. Based on his work, the *capital asset pricing model* (CAPM) was developed one decade later. Today, the results of the CAPM (and its extended versions) are widely used for describing the risks and returns of portfolios and for performance measurement. This chapter is devoted to the theoretical part of asset management and shows the key tests that compare this theory to practice.

Using the mathematical prerequisites on risk and return measurement from the first chapter, we will look at modern portfolio theory in detail. Before we present MPT, Sect. 2.2 provides a review of regression analysis as needed for this chapter. The concept of regression will be shown by extending our business case from the first chapter. The capital asset pricing model is thereafter introduced in Sect. 2.3, a model which allows us to estimate the required return of a risky asset (or a portfolio) based on its β , i.e., its sensitivity to market movements. This model will be presented together with its assumptions and empirical tests. This section will discuss the validity of the CAPM based on the empirical tests and also present some critical views on the simplistic assumptions of the CAPM.

Although the CAPM has found useful applications, for example, in performance measurement, in explaining the benefits of diversification (by introducing the notion of systematic and unsystematic risk), and as a tool for finding undervalued/overvalued securities, empirical evidence suggests that the model describes capital markets returns only incompletely. Some returns can be influenced by other risk factors than the exposure to the market, some by stock market anomalies. Stock market anomalies will be described in detail in Chap. 3, and historical stock market crashes will be summarized in Chap. 4. Anomalies and crashes are not captured by the CAPM but rather by taking psychological factors into account. The irrational

behavior of market participants heavily influences market prices and the returns of assets. This is researched by *behavioral finance*, which will be covered in Chap. 5.

Empirical evidence suggests that the returns of a risky asset/portfolio are not only driven by market movements, but also by other risk factors which are not included in the CAPM. In 1992, Eugene Fama¹ and Kenneth French² developed an extension of the CAPM, the Fama–French three-factor model (FF3M), introduced in Sect. 2.3.6, which incorporates two of these factors: the size of companies and the book-to-market ratio. Like in the discussion about the CAPM, empirical tests of the FF3M will be presented, together with some critical views of this model.

As said before, this chapter looks at the theory while Chaps. 3 and 4 look at the reality of financial markets. Chapter 3 discusses stock market anomalies, i.e., market irregularities which distort the price-return relationship of assets and contradict traditional finance theory. For example, calendar effects have an impact on asset returns while standard finance theory does not distinguish between the holiday season and spring. That is, according to standard finance theory, a calendar effect should not exist. Chapter 4 then looks at behavioral finance, the psychology of investing. Here we will see how psychology plays an important role in finance which again, according to standard finance theory, should not be the case. Yet, behavioral factors were of critical importance for what happened in the worldwide stock markets in 2000–2003, 2008–2009 or 2011.

2.2 A Quick Review of Regression Analysis

Regression analysis is a widely used technique in finance, especially in applications of the CAPM and of the Fama–French three-factor model.³ The goal of regression analysis is to model a variable Y as a function of different input parameters X_1, \dots, X_K . For our purposes, the asset return is the variable Y , and the different factors which contribute to the asset returns serve as the X -parameters, for example,

¹Eugene F. Fama, born in 1939, is an American economist. He is known for his work on portfolio theory and asset pricing, both theoretical and empirical. He won the Nobel Prize in Economics in 2013 together with Robert J. Shiller and Lars Peter Hansen, see Reinganum (2013). His Ph.D. thesis (one of his supervisors was Nobel Prize winner Merton Miller) concluded that stock price movements are unpredictable and follow a random walk. Fama (1970) proposes the groundbreaking concept of *efficient-markets* (see Sects. 3.1 and 5.1) such that Fama is most often thought of as the father of the efficient-market hypothesis. Currently, he is a professor of finance at the University of Chicago, Booth School of Business.

²Kenneth R. French, born in 1954, is an American economist and professor of finance at Dartmouth College, Tuck School of Business. He has previously been a faculty member at MIT, the Yale School of Management, and the University of Chicago Booth School of Business. He obtained his Ph.D. in Finance in 1983 from the University of Rochester. French is an expert on the behavior of security prices and investment strategies and is most famous for his work on asset pricing with Eugene Fama and the Fama–French three-factor model (1992) as an extension of the CAPM (see Sect. 2.4).

³This paragraph is based on Lhabitant (2004, pp. 147–175).

return on the market, return on oil, GDP growth, etc. While the correlation quantifies how consistently two variables vary together, regression analysis describes the specific relationship between two (or more) variables. Therefore, regression analysis can be seen as an extension of the correlation/covariance concept.

2.2.1 Simple Linear Regression

The most basic type of regression is the simple linear regression. In order to understand the concept, let us go back to our business case from Chap. 1 and continue to analyze the stock returns of US Airways (LCC), Delta Airlines (DAL) and the oil price. On page 52 we found that the stock return of US Airways is positively related to the stock return of Delta Airlines, but negatively correlated to oil. The goal of simple linear regression is to find a linear relationship. How much has the price of US Airways changed when oil went up 1 % or when Delta Airlines went up 1 %? And how accurate is this linear relationship?

Let us analyze the period January 2008–June 2010, divided into 30 monthly subperiods. For example, we describe the dependency of US Airways' monthly returns r_{LCC}^k (in the k th month) to oil monthly returns r_{Oil}^k in the form

$$r_{LCC}^k = \underbrace{a + b \cdot r_{Oil}^k}_{\text{linear model}} + \underbrace{\varepsilon_k}_{\text{error term}}. \quad (2.1)$$

The linear relationship, described by a *regression line* of the form

$$r_{LCC}^{\text{monthly}} = a + b \cdot r_{Oil}^{\text{monthly}} \quad (2.2)$$

is what we are looking for, and the numbers a and b have to be calculated. The monthly return of oil is used to *explain* the monthly returns of Delta, this is why we call r_{Oil}^{monthly} an explanatory variable. The error term

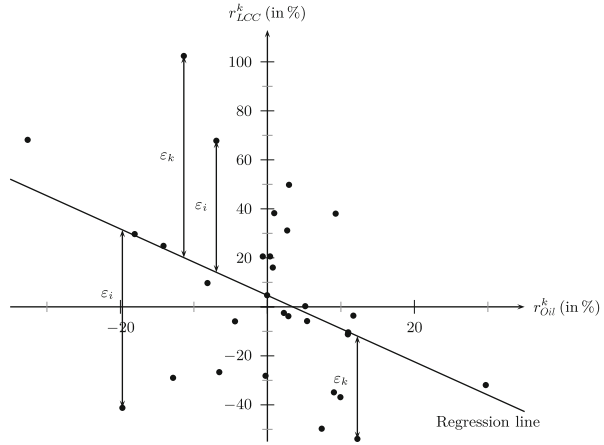
$$\varepsilon_k = r_{LCC}^k - (a + b \cdot r_{Oil}^k) \quad (2.3)$$

accounts for the deviation from our model due to other factors which explain the returns on US Airways, but are not related to oil. Take a look at Fig. 2.1 which is the same as Fig. 1.19 on page 48, but with the regression line (2.2) added (which still has to be calculated). Given the monthly returns $r_{LCC}^1, r_{LCC}^2, \dots, r_{LCC}^{30}$ of US Airways and $r_{Oil}^1, r_{Oil}^2, \dots, r_{Oil}^{30}$ of oil, the graph plots the points

$$(r_{Oil}^1, r_{LCC}^1), (r_{Oil}^2, r_{LCC}^2), \dots, (r_{Oil}^{30}, r_{LCC}^{30}).$$

The line (2.2) is drawn such that the error terms ε_k , i.e., the distances of the points (r_{Oil}^k, r_{LCC}^k) from the line, are kept small on average. We could search for the line which minimizes the average distance from the line, but in practice, the squared sum

Fig. 2.1 Scatter plot of monthly returns of US Airways (LCC) against oil (Crushing, OK Crude Oil Futures Contract) for the period January 2008–June 2010, together with regression line. *Sources:* Yahoo! Finance and U.S. Energy Information Administration



of the error terms is used as the term to be minimized which puts greater weight to big outliers.

We get the numbers a and b for the regression line (2.2)

$$r_{LCC}^{monthly} = a + b \cdot r_{Oil}^{monthly}$$

by solving the following problem: Find the numbers a and b such that the sum of squared errors

$$\sum_{i=k}^{30} \varepsilon_k^2 = \sum_{k=1}^{30} (r_{LCC}^k - (a + b \cdot r_{Oil}^k))^2 \quad (2.4)$$

is minimal. We will present the solution to this problem after we have introduced the general problem and the general solution.

Given data points $(X_1, Y_1), (X_2, Y_2), \dots, (X_N, Y_N)$, in the general problem we want to find the *regression line* (or *best-fit line*)

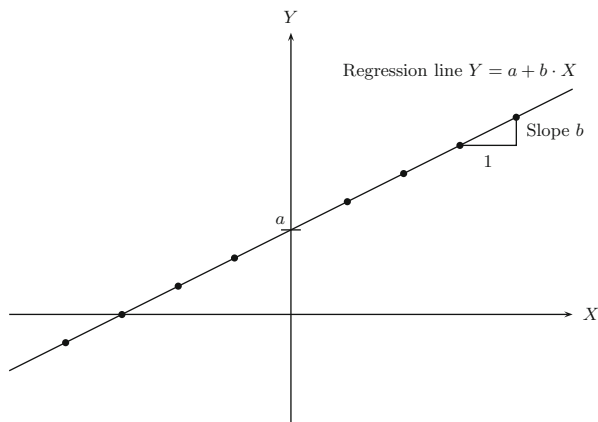
$$Y = a + b \cdot X \quad (2.5)$$

which describes best the relationship between X_k and Y_k .⁴ In this regression equation, we call X the *independent variable*, Y the *dependent variable*, a the *intercept* (where the line crosses the Y -axis) and b the *slope coefficient*.⁵ We refer to a and b as the *regression coefficients*.

⁴For an introduction to linear regression, see also DeFusco, McLeavey, Pinto, and Runkle (2004, pp. 395–420).

⁵The terms are from DeFusco et al. (2004, p. 395).

Fig. 2.2 Regression line for perfect linear relationship between Y and X . *Source:* Own, for illustrative purposes only



X is also called *explanatory variable*⁶ because X is used to explain the variable Y . The larger the absolute value of b , the greater the *explanatory power* of X . If b is close to zero, then changes in X cannot explain changes in Y , and X has no explanatory power.

The graph in Fig. 2.2 shows a scatter plot where all data points (X, Y) lie on the line $Y = a + b \cdot X$. In practice, you usually have a scatter plot like in Figs. 2.3 or 2.4 where the data points are scattered around a line and where you can still see a linear tendency, but in the latter plot, the linear relationship is weak.

For any data point (X_k, Y_k) , the error term ε_k is the difference between the actual Y_k value and the Y value predicted by the line (2.5), hence

$$\varepsilon_k = Y_k - (a + b \cdot X_k). \quad (2.6)$$

The numbers a and b for the regression line $Y = a + b \cdot X$ are chosen such that the sum of the squared error terms

$$\sum_{k=1}^N \varepsilon_k^2 = \sum_{k=1}^N (Y_k - (a + b \cdot X_k))^2 \quad (2.7)$$

is minimized. For that reason, this is also called the ordinary least squares (OLS) method.

⁶For example, the term *explanatory variable* is used in Fama and MacBeth (1973, p. 618).

Fig. 2.3 Regression line for an approximate linear relationship between Y and X . Example for a high coefficient of determination R^2 . *Source:* Own, for illustrative purposes only

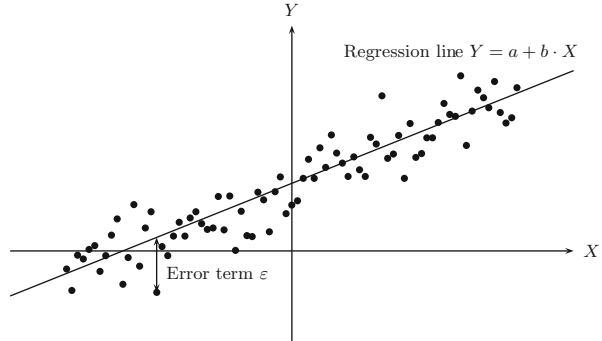
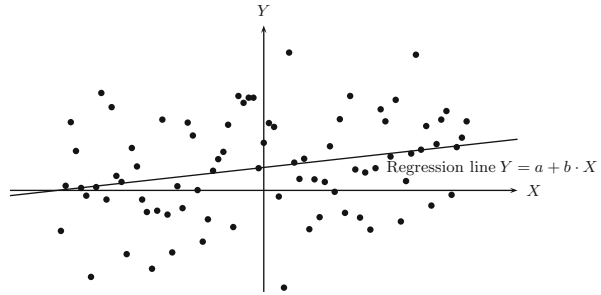


Fig. 2.4 The regression line provides only a weak description of the relationship between Y and X . Example for a low coefficient of determination R^2 . *Source:* Own, for illustrative purposes only



Let

$$\bar{X} = \frac{1}{N} \sum_{k=1}^N X_k \quad \text{and} \quad \bar{Y} = \frac{1}{N} \sum_{k=1}^N Y_k.$$

Then, the solutions for the parameters a and b are

$$b = \frac{\sum_{k=1}^N (X_k - \bar{X}) \cdot (Y_k - \bar{Y})}{\sum_{k=1}^N (X_k - \bar{X})^2} \quad (2.8)$$

and

$$a = \bar{Y} - b \cdot \bar{X}. \quad (2.9)$$

For any plot one can find a regression line, but it does not always describe the relationship between Y and X well. Figure 2.3 shows one example where the data points are close to the regression line, and Fig. 2.4 shows one example where they are far off the line.

In order to describe the quality of the regression $Y = a + b \cdot X$, we use the R^2 measure, also known as the *coefficient of determination* R^2 . Let \hat{Y}_k be the predicted \hat{Y} value based on the X_k value and the regression equation. In the case of the simple linear regression,

$$\hat{Y}_k = a + b \cdot X_k. \quad (2.10)$$

Then R^2 is the ratio between the variation explained by our regression model

$$\sum_{k=1}^N (\hat{Y}_k - \bar{Y})^2$$

and the total variation

$$\sum_{k=1}^N (Y_k - \bar{Y})^2.$$

In other words:

$$R^2 = \frac{\sum_{k=1}^N (\hat{Y}_k - \bar{Y})^2}{\sum_{k=1}^N (Y_k - \bar{Y})^2}. \quad (2.11)$$

In our case of simple linear regression where $Y = a + b \cdot X$, R^2 can also be calculated as⁷

$$R^2 = \frac{\left(\sum_{k=1}^N (X_k - \bar{X}) \cdot (Y_k - \bar{Y}) \right)^2}{\sum_{k=1}^N (X_k - \bar{X})^2 \sum_{k=1}^N (Y_k - \bar{Y})^2}. \quad (2.12)$$

Please note:

- R^2 lies between zero and one. The larger the value of R^2 , the more accurate the regression. R^2 is 1 if and only if all data points lie on the regression line $Y = a + b \cdot X$.
- The value of R^2 provides the percentage of the variation of the variable Y which is explained by the variation of the variable X . For example, $R^2 = 0.75$ means that the variable X explains 75 % of the variation of the variable Y .

⁷DeFusco et al. (2004, p. 403): For simple linear regression, R^2 is the square of the correlation between X and Y .

- In Fig. 2.3, R^2 is large (but not 1) because the data points lie close to the regression line, which describes the relation between Y and X well.
- In Fig. 2.4, R^2 is small because the regression line only vaguely represents the relationship between Y and X .

Business case (cont.)

Let us get back to our business case from Chap. 1 (an Excel® file with the calculations can be downloaded at <http://www.pecundus.com/publications/springer-solutions> [username: *solutions*; password: *springer-book-sle*]). We want

$$r_{LCC}^{monthly} = a + b \cdot r_{Oil}^{monthly}.$$

The equation for the slope coefficient b from (2.8) can be rewritten as

$$b = \frac{\sigma_{LCC, Oil}}{\sigma_{Oil}^2}, \quad (2.13)$$

using formulas for monthly volatility (1.22) and monthly covariance (1.45). We use the known results (1.56) and (1.62) for calculation.

$$b = \frac{\sigma_{LCC, Oil}^{monthly}}{(\sigma_{Oil}^{monthly})^2} = \frac{-0.019952}{0.12133^2} = -1.355. \quad (2.14)$$

The intercept a is calculated using Eq. (2.9), together with the results from Eqs. (1.53), (1.52) and (2.14):

$$\begin{aligned} a &= \bar{r}_{LCC} - b \cdot \bar{r}_{Oil} \\ &= 4.73 \% - (-1.356 \cdot (-0.04 \%)) = 4.68 \%. \end{aligned} \quad (2.15)$$

The resulting regression line is

$$r_{LCC}^{monthly} = 4.68 \% - 1.356 \cdot r_{Oil}^{monthly}. \quad (2.16)$$

The R^2 formula (2.12) can be rewritten as a square of the correlation, using formulas for volatility (1.20) and monthly covariance (1.45) and correlation (1.43):

$$R^2 = \frac{\sigma_{Oil, LCC}^2}{\sigma_{Oil}^2 \cdot \sigma_{LCC}^2} = \rho_{Oil, LCC}^2. \quad (2.17)$$

(continued)

Use the result (1.68) to calculate

$$R^2 = \rho_{LCC,oil}^2 = (-0.435)^2 = 0.189. \quad (2.18)$$

Our interpretation of the result is: For every 1 % increase (decrease) of the oil price in a given month, the US Airways stocks tend to decrease (increase) by 1.356 %. 18.9 % of the variation of US Airways can be explained by the movements of oil prices.

Let us calculate the regression line for the monthly returns $r_{LCC}^{monthly}$ of US Airways (LCC) against the monthly returns $r_{DAL}^{monthly}$ of Delta Airlines (DAL), based on the period January 2008–June 2010 (see Table 1.11). The regression equation is

$$r_{LCC}^{monthly} = a + b \cdot r_{DAL}^{monthly}. \quad (2.19)$$

The slope b is calculated using Eq. (2.8), together with the results from Eqs. (1.54) and (1.60):

$$b = \frac{\sigma_{DAL,LCC}^{monthly}}{(\sigma_{DAL}^{monthly})^2} = \frac{0.065368}{0.21914^2} = 1.361. \quad (2.20)$$

The intercept a is calculated using Eq. (2.9), together with the results from Eqs. (1.52), (1.51) and (2.20):

$$\begin{aligned} a &= \bar{r}_{LCC} - b \cdot \bar{r}_{DAL} \\ &= 4.73 \% - 1.361 \cdot 1.59 \% = 2.57 \%. \end{aligned} \quad (2.21)$$

The resulting regression line is

$$r_{LCC}^{monthly} = 2.57 \% + 1.361 \cdot r_{DAL}^{monthly}. \quad (2.22)$$

R^2 is calculated from Eq. (2.12) using (1.66):

$$R^2 = \rho_{DAL,LCC}^2 = 0.789^2 = 0.623. \quad (2.23)$$

Our interpretation of the result is: For every 1 % increase (decrease) of Delta Airlines stocks in a given month, the US Airways stocks tend to increase (decrease) by 1.361 %. 62.3 % of the variation of the US Airways stocks can be explained by movements of Delta Airlines stocks.

(continued)

Fig. 2.5 Scatter plot of monthly returns of US Airways (LCC) against Delta (DAL) for the period January 2008–June 2010, together with regression line. *Source:* Yahoo! Finance

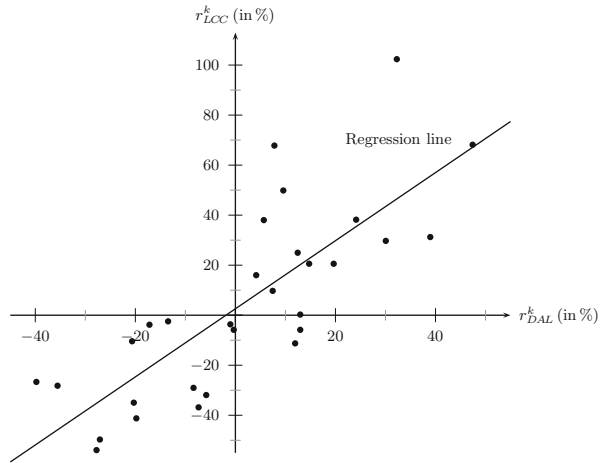


Figure 2.5 on the following page shows the scatter plot of monthly returns of US Airways against Delta Airlines for the period January 2008–June 2010, together with the regression line from Eq. (2.22).

When we compare the scatter plots in Figs. 2.5 and 2.1, we can see that the data points of the former are much closer to the regression line than those of the latter. This can be explained with R^2 being much greater in the former case [0.623, see Eq. (2.23)] than in the latter case [0.189, see Eq. (2.18)].

2.2.2 Multi-Linear and Non-Linear Regression

We can extend the concept of simple linear regression to multi-linear (or non-linear) regression to study the dependency of a variable on multiple variables (or to study non-linear dependencies).⁸

An example for multi-linear regression is

$$Y = a + b \cdot X + c \cdot W + d \cdot Z. \quad (2.24)$$

An example for non-linear regression is

$$Y = a + b \cdot X + c \cdot X^2 + d \cdot Z. \quad (2.25)$$

⁸For an introduction to multi-linear regression, see also DeFusco et al. (2004, pp. 441–494).

For any observed data point, the *error term* ε_k is the difference between the actual observed Y_k value and the value \hat{Y}_k predicted by the regression equation, i.e.,

$$\varepsilon_k = Y_k - (a + b \cdot X + c \cdot W + d \cdot Z) \quad (2.26)$$

in our multi-linear example and

$$\varepsilon_k = Y_k - (a + b \cdot X + c \cdot X^2 + d \cdot Z) \quad (2.27)$$

in our non-linear example. We call X , W and Z the *independent variables*, Y the *dependent variable*, a the *intercept* (where the line crosses the Y -axis) and b the *slope coefficient*.⁹ We refer to a and b as the *regression coefficients*.

In analogy to the simple linear regression, we get the regression equation by finding the *regression coefficients* a , b , c , d (and we may have more parameters for other examples) such that the sum of squares $\sum_{k=1}^N \varepsilon_k^2$ is minimal.

The variables X , W and Z are also called *explanatory variables* because they serve as variables to explain changes in Y . The larger the slope coefficients b , c and d , the stronger the *explanatory power* of the corresponding explanatory variables X , W and Z in describing changes in Y . If a slope coefficient (for example, d) turns out to be close to zero, then the corresponding explanatory variable (Z) is rather useless for describing the variable Y .

The R^2 measure from the simple linear regression [see Eq. (2.11)] is also used here to measure the quality of the regression. Multi-linear regressions also use R_{adj}^2 , the *adjusted R^2* ,¹⁰ to describe the quality of the regression:

$$R_{adj}^2 = 1 - \frac{N-1}{N-I-1}(1-R^2) \quad (2.28)$$

where N is the number of observations and I is the number of independent variables.

We now have all prerequisites in order to understand the basic concepts of MPT as presented in the following section. We will start with the key idea of diversification and then continue and introduce the mean-variance efficient portfolio which will lead us to the famous capital asset pricing model (CAPM). When looking at empirical tests of the CAPM we will see that the CAPM often does not hold in reality. Therefore, we will present an extension of the CAPM: the Fama–French three-factor model.

⁹The terms are from DeFusco et al. (2004, p. 443).

¹⁰DeFusco et al. (2004, p. 457).

2.3 The Capital Asset Pricing Model (CAPM)

2.3.1 Introduction

Harry Markowitz¹¹ laid down the foundation of *modern portfolio theory* (MPT)¹² in his article Markowitz (1952) and his book *Portfolio Selection: Efficient Diversification of Investments*. A decade later, Treynor in 1961 (see French 2002), Sharpe (Sharpe 1964), Lintner (Lintner 1965a) and Mossin (Mossin 1966) built on his work to develop the capital asset pricing model (CAPM).

In Sect. 2.3 of this book, Sect. 2.3.2 presents the assumptions which are necessary for the CAPM to hold. Although these assumptions will turn out to be too idealized and unrealistic which is why the CAPM is not exactly true, the main implications and theories are still valid. Section 2.3.3 introduces the capital asset pricing model. Using a hypothetical *market portfolio*, i.e., a portfolio containing all risky assets, the set of optimal portfolios turns out to be a combination of the risk-free asset and the market portfolio. As a result, the expected return of a portfolio can be calculated by the CAPM equation as the sum of the risk-free rate plus the excess return (market return minus risk-free rate) multiplied by β , the sensitivity of the portfolio to market movements. This equation implies that the market rewards investors who take risk in terms of β . A brief summary of important aspects in Sects. 2.3.2 and 2.3.3 can be found in Schulmerich (2012a) or Schulmerich (2013b), *The Efficient Frontier in Modern Portfolio Theory: Weaknesses and How to Overcome Them (white paper)*.¹³

Various empirical tests of the CAPM have been conducted with mixed results. The tests are presented in Sect. 2.3.4, followed by a discussion about the empirical validity of the CAPM in Sect. 2.3.5. The CAPM is often criticized for its unrealistic assumptions which may lead to questionable results. Section 2.3.6 will discuss the assumptions and show that by relaxing them the results will be slightly changed without changing the main implications.

¹¹Harry M. Markowitz, born August 24, 1927, in Chicago, Illinois, is an American economist who became famous for his pioneering work in modern portfolio theory. He earned his Bachelor of Philosophy from the University of Chicago in 1947 and received his Master and Doctor of Economics at the same university in 1950 and 1954, respectively. He held various positions with RAND corporation (1952–1963), Consolidated Analysis Centers, Inc. (1963–1968), the University of California, Los Angeles (1968–1969), Arbitrage Management Company, (1969–1972), and IBM's T.J. Watson Research Center (1974–1983) before becoming a professor of finance at Baruch College of the City University of New York. He joined the University of California, San Diego in 1994 as a research professor of economics where he became a finance professor at the Rady School of Management in 2006. He received the John von Neumann Theory Prize in 1989 and the Nobel Memorial Prize in Economic Sciences in 1990.

¹²Although he is known as the father of MPT, Markowitz credits Andrew D. Roy with half of the honor in Markowitz (1999).

¹³An extension of these articles on MPT including different risk measures can be found in Schulmerich (2012b), *Extending Modern Portfolio Theory: Efficient Frontiers for Different Risk Measures (white paper)*. A third white paper in this MPT trilogy is *Can the Black–Litterman Framework Improve Asset Management Outcomes?*, see Schulmerich (2013a).

2.3.2 Assumptions

Before we introduce the CAPM, we need to talk about its assumptions first. These are¹⁴:

- **Assumption 1 (A1):**
All investors have homogeneous expectations, i.e., they expect the same probability distribution of returns.
- **Assumption 2 (A2):**
All investors want to invest in an optimal portfolio based on Markowitz's mean-variance framework, i.e., for a given expected return, they target the portfolio with the lowest volatility.
- **Assumption 3 (A3):**
All investors can lend and borrow any amount of money at the risk-free rate.
- **Assumption 4 (A4):**
All investors have the same one-period horizon.
- **Assumption 5 (A5):**
All assets are infinitely divisible.
- **Assumption 6 (A6):**
There are no taxes and transaction costs.
- **Assumption 7 (A7):**
There is no inflation or any change in interest rates, or inflation is fully anticipated.
- **Assumption 8 (A8):**
Capital markets are *efficient*, i.e., they are in equilibrium.

These assumptions are idealized and unrealistic. Does this mean that the CAPM is useless because it is based on “wrong” assumptions? There are two important points to note¹⁵: Many of these assumptions can be relaxed to get closer to the real world, which is discussed in Sect. 2.3.5. This will lead to slight modifications of the CAPM without changing the main conclusions. The other point is that “*a theory should never be judged on the basis of its assumptions, but rather on how well it explains and helps us predict behavior in the real world*”.¹⁶

¹⁴Reilly and Brown (1997, p. 279). This work offers a good introduction and is used as a standard reference work in the CFA curriculum. The sections in this book about the CAPM and its assumptions are based on this reference.

¹⁵Reilly and Brown (1997, p. 279).

¹⁶Reilly and Brown (1997, p. 279).

2.3.3 The Model

After we have defined the set of assumptions of the capital asset pricing model, we now turn to the model itself. Of course, the simplistic assumptions in Sect. 2.3.2 will lead to a rather simplistic model. This model should be thought of as a fundament for financial analysis which helps us to understand the relation between risk and return. It helps us to understand the real world, but does not represent the real world.

You may ask why you should care about an oversimplistic model. Maybe you remember the Bohr¹⁷ atom model from high school physics. In this model, the atom consists of a heavy positively charged nucleus surrounded by electrons which travel around the nucleus on circular orbits. You can compare this model to our solar system, where the planets orbit around the sun. Although this model was a breakthrough to understand atomic physics, it is very simplistic and was later substituted by the theory of quantum mechanics. The point is, that like the Bohr model, the CAPM should just be treated as a starting point to understand mathematical finance. You have to understand the basic model before you can deal with more complex questions.

Like models in physics, models in finance have to be positively tested until they are accepted. The most important tests of the CAPM will be presented in Sect. 2.3.3, followed by a discussion about the empirical validity of the CAPM in Sect. 2.3.4. Section 2.3.5 will then deal with relaxed assumptions for the CAPM which lead to a slightly modified model.

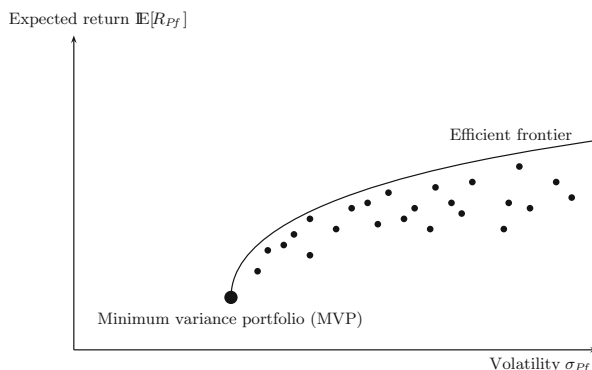
Let us assume a future 1-year horizon $[0, 1]$, for which we have available data about the expected (annual) returns of individual securities, their risks in terms of (annual) volatility, and the correlations between the securities. For every single portfolio, we can calculate the expected annual return $\mathbb{E}[R_{Pf}]$ [see Eq. (1.82)] and its annual volatility σ_{Pf} [see Eq. (1.83)], as outlined in Sect. 1.5.

Please note:

- The future horizon for the CAPM does not have to be 1 year, but we assume this for simplicity. You could also have one quarter year, one month, one week, etc. as a future horizon, and if not dealing with annual returns, the CAPM makes the same statements about quarterly, monthly and weekly returns.
- We use capital letters for random variables, for example the future annual return of a portfolio R_{Pf} which is uncertain, and lower letters are used for numbers like historical returns which we know with certainty.
- For annual returns on the time horizon $[0, 1]$ we use the same notations as for annualized returns because they are the same.

¹⁷Niels H.D. Bohr (October 7, 1885–November 18, 1962) was a Danish physicist who made foundational contributions to understanding atomic structure and quantum theory, for which he received the Nobel Prize in Physics in 1922.

Fig. 2.6 Mean-variance diagram with efficient frontier. *Source:* Own, for illustrative purposes only



The rational investor who is the presumed CAPM protagonist, wants to earn a certain return and tries to identify a portfolio of minimal risk which satisfies this goal. Following this purpose, we plot all possible portfolios of risky assets in a mean-variance diagram,¹⁸ as shown in Fig. 2.6 where the points represent the expected returns $\mathbb{E}[R_{Pf}]$ (vertical axis) and the volatilities σ_{Pf} (horizontal axis) of the portfolios.

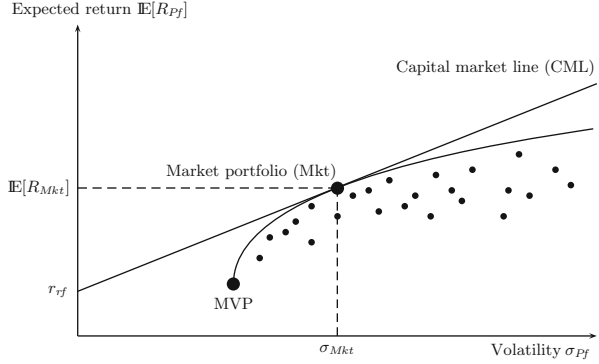
A portfolio is called *mean-variance efficient* (or just *efficient*), if for a given volatility there is no portfolio with a higher return. As Merton¹⁹ notes, the set of efficient portfolios in the mean-variance diagram is called the *efficient frontier* and has the shape of a hyperbola.²⁰ It is the upper boundary of all portfolios in the mean-variance diagram from Fig. 2.6. This is exactly the set of portfolios, that the rational investor in our framework is looking for: They maximize the expected return for a given risk, and they minimize the risk for a given return.

¹⁸Technically a *volatility-mean* diagram, it is called *mean-variance* diagram for historical reasons because when Harry Markowitz introduced MPT in Markowitz (1952, 1999), he plotted the variance against the mean return, although today it is more common to plot mean return against volatility.

¹⁹Robert C. Merton (born July 31, 1944), an American economist and professor at the MIT Sloan School of Management, is known for his significant contributions to continuous-time finance, especially the first continuous-time option pricing model, the Black–Scholes–Merton formula. Together with Myron Scholes, Merton received the Alfred Nobel Memorial Prize in Economic Sciences in 1997 for expanding the Black–Scholes–Merton formula. He earned his Doctor of Economics from the MIT in 1970 under the guidance of Paul Samuelson. He then joined the faculty of the MIT Sloan School of Management where he taught until 1988 before moving to Harvard Business School, where he stayed until 2010. In 2010 he rejoined the MIT Sloan School of Management as the School of Management Distinguished Professor of Finance. He is the past President of the American Finance Association.

²⁰Merton (1972, p. 1856).

Fig. 2.7 The mean-variance diagram shows the capital market line (CML) which connects the risk-free asset with the tangency point on the efficient frontier, the market portfolio. The efficient frontier is based on portfolios without the risk-free asset. *Source:* Own, for illustrative purposes only



The portfolio on the efficient frontier with the lowest volatility is called *minimum-variance portfolio (MVP)*.²¹ If a *risk-free asset*²² exists, that is, an asset with *zero volatility*, then the set of mean-variance efficient portfolios, formed with risk-free and risky assets, is a line from the risk-free asset to the tangency point on the efficient frontier, as shown in Fig. 2.7. We call the tangency point and the corresponding portfolio “Mkt”, and the risk-free asset is labeled “rf”. The reason why we get a line is that the volatility σ_{rf} and also the covariance $\sigma_{rf,Mkt}$ are zero (see definition of covariance in Eq. (1.42)). We use $r_{rf} = \mathbb{E}[R_{rf}]$, so the volatility of the portfolio, with weight w_{Mkt} invested in “Mkt” and $1 - w_{Mkt}$ invested in the risk-free asset, is [see Eq. (1.83)]

$$\begin{aligned} \sigma_{Pf} &= \sqrt{w_{Mkt}^2 \sigma_{Mkt}^2 + (1 - w_{Mkt}^2) \underbrace{\sigma_{rf}^2}_{=0} + 2w_{Mkt}(1 - w_{Mkt}) \underbrace{\sigma_{rf,Mkt}}_{=0}} \\ &= w_{Mkt} \cdot \sigma_{Mkt}. \end{aligned} \quad (2.29)$$

Then

$$\begin{aligned} \mathbb{E}[R_{Pf}] &= w_{Mkt} \mathbb{E}[R_{Mkt}] + (1 - w_{Mkt}) r_{rf} \\ &= r_{rf} + w_{Mkt} (\mathbb{E}[R_{Mkt}] - r_{rf}) \\ &= r_{rf} + \sigma_{Pf} \cdot \frac{\mathbb{E}[R_{Mkt}] - r_{rf}}{\sigma_{Mkt}}. \end{aligned} \quad (2.30)$$

²¹Bodie, Kane, and Marcus (2009, p. 211).

²²Risk-free assets are discussed below on page 118.

This gives us the equation for our line which is called the *capital market line* (CML). All portfolios on the CML are a combination of the tangency portfolio and the risk-free asset. Since it is assumed that capital markets are in equilibrium, every investor holds a portfolio on the CML and therefore a part of the same tangency portfolio, the latter has to be the *market portfolio*.

Black,²³ Jensen²⁴ and Scholes²⁵ were among the first to use the term *market portfolio*.²⁶ The market portfolio is a theoretical portfolio which is central to the CAPM. It contains every risky asset in the market,²⁷ including stocks, bonds, options, real estate, coins, stamps, art, antiques²⁸ and also human capital.²⁹ The assets are weighted according to their market value.

The reason for this is simple³⁰: In our framework, every investor invests in the same risk-free asset and the same risky portfolio. For example, if all investors have 1 % of their risky portfolio invested in Apple Inc., then Apple comprises 1 % of the

²³Fischer S. Black (January 11, 1938–August 30, 1995), an American economist, is one of the authors of the famous Black–Scholes equation. He graduated from Harvard College in 1959 and received a Ph.D. in Applied Mathematics from Harvard University in 1964. He was initially expelled from the Ph.D. program due to his inability to settle on a thesis topic, having switched from physics to mathematics, then to computers and artificial intelligence. In 1971, he began to work at the University of Chicago but later left to work at the MIT Sloan School of Management. In 1984, he joined Goldman Sachs where he worked until his death in August 1995 from throat cancer.

²⁴Michael C. Jensen (born November 30, 1939 in Rochester, Minnesota, U.S.) is an American economist working in the area of financial economics. He is currently the managing director in charge of organizational strategy at Monitor Group, a strategy consulting firm, and the Jesse Isidor Straus Professor of Business Administration, Emeritus, at Harvard University. He received his B.A. in Economics from Macalester College in 1962 and both his M.B.A. (1964) and Ph.D. (1968) degrees from the University of Chicago Booth School of Business, notably working with Professor Merton Miller, the 1990 co-winner of the Nobel Prize in Economics. Jensen is also the founder and editor of the *Journal of Financial Economics*. The Jensen Prize in corporate finance and organizations research is named in his honor.

²⁵Myron S. Scholes (born July 1, 1941), a Canadian-born American financial economist, is one of the authors of the Black–Scholes equation. In 1968, after finishing his dissertation under the supervision of Eugene Fama and Merton Miller, Scholes took an academic position at the MIT Sloan School of Management where he met Fischer Black and Robert C. Merton, who joined MIT in 1970. For the following years Scholes, Black and Merton undertook groundbreaking research in asset pricing, including the work on their famous option pricing model. In 1997 he shared the Nobel Prize in Economics with Robert C. Merton “for a new method to determine the value of derivatives”. Fischer Black, who co-authored with them the work that was awarded, had died in 1995 and thus was not eligible for the prize. In 1981 Scholes moved to Stanford University, where he remained until he retired from teaching in 1996. Since then he holds the position of Frank E. Buck Professor of Finance Emeritus at Stanford.

²⁶Black, Jensen, and Scholes (1972, p. 444).

²⁷Reilly and Brown (1997, p. 284).

²⁸This list of examples is mentioned in Reilly and Brown (1997, p. 284).

²⁹Richard Roll mentioned human capital as part of the market portfolio in Roll (1977, p. 131 and p. 155).

³⁰This reasoning is from Bodie, Kane, and Marcus (1999, p. 253).

whole market, and vice versa. Every investor holds the same risky portfolio which excludes the possibility that Apple stock represents a 1.5 % portion in the portfolio of one investor and a 0.5 % portion in a another one.

The market portfolio cannot only contain stocks or bonds, it has to contain everything. If it did not include any Picasso paintings, then by our theory nobody would own Picasso paintings, because everyone's risky portfolio is supposed to be the same. There would be no demand for those, and they would not be worth millions. According to our theory, every investor who makes a risky investment holds some tiny fractions of Picasso paintings. Of course, this is not realistic, and this clearly shows the limitations of CAPM.

It is common practice to use Treasury bills as *the* risk-free asset and its yield as the risk-free rate r_{rf} ³¹: As a short-term fixed income investment it is insensitive to interest rate fluctuations. It is backed by the U.S. government and has virtually no default risk. In practice, all *money market instruments* can be treated as risk-free assets because their short maturities make them virtually free of interest rate risk. They are fairly safe in terms of default or credit risk, and as one of the most liquid asset types their liquidity risks are low. Money market instruments are short-term debt instruments that have original maturities of 1 year or less.³² They include U.S. Treasury bills, commercial paper, some medium-term notes, bankers acceptances, federal agency discount paper, most certificates of deposit, repurchase agreements, federal funds, and short-lived mortgage- and asset-backed securities.

After having discussed the optimal portfolios which are situated on the CML, we want to take a closer look at the individual securities within the portfolio. Since the rational investor does not hold one single, but many securities in his portfolio, we have to assess the risk of these securities in the portfolio context and not just the volatility which describes the risk of holding only one security alone. Let us take a look at the mean-variance diagram in Fig. 2.8, and analyze security *A*. If you are invested only in this asset and nothing else, then the volatility σ_A is the risk which matters. But if you hold security *A* as part of a portfolio on the CML, then some of the risk, the unsystematic risk $\sigma_{e,A}$, is diversified away, and what is left is the systematic risk $\beta_A \cdot \sigma_{Mkt}$ (β_A is defined in Eq. (2.31)). In this case, security *A* becomes equivalent to a portfolio *B* on the CML with the same expected return as portfolio *A*, but with a lower risk $\beta_A \cdot \sigma_{Mkt}$.

This risk cannot be diversified away. Dividing it by σ_{Mkt} produces the *standardized measure of systematic risk*, *beta* β_A . The market portfolio has beta 1. Since every rational investor is assumed to hold a portfolio on the CML, we only have to take account of the systematic risk, i.e., β .

In equilibrium, *all* securities and *all* portfolios are on the security market line (SML), as shown in Fig. 2.9 which plots the expected return of a portfolio against its *beta*. The beta of the risk-free asset is 0, the beta of the market portfolio is 1, and the SML is the line which connects these portfolios.

³¹Bodie et al. (1999, p. 181).

³²Fabozzi, Mann, and Choudhry (2002, p. 1).

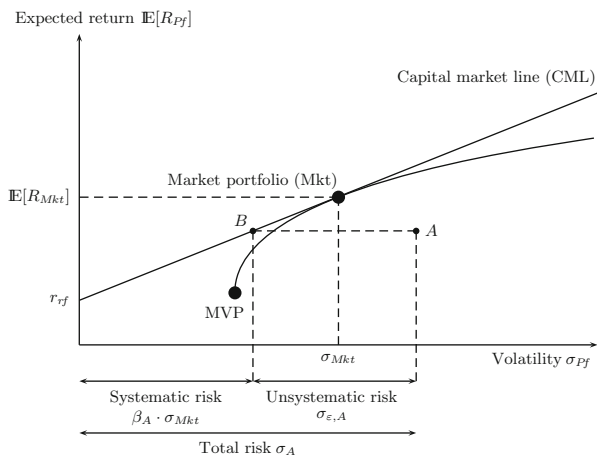


Fig. 2.8 The mean-variance diagram shows the capital market line (CML) which connects the risk-free asset with the tangency point on the efficient frontier, the market portfolio. *Source:* Own, for illustrative purposes only

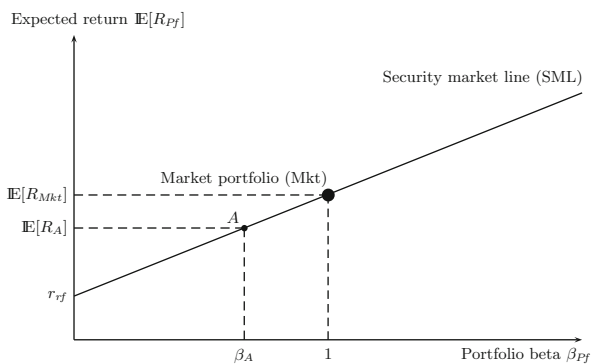


Fig. 2.9 The diagram shows the security market line (SML) which connects the risk-free asset with the market portfolio. In equilibrium, all assets and portfolios plot on the SML. *Source:* Own, for illustrative purposes only

The *beta* (β_{Pf}) of a portfolio is the covariance between the portfolio return and the market return divided by the variance of the market return:

$$\beta_{Pf} = \frac{\sigma_{Pf,Mkt}}{\sigma_{Mkt}^2}. \quad (2.31)$$

In an efficient market, the expected value \mathbb{E} of the portfolio return R_{Pf} satisfies the CAPM equation for any possible portfolio

$$\mathbb{E}[R_{Pf}] = r_{rf} + \beta_{Pf} \cdot (\mathbb{E}[R_{Mkt}] - r_{rf}), \quad (2.32)$$

where the portfolio beta is β_{Pf} , the risk-free return is r_{rf} , and the market return is r_{Mkt} . The difference $E[R_{Mkt}] - r_{rf}$ is called the *market risk premium*. This equation also shows that the investor is directly rewarded for taking risk in terms of β_{Pf} , but not for volatility σ_{Pf} .

There are big differences between the two risk measures beta and volatility:

- While the volatility of a portfolio is an absolute measure which depends only on its returns, beta is a relative measure, since it measures how the portfolio moves relative to the market. A β_{Pf} of 0.8 means that a 1 % increase in the market return increases the portfolio return by 0.8 %. Contrary to the volatility, β_{Pf} could be negative which would mean that market returns and portfolio returns tend to move in opposite directions.
- While computing the volatility σ_{Pf} of a portfolio with M securities (with portfolio weights w_1, \dots, w_M), one has to take into account the pairwise covariances

$$\sigma_{i,j} \quad (1 \leq i, j \leq M, i \neq j)$$

among these M securities besides their volatilities

$$\sigma_i \quad (1 \leq i \leq M),$$

see Eq. (1.83). The relation for beta is linear:

$$\beta_{Pf} = w_1 \cdot \beta_1 + w_2 \cdot \beta_2 + \dots + w_M \cdot \beta_M. \quad (2.33)$$

According to the CAPM, all optimal portfolios are a combination of the risk-free portfolio and the market portfolio, so every investor—independent of his risk-preference—will always hold a combination of those two portfolios. If w_{Mkt} is the portfolio weight invested in the market portfolio and $1 - w_{Mkt}$ the proportion invested in the risk-free asset, then the portfolio beta β_{Pf} is w_{Mkt} . In order to construct individually tailored optimal portfolios, two tasks have to be completed, which can be done separately (this is also called *Tobin separation*³³):

³³Spremann (2008, pp. 227–228), initially discussed in Tobin (1958, p. 66). In Markowitz (1999, p. 10), Harry Markowitz refers to the Tobin (1958) article as the *first CAPM*, and summarizes Tobin's work. In Sect. 3.6. of Tobin's article, the author presented his seminal result, today known as the Tobin separation theorem. Tobin assumed a portfolio selection model with M risky assets and one riskless asset: cash. Because these assets were monetary, i.e., “*marketable, fixed in money value, free of default risk*” (see Tobin 1958, p. 66), the risk was market risk, not default risk. Holdings had to be nonnegative. Borrowing was not permitted. Implicitly, Tobin assumed that the covariance matrix for risky assets is nonsingular. The result was that “*the proportionate composition of the non-cash assets is independent of their aggregate share of the investment balance. This fact makes it possible to describe the investor's decisions as if there were a single non-cash asset, a composite formed by combining the multitude of actual non-cash assets in fixed proportions.*” (see Tobin 1958, p. 84).

Table 2.1 Example for calculating the expected return of a portfolio using CAPM

i	Weight w_i	Beta β_i
1	15 %	1.2
2	20 %	0.6
3	25 %	0.5
4	15 %	2.4
5	25 %	−0.4

Source: Own, for illustrative purposes only

- Construct the risk-free portfolio (with money market instruments) and the market portfolio. This is the task of the portfolio manager.
- Determine the correct risk profile of the client and find the optimal allocation between risk-free and risky investments. This is the role of the investment advisor.

In order to find the optimal portfolio, one would have to know the composition of the market portfolio. In theory, this would also include non-tradable assets like human capital, rarely traded assets like art collections, and assets like real estate which are difficult to value. Their respective market capitalization has to be known in order to set up the market portfolio. But given that human capital makes up a large part of the market portfolio with estimates of its proportion within the portfolio ranging from 50 to 90 %, ³⁴ the market portfolio cannot be measured as long as the valuation of human capital remains questionable. ³⁵

Example

Let us consider the following example: We want to calculate the expected return of a portfolio containing five different assets. Table 2.1 shows the weights of the assets in our portfolio. Assume that the risk-free rate r_{rf} is 5 % and the expected market return $E[R_{Mkt}]$ is 13 %.

The first step is to calculate the portfolio beta β_{Pf} using Eq. (2.33) with $M = 5$:

$$\begin{aligned}
 \beta_{Pf} &= w_1 \cdot \beta_1 + \dots + w_5 \cdot \beta_5 \\
 &= 0.15 \cdot 1.2 + 0.20 \cdot 0.6 + 0.25 \cdot 0.5 + 0.15 \cdot 2.4 + 0.25 \cdot (-0.4) \\
 &= 0.685.
 \end{aligned} \tag{2.34}$$

We use this result to calculate the expected return $E[R_{Pf}]$ of the portfolio from the CAPM equation—Eq. (2.32):

³⁴In the studies Kendrick (1974, 1976, 1994) and Eisner (1985, 1989) which used the cost-based approach, the size of human capital was about the size of non-human capital. In the study Jorgenson and Fraumeni (1989, 1992) which used the income-based approach, the share of human capital in total wealth was over 90 %.

³⁵A good summary about the market portfolio is provided in Le, Gibson, and Oxley (2003).

$$\begin{aligned}
\mathbb{E}[R_{Pf}] &= r_{rf} + \beta_{Pf} \cdot (\mathbb{E}[R_{Mkt}] - r_{rf}) \\
&= 5\% + 0.685 \cdot (13\% - 5\%) = 10.48\%. \quad (2.35)
\end{aligned}$$

The expected return of our portfolio is 10.48 %.

End of Example

Remark. We have derived the CAPM equation—Eq. (2.32)—for a 1-year period and for annualized returns. But the CAPM also holds for arbitrary periods, for example, quarters, months, weeks, days, and you also get CAPM equations with quarterly, monthly, weekly and daily returns. For example, for monthly returns, the CAPM formula is

$$\mathbb{E}[R_{Pf}^{monthly}] = r_{rf}^{monthly} + \beta_{Pf} \cdot (\mathbb{E}[R_{Mkt}^{monthly}] - r_{rf}^{monthly}). \quad (2.36)$$

2.3.4 Empirical Tests

We have described the CAPM after starting with very simplistic and unrealistic assumptions in Sect. 2.3.1 and made two points about the usefulness of the theory³⁶: First, many assumptions can be relaxed to approximate real-world conditions. This would slightly modify the CAPM without changing the main implications. Second, a theory should “*not be judged on the basis of its assumptions, but on how well it explains relationships that exist in the real world*”.³⁷

Therefore, in this section, we will discuss the empirical tests of the CAPM. This section is based on the paper Fama and French (2004) and Reilly and Brown (1997, pp. 310–311).

2.3.4.1 The Testable Hypotheses

Let us recall the CAPM equation (2.32) which should be tested:

Given the risk-free return r_{rf} and the market return R_{Mkt} , the expected return $\mathbb{E}[R_{Pf}]$ of the portfolio depends only on the systematic risk, the portfolio beta β_{Pf} (i.e., the covariance of the portfolio Pf with the market portfolio Mkt divided by the variance of the market portfolio), satisfying the equation

$$\mathbb{E}[R_{Pf}] = r_{rf} + \beta_{Pf} \cdot (\mathbb{E}[R_{Mkt}] - r_{rf}). \quad (2.37)$$

The first question which arises is if beta is a stable measure of systematic risk.³⁸ Can historical betas be used as an estimate for future betas? If betas change too

³⁶Reilly and Brown (1997, p. 279).

³⁷Reilly and Brown (1997, p. 310).

³⁸Reilly and Brown (1997, p. 310).

much over time, then they are no useful measures for future risks and for estimating expected returns with the CAPM formula.

Equation (2.37) has four testable implications which are to be tested³⁹:

- **Hypothesis 1 (C1)—Linearity:**

Expected returns of all assets are linearly related to their betas.

- **Hypothesis 2 (C2)—No systematic non-beta risks:**

Beta is a complete measure of risk for an asset in the market portfolio. No other variable has marginal explanatory power to explain returns.

- **Hypothesis 3 (C3)—Positive beta premium:**

The beta premium,⁴⁰ i.e., the difference between the expected return of the market portfolio and the expected return of assets uncorrelated to the market, is greater than zero. In other words, the expected return of the market portfolio exceeds the expected return of assets uncorrelated to the market.

- **Hypothesis 4 (C4)—Risk-free return on zero-beta assets:**

Assets uncorrelated to the market portfolio have expected returns equal to the risk-free rate.

2.3.4.2 Regressions

There are two different simple linear regressions which are used to test Eq. (2.37):

- The *cross-sectional regressions* test the CAPM across assets with different betas. The returns of different assets are regressed over the betas.
- The *time-series regressions* test the CAPM equations for each individual asset over time. The excess portfolio returns in the subperiods are regressed over the respective excess return of the market portfolio.

Cross-Sectional Regression

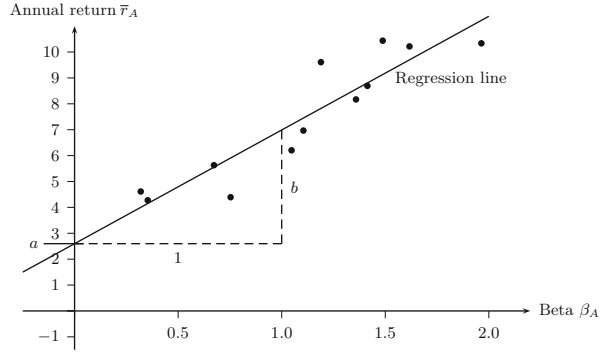
The cross-sectional regression is the main tool used to test Eq. (2.37) over a period $[0, T]$ of T years. CAPM tests do not exactly test Eq. (2.37) because it includes *expected returns* which are not measurable. Instead, the question is if the relation holds for *realized returns*, i.e., if the relation has been correct in the past. The cross-sectional regression tests the CAPM equation by regressing the (arithmetic) average annual return \bar{r}_A of an asset A over beta.

Figure 2.10 shows a typical cross-sectional regression. For every asset A , the data point (β_A, r_A) is plotted with β_A its beta and \bar{r}_A as its (arithmetic) average annual return. We then get the regression line which has the form

³⁹These are from Fama and French (2004, p. 30), and Fama and MacBeth (1973, p. 610 and p. 613).

⁴⁰Literally, the beta premium is the premium per unit of beta. The CAPM implies that the beta premium is the excess market return, i.e., the difference between the expected return on the market portfolio and the risk-free rate. But this equality is equivalent to the fourth hypothesis, that zero-beta assets expect to earn the risk-free rate. The *positive beta premium* does not test the equality with the excess market return, only the positivity.

Fig. 2.10 Cross-sectional regression. Annualized returns of asset returns are regressed over their respective betas. *Source:* Own, for illustrative purposes only



$$\bar{r}_A = a + b \cdot \beta_A. \quad (2.38)$$

This diagram looks very similar to Fig. 2.9 where the expected returns of assets are plotted against their betas. They lie on the security market line (SML) which is the line *predicted* by the CAPM, whereas the regression line is the *empirical* one we observe, based on historical data. The empirical test of the CAPM examines if both lines are equal, or in other words, if the data fits the predicted line, the SML. The intercept a in Eq. (2.38) and the slope coefficient b (the beta premium) correspond to the risk-free rate r_{rf} in Eq. (2.37) and the market risk premium $R_{Mkt} - r_{rf}$, respectively. With this regression, we can check hypotheses (C3) and (C4) from the list on page 123:

- *Positive beta premium:* The beta premium b is positive.
- *Risk-free return on zero-beta assets:* The intercept a equals the risk-free rate r_{rf} .

Given the market data, we still need the betas β_A to plot a diagram like Fig. 2.10 and to perform our regression analysis. We get the betas from a time-series regression.

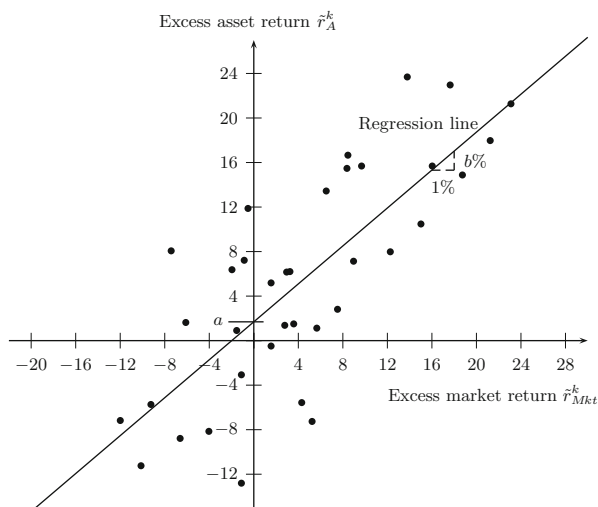
Time-Series Regression

The time-series regression tests the CAPM equation for each individual asset separately on N equidistant subperiods of a time horizon $[0, T]$. The excess asset return in each subperiod k is regressed over the excess market return from the same subperiod. Figure 2.11 shows a typical time-series regression. Given an asset A , for every subperiod k , the data point $\tilde{r}_{Mkt}^k, \tilde{r}_A^k$ is plotted where $\tilde{r}_{Mkt}^k = r_{Mkt}^k - r_{rf}^k$ is the excess return of the market portfolio in subperiod k , i.e., the difference between the market return and the return on a risk-free asset, and $\tilde{r}_A^k = r_A^k - r_{rf}^k$ is the excess return of the asset A in the subperiod k .

We get a regression line of the form

$$\tilde{r}_A^{subperiod} = a + b \cdot \tilde{r}_{Mkt}^{subperiod} \quad (2.39)$$

Fig. 2.11 Time-series regression. Excess returns of an asset A in subperiod i are regressed over the excess market return. *Source:* Own, for illustrative purposes only



where *subperiod* can be *quarterly*, *monthly*, *weekly*, *daily*, etc. and refers to the length of the subperiods. For example, if we choose monthly subperiods, then we have a regression on monthly returns:

$$\underbrace{\tilde{r}_A^{\text{monthly}}}_{\text{monthly excess return}} = a + b \cdot \tilde{r}_{Mkt}^{\text{monthly}}. \quad (2.40)$$

Let us recall the CAPM equation using monthly returns (Eq. (2.36)):

$$\mathbb{E}[R_A^{\text{monthly}}] = r_{rf}^{\text{monthly}} + \beta_A \cdot (\mathbb{E}[R_{Mkt}^{\text{monthly}}] - r_{rf}^{\text{monthly}}).$$

Rewrite this as

$$\underbrace{\mathbb{E}[R_{Pf}^{\text{monthly}}] - r_{rf}^{\text{monthly}}}_{\text{expected monthly excess return}} = \beta_A \cdot (\mathbb{E}[R_{Mkt}^{\text{monthly}}] - r_{rf}^{\text{monthly}}), \quad (2.41)$$

and we can see that in the regression equation—Eq. (2.40)— a should be zero and b is the beta β_A . For any asset A , we call the intercept a from the above regression the *alpha* of A . In case of monthly subperiods, we write $\alpha_A^{\text{monthly}}$, similarly to other subperiods. The alpha represents the incremental rate of return which exceeds the theoretical rate of return implied by the CAPM. It is introduced in Jensen (1967) as the intercept of the linear time-series regression and as a performance measure to evaluate a portfolio manager who invests in portfolio A .⁴¹

⁴¹Jensen (1967, p. 8).

Regarding the CAPM hypotheses on page 123, the time-series regression can be used to test hypothesis (C4):

Risk-free return on zero-beta assets: The alphas of all assets are zero.

Note that for our regressions, we do not use the real market portfolio *Mkt* because it is hardly observable. Instead, we use a proxy like a stock market index to mimic the market return.

2.3.4.3 Beta Stability

In the CAPM, the only relevant risk measure is the systematic risk, beta. For any asset with historical data, we can calculate the historical beta with a time-series regression. But is the historical beta reliable enough to assess future risk? Figure 2.12 depicts the following scenario for an investor: Assume that you are looking for a rather conservative investment with a low beta of around 0.5, and you have found a portfolio with a historical beta of 0.5. Your natural question to ask is if the portfolio's beta will only change little over time, i.e., whether it will remain stable.

There have been lots of studies on this topic, with similar results. They are summarized in Reilly and Brown (1997, pp. 310–311), and this section about beta stability is based on this book. The two researchers tested the correlation between two consecutive time periods, as illustrated in Fig. 2.13: The earlier period is called *estimation period*, the latter is termed *subsequent period*.⁴² If the correlation between the betas of both periods is high, then beta is stable, and the investor can rely on the historical beta to provide a good estimate of the future beta. The correlation of betas of consecutive time periods is therefore used as a measure for beta stability.

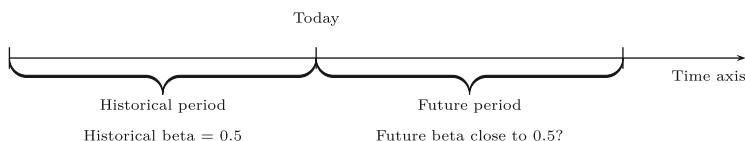


Fig. 2.12 Illustration of historical beta vs. future beta. *Source:* Own

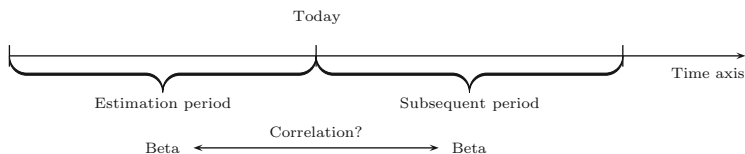


Fig. 2.13 Illustration of two consecutive periods: estimation period and subsequent period. *Source:* Own

⁴²This terminology is from Roenfeldt, Griepentrog, and Pflaum (1978).

Marshall E. Blume studied the variation of the beta measure for all NYSE common stocks (the number of stocks ranged from 415 to 890)⁴³ in the period July 1926–June 1968 in his paper Blume (1971). He split this 42-year period into seven consecutive 6-year periods for which he calculated the betas (based on monthly returns) of portfolios with 1, 2, 4, 7, 10, 20, 35, 50, 75 and 100 stocks. He then calculated the correlations of betas between consecutive periods, as illustrated on the timeline in Fig. 2.14.

His result is that the correlations of betas are low for one single stock (around 0.6), but these numbers increase with the number of stocks in the portfolio. With 50 stocks, the correlation becomes very high (0.98). The beta is unstable for individual stocks, but it is very stable for a larger portfolio with 50 stocks.⁴⁴

Robert A. Levy did a similar study in Levy (1971). Here, the time periods were much shorter: 13 weeks, 26 weeks and 52 weeks. The betas were calculated based on weekly returns. Short-term betas are more important for portfolio managers with shorter time horizons.⁴⁵ Again, the beta of the single stock was unstable. The longer the time period (at least 26 weeks) and the more stocks (at least 25 stocks) in the portfolios, the more stable the beta of the portfolio.

R.B. Porter and John R. Ezzell argue in Porter and Ezzell (1975) that the high correlation of betas in Blume’s study is only due to his portfolio selection methodology. Blume selected the portfolios in his study based on the betas⁴⁶: The stocks with the lowest betas are assigned to portfolio 1, the next smallest ones to portfolio 2, etc. Porter and Ezzell repeated his study with randomly selected portfolios and found that in contrast to Blume’s results, the stability of beta is “*relatively light and [...] totally unrelated to the number of securities in the portfolio*”.⁴⁷

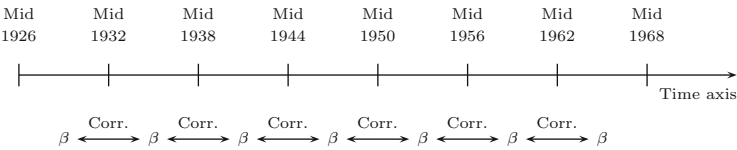


Fig. 2.14 Illustration of the timeline used in Blume (1971) to study the stability of beta. The time period July 1926–June 1968 is split in seven equidistant subperiods for which the betas are measured. The correlation between the betas of consecutive subperiods are calculated and used as a measure for beta stability. *Source:* Own

⁴³Blume (1971, p. 4).

⁴⁴Blume (1971, p. 7).

⁴⁵Levy (1971, p. 57).

⁴⁶Blume (1971, p. 6).

⁴⁷Quote from Porter and Ezzell (1975, p. 369). See p. 370, Table 2, which compares Blume’s to Porter’s & Ezzell’s correlation numbers.

T.M. Tole examined the standard deviation of betas as another measure of beta stability in Tole (1981). He found a significant decrease in the standard deviation (and therefore an increase in beta stability) with increasing portfolio size,⁴⁸ even beyond 100 stocks.⁴⁹

To which extent does beta stability depend on the lengths of the estimation and subsequent periods? In Baesel (1974), the beta stability of single U.S. stocks is examined by using different lengths for the two periods. Baesel found that an increase in these lengths also increases the beta stability.⁵⁰ Altman, Jacquillat, and Levasseur (1974) presents the same results for French stocks. While both periods were equally long in these two studies, Roenfeldt et al. (1978) compares the beta of a 4-year estimation period to the betas of subsequent 1-year, 2-year, 3-year and 4-year periods. They found that estimated betas based on the 4-year period were more reliable as a forecast for subsequent 2-year, 3-year and 4-year periods than for a subsequent 1-year period.⁵¹

According to Reilly and Brown (1997), the overall conclusion of the empirical tests is that individual betas are generally volatile, whereas portfolio betas are stable. The estimation period should be at least 36 months to forecast beta.⁵²

2.3.4.4 Tests of the Main Hypotheses

Now we discuss tests of the main hypotheses which were formulated on page 123. Because the beta stability was low for individual securities and high for large portfolios (as discussed in the Section “*Beta Stability*” above), the CAPM was only tested for portfolios. This is plausible, since the CAPM is only interesting for portfolios where the beta is supposed to be the significant risk measure, in contrast to an individual security. An application to single securities does not make sense.

A few years after the CAPM was developed in Sharpe (1964), Lintner (1965a) and Mossin (1966), the first empirical tests were performed. We want to focus on three groundbreaking empirical studies which shaped the discussion about the empirical CAPM and led to modifications of this model: Black et al. (1972), Fama and MacBeth (1973) and Fama and French (1992).

Black, Jensen and Scholes (1972): *The Capital Asset Pricing Model: Some Empirical Tests*

The early empirical cross-sectional tests in Douglas (1969) and Miller and Scholes (1972) showed that zero-beta assets earned more than the risk-free rate and that the beta premium was lower than the market excess return, in violation with hypothesis (C3).⁵³ Friend and Blume (1970) also showed that low-beta assets earned positive alpha and high-beta assets earned negative alpha. Black et al. (1972) provided

⁴⁸Tole (1981, p. 47, Exhibit 1).

⁴⁹This is pointed out in Reilly and Brown (1997, p. 310).

⁵⁰Conclusion in Baesel (1974, p. 1493). Compare Table 1 on p. 1492 with Table 2 on p. 1493.

⁵¹Conclusion in Roenfeldt et al. (1978, p. 120 and Table 1 on p. 119).

⁵²Reilly and Brown (1997, p. 311)

⁵³Fama and French (2004, p. 32).

additional time-series and cross-sectional tests in support of these results and, as a consequence, postulated a modified version of the CAPM, the *zero-beta CAPM*, which accommodated zero-beta returns above the risk-free rate.

The data used in the tests to be described were taken from the University of Chicago Center for Research in Security Prices monthly price relative file, which contains monthly price, dividend, and adjusted price and dividend information for all securities listed on the New York Stock Exchange in the period January 1926–March 1966.⁵⁴ The monthly returns on the market portfolio $R_{Mkt}^{monthly}$ were defined as the returns that would have been earned on a portfolio consisting of an equal investment in every security listed on the NYSE at the beginning of each month.⁵⁵ The risk-free rate was defined as the 30-day rate on U.S. Treasury bills for the period 1948–1966. For the period 1926–1947, where Treasury bill rates were not available, the dealer commercial paper rate was used.

Portfolio selection process⁵⁶: The tests used portfolios with a wide range of betas. This was done by ranking individual securities by their betas and then assigning the ones with the highest beta to portfolio 1, the next highest to portfolio 2, etc. But if these betas were used as an input for the regression analysis, this would cause a bias.⁵⁷ This is why the analysis is split in two phases:

1. Portfolio formation period: In this period, the betas of individual securities are calculated and ranked. Based on the rankings, the portfolios are formed. The betas from this period are not used in the test period.
2. Test period: The returns and betas of portfolios which are used for regression analysis are computed. The beta is calculated only on the basis of the test period.

For the beginning of January 1931, only stocks with at least 24 months of available data were considered, and their individual betas were calculated based on the monthly returns in the 5-year period January 1926–December 1930 (or from a shorter period if less than 5 years of data were available). These securities were ranked according to the calculated betas and then assigned to ten portfolios, with an equal investment in each security⁵⁸: the 10 % with the largest beta to the first portfolio, the next 10 % to the second portfolio, etc. The return in each of the next 12 months for each of the ten portfolios was calculated.

After 12 months, for the beginning of January 1932, the ten portfolios were rebalanced using the same procedure: Only stocks with at least 24 months of previous monthly returns were considered, their betas were calculated based on the

⁵⁴Black et al. (1972, p. 10).

⁵⁵Black et al. (1972, p. 8).

⁵⁶Black et al. (1972, p. 11).

⁵⁷Black et al. (1972, p. 9).

⁵⁸Black et al. (1972, p. 8): An equal investment is indicated by using the average return and average risk (beta) of all securities in a portfolio.

Table 2.2 Summary of portfolio formation period and testing period

Period	1	2	...	35
Portfolio formation period	1926–1930	1927–1931	...	1960–1964
Testing period	1931	1932	...	1965

Source: Black et al. (1972, p. 11)

Table 2.3 Summary of regression statistics for the time-series regression for ten different portfolios based on monthly returns in the period January 1931–December 1965

Portfolio #	1	2	3	4	5	6	7	8	9	10
β	1.561	1.384	1.248	1.163	1.057	0.923	0.853	0.753	0.629	0.499
α (in %)	−0.083	−0.194	−0.065	−0.017	−0.054	0.059	0.046	0.081	0.197	0.201

Source: Black et al. (1972, Table 2, p. 14)

5-year period January 1927–December 1931 (or on a shorter period if less than 5 years of data were available), the stocks were ranked and assigned to the ten portfolios based on the rankings.

This process was repeated for the beginnings of January 1933, January 1934, etc. through January 1965. Table 2.2 summarizes the periods:

The total number of stocks in the portfolios ranged from 582 to 1,094. For each portfolio, we get 35 years of monthly data.

Time-series regression was run on the ten portfolios based on 35 years of monthly returns in the period January 1931–December 1965 (420 observations). The results are shown in Table 2.3.

The table shows the alphas and betas of the ten portfolios, calculated with a linear time-series regression. Portfolio number 1 contains the highest-beta securities, portfolio number 10 consists of the lowest-beta securities. The portfolio betas range from 0.499 to 1.561. The critical result is that the high-beta portfolios ($\beta > 1$) consistently show negative alphas while low-beta portfolios ($\beta < 1$) show positive alphas. In other words, high-beta portfolios yield lower returns and low-beta portfolios yield higher returns than predicted by the CAPM. This rejects the CAPM hypothesis (C4) (see the list of hypotheses on page 123) that alpha should be zero. Furthermore, the study shows that the incremental return per unit of risk β (the beta premium) is smaller than implied by the CAPM.

Black et al. (1972) also performed a cross-sectional regression analysis with the same ten portfolios for the period January 1931–December 1965, which is illustrated in Fig. 2.15.

For each of the ten portfolios, the average excess monthly return⁵⁹ (vertical axis) and the beta (horizontal axis) are plotted (represented by dots), together with the (bold) regression line. The square represents the market portfolio and has beta 1.

⁵⁹The excess return of a portfolio is the difference between the return of a portfolio and the return on a risk-free asset.

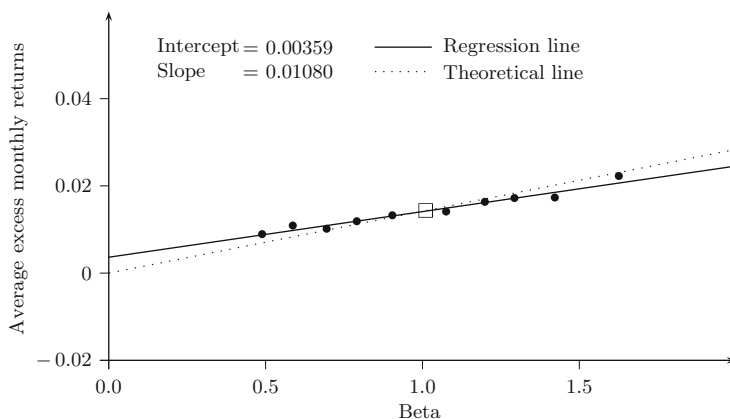


Fig. 2.15 Average excess monthly returns of ten portfolios (denoted by *dots*) and the market portfolio (denoted by the *square*) are plotted against their betas for the 35-year period 1931–1965. The *bold line* is the regression line, the *dotted line* is the theoretical line implied by the CAPM. Source: Black et al. (1972, p. 21)

The dotted line is the theoretical CAPM line (i.e., the SML) which goes through $(0, 0)$ and the market portfolio is also drawn to compare it with the regression line. The regression line has a flatter slope than the theoretical line and a higher intercept which is significantly greater than zero. The latter contradicts⁶⁰ the CAPM hypothesis (C4) (see the list of hypotheses on page 123). On the other hand, the ten data points representing the portfolios plot close to the line, indicating a linear relationship between return and beta (in support of (C1)).

Black et al. (1972) proceeded by examining the time-dependency of the regression line and its variation over time. A 35-year period was divided in four equal subperiods of 105 months length, and the same cross-sectional regression analysis was done for these subperiods. Figure 2.16 shows the regression for the period January 1931–September 1939. The intercept was significantly less than zero and the regression line was steeper than the theoretical line. Figure 2.17 shows the regression for October 1939–June 1948. The intercept is significantly greater than zero, and the regression line is flatter than the theoretical line. Figure 2.18 shows how the regression line became even flatter in the period July 1948–March 1957. From April 1957 to December 1965, the regression line in Fig. 2.19 even shows a negative slope, implying a negative beta premium which contradicts hypothesis (C3).

⁶⁰In the general cross-sectional regression (2.38) that we described earlier, the average return, not the average *excess* return, is regressed on beta. The only difference is that we have to shift the graph from Fig. 2.16 down by the average risk-free rate r_{rf} . So the second hypothesis stated on page 124, i.e., *risk-free return on zero-beta assets*, translates into: “The intercept a is zero.”

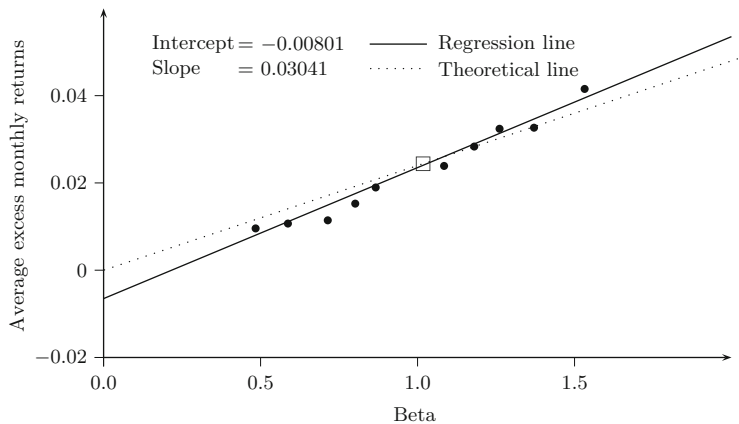


Fig. 2.16 Average excess monthly returns of ten portfolios (denoted by *dots*) and the market portfolio (denoted by the *square*) are plotted against their betas for the 105-month period January 1931–September 1939. The **bold line** is the regression line, the **dotted line** is the theoretical line implied by the CAPM. *Source:* Black et al. (1972, p. 24)

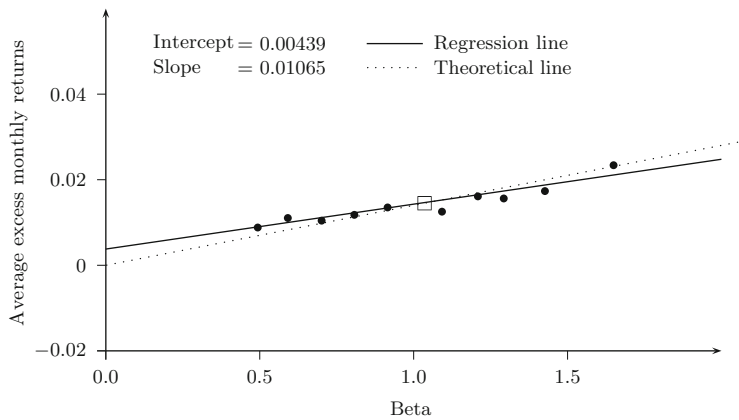


Fig. 2.17 Average excess monthly returns of ten portfolios (denoted by *dots*) and the market portfolio (denoted by the *square*) are plotted against their betas for the 105-month period October 1939–June 1948. The **bold line** is the regression line, the **dotted line** is the theoretical line implied by the CAPM. *Source:* Black et al. (1972, p. 24)

On the other hand, in all four subperiods which show different behaviors of the regression line (the slope was greater than the theoretical line in one subperiod and negative in an other subperiod), the data points plot close to the regression line, supporting hypothesis (C1) about the linearity between return and beta.

The overall conclusion of the empirical tests is that linearity between return and beta is supported by the data, but the hypothesis that zero-beta assets earn the

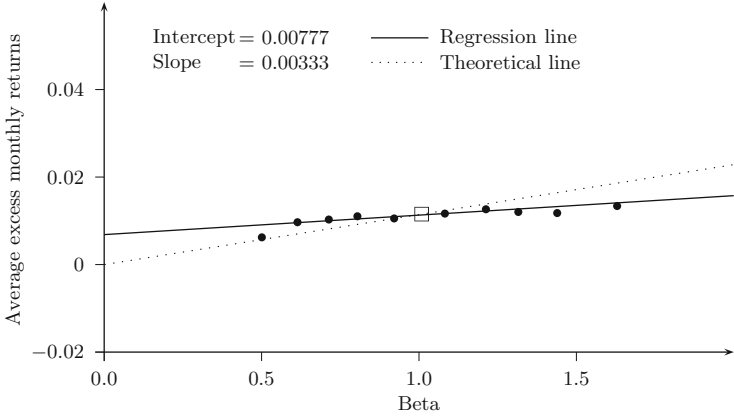


Fig. 2.18 Average excess monthly returns of ten portfolios (denoted by *dots*) and the market portfolio (denoted by the *square*) are plotted against their betas for the 105-month period July 1948–March 1957. The *bold line* is the regression line, the *dotted line* is the theoretical line implied by the CAPM. *Source:* Black et al. (1972, p. 24)

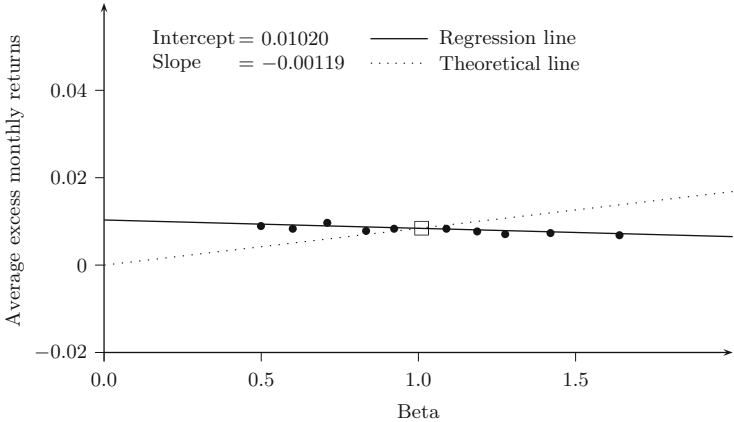


Fig. 2.19 Average excess monthly returns of ten portfolios (denoted by *dots*) and the market portfolio (denoted by the *square*) are plotted against their betas for the 105-month period April 1957–December 1965. The *bold line* is the regression line, the *dotted line* is the theoretical line implied by the CAPM. *Source:* Black et al. (1972, p. 24)

risk-free rate is rejected. The paper therefore suggests a modified CAPM without the assumption of risk-free borrowing and lending, the *zero-beta CAPM* (see also Sect. 2.3.5):

Let R_Z be the return on an asset Z which has zero beta. Then

$$R_{Pf} = \mathbb{E}[R_Z] + \beta_{Pf} \cdot (\mathbb{E}[R_{Mkt}] - \mathbb{E}[R_Z]). \quad (2.42)$$

The zero-beta CAPM was introduced in Black (1972), where it was shown that Eq. (2.42) holds when all CAPM assumptions from Sect. 2.3.1 are true except the assumption of risk-free borrowing and lending (A3). The empirical tests from Black et al. (1972) are consistent with this model.⁶¹

Fama and MacBeth (1973): Risk, Return, and Equilibrium: Empirical Tests

Before Eugene F. Fama and James D. MacBeth published their study in 1973, empirical tests of the CAPM were focused on the hypothesis that zero-beta assets earn the risk-free rate (C4, see page 123).⁶² This hypothesis was rejected in several papers, including Douglas (1969), Friend and Blume (1970), Miller and Scholes (1972) and Black et al. (1972). This led to the introduction of the zero-beta CAPM (2.42).

Fama and MacBeth (1973) made the next step: They formulated⁶³ the list of testable hypotheses (C1)–(C4) implied by the CAPM as shown on page 123 and tested these hypotheses by extending the cross-series regression with beta as the only explanatory variable to a multi-linear regression with three explanatory variables:

- **Beta:**
As applied in the cross-sectional regressions above.
- **Beta squared:**
Beta squared is used to test linearity (C1). If it turns out to have explanatory power in explaining returns, then the relation between return and beta cannot be linear. Beta squared serves as an explanatory variable for possible low-beta or high-beta tilts.⁶⁴ If the slope coefficient of beta squared is positive, then high-beta securities have higher expected returns and low-beta securities have lower expected returns than predicted by the CAPM. If the slope coefficient is negative, then it is the other way round.
- **Unsystematic risk:**
Total risk (in terms of volatility) is the sum of systematic and unsystematic risk which are not correlated with each other.⁶⁵ Unsystematic risk is used as an explanatory variable in the regression to test (C2), i.e., if non-beta risks also explain returns. If the CAPM is valid, then unsystematic risk should have no explanatory power.

The data used in the study are the monthly returns (including dividends and capital gains with the appropriate adjustments for capital changes such as splits and stock dividends) for all common stocks traded on the New York Stock Exchange

⁶¹Black et al. (1972, p. 25).

⁶²Fama and MacBeth (1973, p. 614).

⁶³Fama and MacBeth (1973, p. 610 and p. 613).

⁶⁴Fama and MacBeth (1973, p. 614).

⁶⁵Fama and MacBeth (1973, p. 616).

during the period January 1926–June 1968.⁶⁶ The data are taken from the Center for Research in Security Prices of the University of Chicago. *Fisher’s arithmetic index* is chosen as the market portfolio, an equally-weighted index on all stocks listed on the New York Stock Exchange.⁶⁷ The monthly risk-free rate is taken to be the yield on the 1-month Treasury bills.⁶⁸

The portfolio selection process for the study is similar to Black et al. (1972), as described on page 129. 20 portfolios are selected based on the beta rankings of the securities, with equal investment in all securities after a *portfolio formation period* of 7 years (except for the first period which is 4 years long, see Table 2.4). Data which is used for beta rankings is not reused for regression analysis because this introduces statistical biases.⁶⁹ The next 5 years of data in the *initial estimation period* is used to estimate the betas and the unsystematic risk (the average of the unsystematic risks of all securities in the portfolio) of the portfolios. After that, in the *testing period*, monthly returns of the 20 portfolios are measured. For every month in the testing period, the monthly returns of the 20 portfolios are regressed over beta, beta squared and unsystematic risk which are measured based on the preceding 5 years of data. Then the regression coefficients are averaged to get the result. Table 2.4 shows the different periods.

Let us illustrate the methodology for period 1: First, the betas of the securities are measured based on the period 1926–1929 and ranked. The 5 % of the securities with the lowest betas go to portfolio 1, the next lowest 5 % to portfolio 2, etc., until 20 portfolios are established. The first month in the test period is illustrated in Fig. 2.20: In January 1935, the monthly returns of the 20 portfolios are measured, and they are cross-sectionally regressed over beta, beta squared and unsystematic risk which were measured based on the preceding 60 months of data (January 1930–December 1934).

Table 2.4 Portfolio formation, estimation and testing period

Period	1	2	3	4	5
Portfolio formation period	1926–1929	1927–1933	1931–1937	1935–1941	1939–1945
Initial estimation period	1930–1934	1934–1938	1938–1942	1942–1946	1946–1950
Testing period	1935–1938	1939–1942	1943–1946	1947–1950	1951–1954

Period	6	7	8	9
Portfolio formation period	1943–1949	1947–1953	1951–1957	1955–1961
Initial estimation period	1950–1954	1954–1958	1958–1962	1962–1966
Testing period	1955–1958	1959–1962	1963–1966	1967–1968

Source: Fama and MacBeth (1973, Table 1, pp. 618–619)

⁶⁶Fama and MacBeth (1973, p. 614).

⁶⁷Fama and MacBeth (1973, p. 614).

⁶⁸Fama and MacBeth (1973, p. 626).

⁶⁹Fama and MacBeth (1973, p. 615).

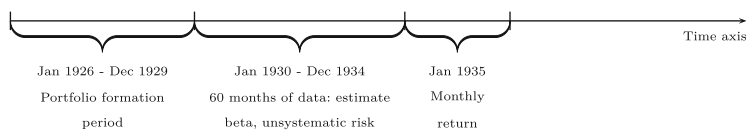


Fig. 2.20 Fama–MacBeth regression illustrated: 1st month of period 1. *Source:* Own

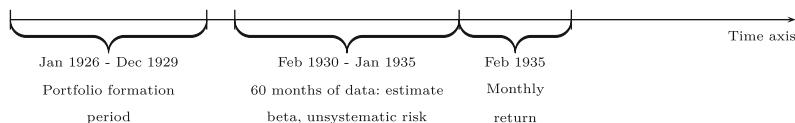


Fig. 2.21 Fama–MacBeth regression illustrated: 2nd month of period 1. *Source:* Own

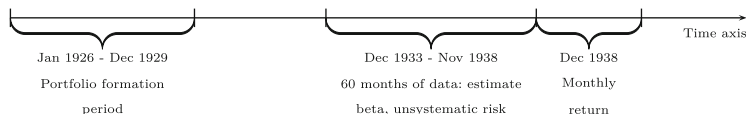


Fig. 2.22 Fama–MacBeth regression illustrated: last month of period 1. *Source:* Own

The second month in the test period is illustrated in Fig. 2.21: In February 1935, the monthly returns of the 20 portfolios are measured, and they are cross-sectionally regressed over beta, beta squared and unsystematic risk which were measured based on the preceding 60 months of data (February 1930–January 1935).

This procedure is continued until the last month of period 1, illustrated in Fig. 2.22: In December 1938, the monthly returns of the 20 portfolios are measured, and they are cross-sectionally regressed over beta, beta squared and unsystematic risk which were measured based on the preceding 60 months of data (December 1933–November 1938).

For each of the 48 months in the testing period 1935–1938, a cross-sectional regression is performed and regression coefficients are calculated. These are averaged for the whole period 1935–1938. This process is repeated for the other eight testing periods (see Table 2.4), the regression coefficients are averaged over all months of the period 1935–1968, and the result for the entire period is obtained.

This regression method, which is also known as the *Fama–MacBeth regression*, has become standard in the literature.⁷⁰ Fama and MacBeth performed their tests on the 20 portfolios for the period January 1935–June 1968 and its subperiods (see Table 2.4). The results of Fama’s and MacBeth’s tests were the following⁷¹:

- The beta premium is positive for the overall period, which validates hypothesis (C3) that beta is positively related to return.⁷²

⁷⁰Fama and French (2004, p. 31).

⁷¹Fama and MacBeth (1973, p. 624).

⁷²The beta premium turned negative during a short subperiod (1956–1960), but this does not invalidate the long-term result.

- Beta squared showed no explanatory power for returns. Therefore, the test supports hypothesis (C1) that the relation between return and beta is linear.
- Unsystematic risk showed no explanatory power for returns, which supports hypothesis (C2) that no variable other than beta explains returns.

The results support the zero-beta CAPM (2.42). In addition, the cross-sectional regression analysis with beta as the only explanatory variable shows that the returns on zero-beta assets are higher than the risk-free rate, thus invalidating hypothesis (C4).

There is one big difference between the cross-sectional regressions used in the tests in Black et al. (1972) and Fama and MacBeth (1973): In Black et al. (1972), monthly returns and betas are measured in the *same* testing period. While this is a test of the CAPM, this is not a test if estimated betas can be used for predicting returns because beta is unknown before the returns are measured. The Fama–MacBeth regression is an improvement because betas are measured *before* the monthly returns are realized. So this empirical test also supports the assertion that estimated betas can be used for making predictions and decisions.⁷³ Another difference is that while the early cross-sectional tests treated the zero-beta returns as constant (because the CAPM assumes a constant and deterministic risk-free rate), the Fama–MacBeth regression uses time-varying zero-beta returns which come closer to the stochastic nature of zero-beta returns, as postulated in the zero-beta CAPM.

Other tests like Blume and Friend (1973) and Stambaugh (1982) also positively tested the linearity assumption (C1) while rejecting (C4).

Fama and French (1992): *The Cross-Section of Expected Stock Returns*

After the CAPM was developed, some researchers started to doubt the relation between beta and return. Reinganum (1981) found no significant relationship between the beta and the returns of NYSE/AMEX stocks in the period 1964–1979. Lakonishok and Shapiro (1986) arrived at the same result for NYSE stocks in the period 1962–1981.

While more and more doubts were raised about beta as a basis to predict returns, other factors like size and price ratios were discovered to play a role as well.⁷⁴ Basu (1977) showed that high P/E stocks earned lower returns and low P/E stocks earned higher returns than predicted by the CAPM. Banz (1981) showed a size effect: small firm stocks earned a higher return and large firm stocks earned a lower return than indicated by the CAPM. Bhandari (1988) found a positive relation between leverage (in terms of debt-equity ratio, the book value of debt over market value of equity) and returns, controlling for beta and size.

Rosenberg, Reid, and Lanstein (1985) showed that U.S. stocks with high book-to-market equity ratios (the book value of common stock over its market value)

⁷³Fama and MacBeth (1973, p. 618).

⁷⁴This summary is from Fama and French (2004, p. 35).

earned higher returns than stocks with lower book-to-market ratios, controlling for betas. Chan, Hamao, and Lakonishok (1991) came to the same conclusion for Japanese stocks. Note that all of these new factors (size, P/E, leverage) contain the stock price which reflects the expected future returns,⁷⁵ so the fact that these factors explain returns should not be seen as a big surprise.

Fama and French (1992) provided an extensive test on the role of beta, size, book-to-market (B/M) ratio, leverage, earnings-to-price (E/P) and their combinations in explaining average returns. The study covered the period July 1963–December 1990 and confirmed the results of the other studies mentioned above.

The authors used market data of non-financial stocks traded on the NYSE, AMEX and NASDAQ which they obtained from the Center for Research in Security Prices (CRSP).⁷⁶ The accounting data was taken from COMPUSTAT, also maintained by CRSP. The 1962 start date reflects the fact that book values were not generally available before. The market portfolio is the value-weighted portfolio of all NYSE, AMEX and NASDAQ stocks.⁷⁷

For examining the effect of beta and size on returns, 12 portfolios were constructed based on pre-ranking betas and sizes. They were rebalanced for July in every year. All portfolios were equally weighted.⁷⁸

Construction of Beta-Based Portfolios

To set up the portfolios for the 1-year period July 1963–June 1964, the securities were ranked based on the beta estimates of the preceding 60 months (July 1958–June 1962). The distribution scheme of the portfolios is shown in Table 2.5, starting with the lowest betas on the left (1A) and ending with the highest betas on the right (10B).

The 5 % of the securities with the lowest beta are in portfolio 1A, the next 5 % in 1B, the following 10 % in portfolio 2, etc.

Table 2.5 Distribution scheme for β -based portfolios from low beta (1A) to high beta (10B) which is also used for size-based portfolios

Portfolio	1A (low)	1B	2	3	4	5	6	7	8	9	10A	10B (high)
Distribution	5 %	5 %	10 %	10 %	10 %	10 %	10 %	10 %	10 %	10 %	5 %	5 %

Source: Fama and French (1992, Table II, pp. 436–437)

⁷⁵This argument was initially used for the E/P ratio in Ball (1978). But it was generalized to other factors in Fama and French (1992, p. 428).

⁷⁶Fama and French (1992, p. 429). Financial stocks were excluded because the high leverage that is normal for these firms probably does not have the same meaning for non-financial firms, where high leverage more likely indicates distress.

⁷⁷Fama and French (1992, p. 431).

⁷⁸Fama and French (1992, p. 431 and p. 433).

Table 2.6 Portfolios formed on pre-ranking β from lowest β (1A) to highest β (10B) with average monthly returns and post-ranking β in the period July 1963–December 1990

Portfolio	1A (low)	1B	2	3	4	5	6	7	8	9	10A	10B (high)
Return	1.20	1.20	1.32	1.26	1.31	1.30	1.30	1.23	1.23	1.33	1.34	1.18
β	0.81	0.79	0.92	1.04	1.13	1.19	1.26	1.32	1.41	1.52	1.63	1.73

Source: Fama and French (1992, Table II, pp. 436–437)

After 1 year, the portfolios for the period July 1964–June 1965 were reconstructed, again based on the beta estimates of the preceding 60 months (July 1959–June 1963). We continued this procedure for every year until 1990 to get the 12 portfolios for the period July 1963–December 1990. Table 2.6 lists the average monthly returns of the portfolios, together with their post-ranking betas which are based on the whole period July 1963–December 1990.⁷⁹

Construction of Size-Based Portfolios

A similar procedure was applied to construct 12 size-based portfolios. To set up the portfolios for the 1-year period July 1963–June 1964, the securities were ranked based on their size (in terms of market equity, or ME, i.e., stock price times number of shares outstanding) at the end of June 1963. The distribution scheme of the size-based portfolios was the same as for the beta-based portfolios shown in Table 2.5, starting with the lowest size on the left (1A) and ending with the highest size on the right (10B).

After 1 year, the portfolios for the period July 1964–June 1965 were reconstructed, based on the sizes at the end of June 1964. This procedure was continued for every year until 1990 to establish the 12 portfolios for the period July 1963–December 1990. Table 2.7 lists the average monthly returns of the portfolios, together with their post-ranking betas which are based on the whole period July 1963–December 1990.⁸⁰

In Table 2.7, we can observe that when forming portfolios based on size, the average return decreases with increasing size (portfolio 1A contains the smallest companies, 10B the largest), from 1.64 % per month for the smallest-size portfolio to 0.90 for the largest. This shows the *size effect*, i.e., size is negatively related to return. We can also see that with increasing size, β decreases like the return, from 1.44 for the smallest-size portfolio to 0.90 for the largest, suggesting a strong positive relationship between return and β . But for portfolios based on pre-ranked betas, Table 2.6 shows a rather flat relationship between return and β : While the

⁷⁹The betas are calculated based on the monthly returns which were realized by the portfolios after they were set up. They have nothing to do with the beta used for the pre-ranking.

⁸⁰The beta is calculated based on the monthly returns which were realized by the portfolios after they were set up. They have nothing to do with the beta used for the pre-ranking.

Table 2.7 Portfolios formed on size, from lowest size (1A) to highest size (10B) with average monthly returns and post-ranking β in the period July 1963–December 1990

Portfolio	1A (low)	1B	2	3	4	5	6	7	8	9	10A	10B (high)
Return	1.64	1.16	1.29	1.24	1.25	1.29	1.17	1.07	1.10	0.95	0.88	0.90
β	1.44	1.44	1.39	1.34	1.33	1.24	1.22	1.16	1.08	1.02	0.95	0.90

Source: Fama and French (1992, Table II, p. 436)

betas range from 0.81 to 1.73 (and this range of betas is larger than the range of betas for the size-based portfolios), the range of average returns is quite small, from 1.18 to 1.34 % per month. The lowest-beta portfolio (1A) even shows a higher return (1.20 %) than the highest-beta portfolio (10B) (1.18 %). So a variation in β which is tied to size is positively related to return, while a variation in β alone does not explain a variation of returns.

To examine the pure effect of β on returns without any size effects on β , we form 100 size- β portfolios.

Construction of Size- β Portfolios

For the end of June 1963, we rank the stocks based on size (market equity). We divide the stocks in ten size groups based on data for the end of June 1963, i.e., ME-1 contains the smallest 10 % stocks, ME-2 the next smallest 10 %, etc. Next, for each of the size groups we form ten portfolios based on the pre-ranking β (calculated on the basis of the preceding 60 months, i.e., July 1958–June 1963). For example, within the size group ME-2, we put the 10 % lowest-beta stocks into portfolio ME-2/ $\beta - 1$, the next 10 % into ME-2/ $\beta - 2$, etc. We hold these portfolios for the period July 1963–June 1964.

For the end of June 1964 the portfolios are rebalanced: We rank the stocks based on size at the end of June 1964, divide them in ten size groups, and for each size group form ten portfolios based on the pre-ranking β (calculated on the basis of the preceding 60 months, i.e., July 1959–June 1964). We hold these portfolios for the period July 1964–1965.

We repeat this process year after year and set up 100 portfolios (ten in each size group) with different size-beta characteristics for the period June 1963–December 1990. Since the ten portfolios from the same size group have a wide range of betas, but similar stock sizes, we can always use the ten portfolios from the same size group to test the effects of β which are unrelated to size. Table 2.8 shows the post-ranking betas and Table 2.9 shows the average monthly returns (in %) of the 100 size- β portfolios. Each row in these tables represents a certain size group, and when we read the numbers within a row from left to right, we can observe the results for increasing betas within a size group.

We can see that the betas in Table 2.8 increase from left to right: This is how we constructed the portfolios. In Table 2.9 when reading each row from left to right, we can see how increasing beta—while keeping size constant—effects returns:

Table 2.8 Post-ranking betas of the size- β portfolios formed on size (down) and then β (across) in the period July 1963–December 1990

	β -1 (low)	β -2	β -3	β -4	β -5	β -6	β -7	β -8	β -9	β -10 (high)
	Post-ranking β s									
ME-1 (small)	1.05	1.18	1.28	1.32	1.40	1.40	1.49	1.61	1.64	1.79
ME-2	0.91	1.15	1.17	1.24	1.36	1.41	1.43	1.50	1.66	1.76
ME-3	0.97	1.13	1.13	1.21	1.26	1.28	1.39	1.50	1.51	1.75
ME-4	0.78	1.03	1.17	1.16	1.29	1.37	1.46	1.51	1.64	1.71
ME-5	0.66	0.85	1.12	1.15	1.16	1.26	1.30	1.43	1.59	1.68
ME-6	0.61	0.78	1.05	1.16	1.22	1.28	1.36	1.46	1.49	1.70
ME-7	0.57	0.92	1.01	1.11	1.14	1.26	1.24	1.39	1.34	1.60
ME-8	0.53	0.74	0.94	1.02	1.13	1.12	1.18	1.26	1.35	1.52
ME-9	0.58	0.74	0.80	0.95	1.06	1.15	1.14	1.21	1.22	1.42
ME-10 (large)	0.57	0.71	0.78	0.89	0.95	0.92	1.02	1.01	1.11	1.32

Source: Fama and French (1992, Table I, p. 435)

Table 2.9 Average monthly returns of the size- β portfolios formed on size (down) and then β (across) in the period July 1963–December 1990

	β -1 (low)	β -2	β -3	β -4	β -5	β -6	β -7	β -8	β -9	β -10 (high)
	Average monthly return (in %)									
ME-1 (small)	1.71	1.57	1.79	1.61	1.50	1.50	1.37	1.63	1.50	1.42
ME-2	1.25	1.42	1.36	1.39	1.65	1.61	1.37	1.31	1.34	1.11
ME-3	1.12	1.31	1.17	1.70	1.29	1.10	1.31	1.36	1.26	0.76
ME-4	1.27	1.13	1.54	1.06	1.34	1.06	1.41	1.17	1.35	0.98
ME-5	1.34	1.42	1.39	1.48	1.42	1.18	1.13	1.27	1.18	1.08
ME-6	1.08	1.53	1.27	1.15	1.20	1.21	1.18	1.04	1.07	1.02
ME-7	0.95	1.21	1.26	1.09	1.18	1.11	1.24	0.62	1.32	0.76
ME-8	1.09	1.05	1.37	1.20	1.27	0.98	1.18	1.02	1.01	0.94
ME-9	0.98	0.88	1.02	1.14	1.07	1.23	0.94	0.82	0.88	0.59
ME-10 (large)	1.01	0.93	1.10	0.94	0.93	0.89	1.03	0.71	0.74	0.56

Source: Fama and French (1992, Table I, p. 434)

The relationship between average monthly returns and betas is flat. More surprisingly, in each size group, the lowest-beta portfolio (β -1) earned a higher return than the highest-beta portfolio (β -10)! For example, within the size group ME-5, the lowest-beta portfolio (ME-5/ β -1) earned 1.34 % per month and outperformed the highest-beta portfolio (ME-5/ β -10) with 1.08 % per month.

The overall conclusion about beta is “*that variation in beta that is tied to size is positively related to return, but variation in beta unrelated to size is not compensated*”

in the average returns in 1963–1990”.⁸¹ Fama and French did the same test on NYSE stocks in the 50-year period 1941–1990 and found the same result: There is a “reliable size effect [...] but little relation between beta and average return”. On the other hand, there was a positive relation between beta and average return in the period 1941–1965, which is exactly the period which was used in the early CAPM tests. But “even for the 1941–1965 period, however, the relation between beta and average return disappears when we control for size.”⁸² Due to this striking discovery that beta appears to have no relation to average returns, the paper raised the question “Can β Be Saved?”⁸³

Construction of B/M-based Portfolios

By constructing size-based portfolios, we have observed a negative relation between size and average return. We apply the same approach to examine the relation between the book-to-market ratio and average returns. To set up portfolios for the 1-year period July 1963–June 1964, we rank the stocks based on the book-to-market value at the end of December 1962 using accounting data from the latest fiscal year.⁸⁴ The stocks are assigned to the portfolios based on the scheme explained in Table 2.5, from smallest B/M (1A) to highest (10B). After 1 year, the portfolios are rebalanced for the period July 1964–June 1965 based on the book-to-market value at the end of December 1963. This procedure is continued for every year until 1990.

According to Table 2.10, the average monthly return of portfolios based on B/M increases with increasing B/M ratio, from 0.30 % per month for the smallest B/M ratio to 1.83 % per month for the largest B/M ratio. The difference between the largest and the smallest B/M portfolio is 1.53 % and even twice as large as the difference between the largest- and smallest-size portfolio of 0.74 % (see Table 2.7). Note that beta stays flat across different B/M ratios, so the difference in return cannot be explained by beta!

Table 2.10 Portfolios formed on book-to-market (B/M) ratio, from lowest B/M (1A) to highest B/M (10B) with average monthly returns and post-ranking β of the portfolios

Portfolio	1A (low)	1B	2	3	4	5	6	7	8	9	10A	10B (high)
Return	0.30	0.67	0.87	0.97	1.04	1.17	1.30	1.44	1.50	1.59	1.92	1.83
β	1.36	1.34	1.32	1.30	1.28	1.27	1.27	1.27	1.27	1.29	1.33	1.35

Source: Fama and French (1992, Table IV, p. 442)

⁸¹Fama and French (1992, p. 433).

⁸²Fama and French (1992, p. 440).

⁸³Fama and French (1992, p. 439).

⁸⁴Fama and French (1992, Table IV, p. 442).

Fama–MacBeth Regression

The Fama–MacBeth regression serves to determine the effects of beta, size, leverage and E/P ratio (as single variables and also in combination) on average return. Fama and French (1992) describes the regression of monthly returns of stocks over β , $\ln(ME)$ (ME = market value of common equity in millions of dollars, as a proxy for size), $\ln(BE/ME)$ ratio (BE = book value of common equity), $\ln(A/ME)$ and $\ln(A/BE)$ (as proxies for leverage, A = value of total assets) and E/P ratio. The regression coefficients were averaged over the period July 1963–December 1990. In particular, for each of the 12 months in the period July 1963–June 1964, the monthly returns of the stocks were regressed over

- the post-ranking beta of the size- β portfolio to which the stock was assigned at the end of June 1963. This was used as an approximation of the stock's beta because it is much more stable than the individual stock's beta.
- the size (measured in $\ln(ME)$) where the market value was taken as of end of June 1963.
- leverage, BE/ME and E/P ratio: the accounting variables were taken from the fiscal year ending in December 1962.

After 12 months, the data was updated. For the period July 1964–June 1965, the monthly returns of the stocks were regressed over post-ranking beta at the end of June 1964, size at the end of June 1964, and leverage, $\ln(BE/ME)$ and P/E ratio from the fiscal year ending in December 1963. This process was repeated for every year and the result was averaged. The conclusions of the Fama–MacBeth regression are that

- beta⁸⁵ does not help explaining average stock returns in 1963–1990, neither alone nor in combination with size, leverage and E/P ratio.
- as single explanatory variables, size⁸⁶ and total assets-to-book ratio⁸⁷ are negatively related to average returns, whereas book-to-market ratio,⁸⁸ total assets-to-market ratio⁸⁹ and earnings-to-price ratio⁹⁰ are positively related to average returns.
- when combining all explanatory variables, the size and book-to-market effect turn out to be most significant, making the other variables redundant. The book-to-market effect is even more powerful than the size effect.⁹¹

⁸⁵Fama and French (1992, p. 438).

⁸⁶Fama and French (1992, p. 438).

⁸⁷Fama and French (1992, pp. 441–442).

⁸⁸Fama and French (1992, p. 441).

⁸⁹Fama and French (1992, pp. 441–442).

⁹⁰Fama and French (1992, pp. 442–443).

⁹¹Fama and French (1992, p. 440).

Other Empirical Tests and the Revival of Beta

As a response to the studies which showed that the relationship between return and beta is flat, Pettengill, Sundaram, and Mathur (1995) argued that previous tests of the CAPM were flawed.⁹² They offered a modification: Although the CAPM predicts that the *expected* return should be positively related to beta, this should not be true for all *realized* returns. When markets go up, then high-beta assets overperform low-beta assets. But when the market goes down, then assets with high beta (and therefore with higher risk) should underperform conservative low-beta assets. This kind of behavior of high-beta stocks illustrated why they carry higher risk than low-beta stocks. Previous tests like Fama and French (1992) which only looked for a general positive relationship between beta and returns did not take this implication of the CAPM into account. The empirical test by Pettengill et al. (1995) which covered U.S. stocks (which were available in the Center for Research in Security Prices (CRSP) monthly returns file) in 1936–1990 used a slightly different approach: They tested the relationship between beta and return for up markets (i.e., months where the monthly return of the market portfolio exceeded the monthly risk-free rate) and down markets (i.e., months where the monthly returns of the market portfolio was below the monthly risk-free rate) separately. The sensitivity of returns to up markets/down markets is also known as *bull beta* and *bear beta*, see also Sect. 1.3.6.

The empirical result is that during the period 1936–1990 and also during the subperiods 1936–1950, 1951–1970, 1971–1990, the relationship between beta and return was positive in up markets and negative in down markets.⁹³ When running the usual CAPM test without distinguishing down and up markets, Pettengill et al. (1995) found a positive relationship between beta and return only in the subperiod 1936–1950, but a flat relationship in the subperiods 1951–1970 and 1971–1990. The authors suggested that these “*results are biased due to the aggregation of positive and negative market excess return periods*”.⁹⁴ The separate CAPM tests for up markets and down markets became increasingly popular and this method was used in several other publications (see also Table 2.11).

2.3.4.5 Roll's Critique: The Market Portfolio Problem

Researchers have done several empirical tests which have supported and rejected implications of the CAPM. But in Roll (1977), Richard Roll⁹⁵ questions their validity because the tests only used stocks (or stock market indices like the S&P

⁹²Pettengill et al. (1995, pp. 102–104) for the argument.

⁹³Pettengill et al. (1995, p. 110).

⁹⁴Pettengill et al. (1995, p. 109).

⁹⁵Richard Roll (born October 31, 1939) is an American economist, best known for his work on portfolio theory and asset pricing, both theoretical and empirical. In 1968, he received his Ph.D. from the Graduate School of Business at the University of Chicago in economics, finance, and statistics. In 1976, Roll joined the faculty at UCLA, where he remains as Japan Alumni Chair Professor of Finance. In 1987, Roll was elected President of the American Finance Association.

500) to represent the whole market. However, the market portfolio of the CAPM should contain every risky asset in the market, including stocks, bonds, options, real estate, coins, stamps, art, antiques,⁹⁶ and also human capital.⁹⁷ Since a lot of assets have been ignored, Roll (1977) argues that the CAPM has never been tested.

The trouble is that the zero-beta CAPM

$$R_{Pf} = \mathbb{E}[R_Z] + \beta_{Pf} \cdot (\mathbb{E}[R_m] - \mathbb{E}[R_Z]) \quad (2.43)$$

holds for *any* mean-variance efficient portfolio m and vice versa.

The zero-beta CAPM is tautological to the market portfolio Mkt being mean-variance efficient, and so are the implications (C1)–(C3) from page 123 (linearity in beta, no non-beta risk, positive beta premium) which were empirically tested. So the hypotheses (C1)–(C3) are not independently testable, and it just comes down to the question if the market portfolio is mean-variance efficient or not. This cannot be answered by using stock-only market proxies, and the market is unobservable. Roll argues in Roll (1977, p. 130): “*The theory is not testable unless the exact composition of the true market portfolio is known and used in the tests. This implies that the theory is not testable unless all individual assets are included in the sample.*”⁹⁸

The equation

$$R_{Pf} = \mathbb{E}[R_Z] + \beta_{Pf} \cdot (\mathbb{E}[R_m] - \mathbb{E}[R_Z]) \quad (2.44)$$

holds for *any* mean-variance efficient portfolio m .⁹⁹ This is a mathematical fact without any model assumptions.¹⁰⁰ Even if the CAPM hypotheses (C1)–(C3) are positively tested for a market proxy m , this only implies that the market proxy m is mean-variance efficient, but the true market portfolio might be still inefficient (therefore rendering CAPM invalid), leading to the wrong validation of CAPM. Or the CAPM hypotheses (C1)–(C3) are negatively tested with a wrong proxy m (which is inefficient), while the true market portfolio is efficient, leading to a wrong rejection of CAPM. Therefore, CAPM tests with wrong market proxies are inconclusive.

2.3.5 Evaluation of the CAPM

Let us summarize the results of the empirical tests: Early tests (Black et al. 1972; Douglas 1969; Miller and Scholes 1972) rejected the hypothesis (C4) that zero-beta

⁹⁶This list of examples is mentioned in Reilly and Brown (1997, p. 284).

⁹⁷Richard Roll mentions human capital as part of the market portfolio in Roll (1977, p. 131 and p. 155).

⁹⁸Roll (1977, p. 136).

⁹⁹ β_{Pf} is the portfolio beta measured relative to the portfolio m . The zero-beta asset Z has zero beta relative to m .

¹⁰⁰Roll (1977, p. 130 and p. 136).

Table 2.11 Empirical tests of the capital asset pricing model

Study	Data used	Results
Douglas (1969): <i>Risk in the Equity Markets: An Empirical Appraisal of Market Efficiency</i>	1954–1963: All 301 stocks from the S&P 425, for which all the data were in the broader study where available for all years. ^a	The test showed that <ul style="list-style-type: none">• zero-beta return was greater than risk-free rate.• beta premium was positive, but lower than market excess return.• the residual from the regression which was used to estimate beta had a significant positive impact on return.
Friend and Blume (1970): <i>Measurement of Portfolio Performance under Uncertainty</i>	January 1960–June 1968: All stocks from the New York Stock Exchange (NYSE)	For the whole period January 1960–June 1968, the rate of return was positively related to beta, but the Jensen alpha and other performance measures like Sharpe ratio and Treynor ratio were negatively related to beta. However, in the subperiod April 1964–June 1968 the performance measures and beta were positively related.
Miller and Scholes (1972): <i>Rates of Return in Relation to Risk: A Reexamination of Some Recent Findings</i>	1954–1963: 631 stocks from the New York Stock Exchange (NYSE)	The test showed that <ul style="list-style-type: none">• zero-beta return was greater than risk-free rate.• beta premium was positive, but lower than market excess return.• the residual from the regression which was used to estimate beta had a significant positive impact on return. The authors mention two possible explanations for that: positive association between systematic risk and residual risk, skewness of the underlying distribution of returns.
Black et al. (1972): <i>The Capital Asset Pricing Model: Some Empirical Tests</i>	January 1931–December 1965: All stocks from the New York Stock Exchange (NYSE)	In the period January 1931–December 1965, high-beta portfolios ($\beta > 1$) earned negative alphas while low-beta portfolios ($\beta < 1$) earned positive alphas. In the same period, there was also a positive linear relation between beta and the rate of return. This period was divided in four equidistant subperiods for further study: In January 1931–September 1939, the rate of returns for zero-beta

Fama and MacBeth (1973): <i>Risk, Return, and Equilibrium: Empirical Tests</i>	January 1926–June 1968: All stocks from the New York Stock Exchange (NYSE)	<p>assets was below the risk-free rate and the empirical CAPM line was steeper than the theoretical line. Over time, the empirical CAPM line became flatter and even had a negative slope in the subperiod April 1957–December 1965.</p> <p>The test showed that</p> <ul style="list-style-type: none">• beta premium was positive.• beta squared showed no explanatory power, therefore supporting the linearity between beta and return.• unsystematic risk showed no explanatory power, therefore supporting the hypothesis that no variable other than beta explains returns.
Blume and Friend (1973): <i>A New Look at the Capital Asset Pricing Model</i>	January 1955–June 1968: All stocks from the New York Stock Exchange (NYSE)	<p>The test showed that the returns of zero-beta portfolios assets were different from the risk-free rate. It confirmed the linearity between beta and the rate of return by showing that beta squared had no explanatory power.</p>
Basu (1977): <i>The Investment Performance of Common Stocks in Relation to their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis</i>	April 1957–March 1971: All stocks from the New York Stock Exchange (NYSE) from firms with December as the fiscal-year end	<p>High P/E stocks had lower returns, low P/E stocks had higher returns than predicted by the CAPM.</p>
Reinganum (1981): <i>A New Empirical Perspective on the CAPM</i>	January 1964–December 1979: All stocks from the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX)	<p>No significant relationship between beta and return.</p>
(continued)		

Table 2.11 (continued)

Study	Data used	Results
Banz (1981): <i>The Relationship Between Returns and Market Value of Common Stock</i>	January 1936–December 1975: All stocks from the New York Stock Exchange (NYSE)	Small size firms had higher returns, large size firms had lower returns than predicted by the CAPM.
Stambaugh (1982): <i>On the Exclusion of Assets from Tests of the Two-Parameter Model</i>	February 1953–December 1976: All stocks from the New York Stock Exchange, four preferred stocks and five bond portfolios	Confirmed linearity between beta and return. Beta premium is positive, zero-beta return is different from risk-free rate. The empirical test was done with different market proxies, yielding similar results and the same conclusions about CAPM.
Rosenberg et al. (1985): <i>Persuasive Evidence of Market Inefficiency</i>	January 1973–September 1984: 1,400 of the largest U.S. companies in the COMPUSTAT database, containing largely stocks from the New York Stock Exchange (NYSE), and a few from other exchanges like the AMEX and NASDAQ	High P/E stocks had higher returns than low P/E stocks, controlling for betas.
Lakonishok and Shaprio (1986): <i>Systematic Risk, Total Risk and Size as Determinants of Stock Market Returns</i>	January 1962–December 1981: All stocks from the New York Stock Exchange (NYSE)	Among the three variables—systematic risk, total risk and size—only the size matters when describing returns. If January data are excluded, even size loses its significance. When considering up markets/down markets only, the average return was positively/negatively related to beta.
Bhandari (1988): <i>Debt/Equity Ratio and Expected Common Stock Returns</i>	January 1948–December 1979: All stocks from the New York Stock Exchange (NYSE)	Debt/equity (more precisely: debt/market equity) ratio was positively related to returns, even when controlling for size and beta. The study was also performed on manufacturing firms to remove biases from high-leverage industries like finance. Here, the positive effect of the debt/equity ratio on return

Chan et al. (1991): <i>Fundamentals and Stock Returns in Japan</i>	January 1971–December 1988: Stocks from the Tokyo Stock Exchange (TSE)	<p>Four explanatory variables for returns were tested: size, book/market ratio, earnings/price ratio, cash flow/price ratio. Book/market and cash flow/price ratios had a positive impact on returns. Earnings/price ratio showed little impact after controlling for other variables. The impact of size depended on the regression model used.</p>	<p>was even larger. Size had a negative impact on return, but only in January. The relationship between return and beta was positive only in January.</p>
Fama and French (1992): <i>The Cross-Section of Expected Stock Returns</i>	July 1963–December 1990: Non-financial stocks which were traded on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and NASDAQ	<p>The following explanatory variables for returns were tested: beta, size, book/market ratio, leverage (total assets/book equity and total assets/market equity ratio), earnings/price ratio. Conclusions:</p> <ul style="list-style-type: none">• Beta does not explain returns.• As single explanatory variable, size and total assets/book equity are negatively related to average returns, whereas book/market, total assets/market equity and earnings/price ratios are positively related to average returns.• Of all explanatory variables, size and book/market ratio turn out to be most significant, making the other variables redundant. Book/market ratio has a stronger effect on returns than size.	
Herrera and Lockwood (1994): <i>The Size Effect in the Mexican Stock Market</i>	January 1987–December 1992: All stocks from the Mexican stock exchange and similar stocks from the NASDAQ. To create the sample of NASDAQ stocks, one NASDAQ stock with similar industry characteristics was selected for each Mexican stock.	<p>For Mexican stocks and the NASDAQ sample, a positive beta premium and a negative size premium were found, as single explanatory variables and when both beta and size are examined together.</p>	

(continued)

Table 2.11 (continued)

Study	Data used	Results
Pettengill et al. (1995): <i>The Conditional Relation between Beta and Returns</i>	January 1936–December 1990: U.S. stocks from the Center for Research in Security Prices (CRSP) monthly returns file	In up markets/down markets, the relation between beta and average returns was positive/negative for the periods 1936–1950, 1951–1970 and 1971–1990. Without distinguishing between up markets and down markets, a positive relationship between beta and average returns was found in 1936–1950, while it was flat in the periods 1951–1970 and 1971–1990.
Strong and Xu (1997): <i>Explaining the Cross-Section of U.K. Expected Stock Returns</i>	July 1960–June 1992: U.K. stocks from the London Share Price Database (LSDP). For the period in July 1973–June 1992 where accounting data were also used, only stocks from companies on both Exstat (which contains accounting data) and LSDP databases were included.	For July 1960–June 1992, the study found a positive beta premium and negative size premium (as single explanatory variables). For the time period of July 1973–June 1992 the same effects for beta and size were observed. Furthermore, as single explanatory variables, the premium on total assets/book equity ratio was negative, whereas the premiums for book/market and total assets/market equity were positive. The earnings/price ratio had a positive, albeit not significant, effect on price. The book/market ratio and leverage (total assets/book equity and total assets/market equity ratio) had significant explanatory power in combination with other variables.
Fletcher (1997): <i>An Examination of the Cross-Sectional Relationship of Beta and Return: U.K. Evidence</i>	January 1975–December 1994: U.K. stocks from the London Stock Exchange and Unlisted Securities Market, from the London Business School Share Price Database (LBS)	In 1975–1994 and in both subperiods 1975–1984 and 1985–1994, the relationship between beta and return was flat. When looking at up markets/down markets only, there was a positive/negative relationship between beta and return in 1975–1994 and in both subperiods 1975–1984 and 1985–1994. The negative beta premium in down markets was higher than the positive beta premium in up markets.

Chui and Wei (1998): <i>Book-to-market, Firm Size, and the Turn-of-the-Year Effect: Evidence from Pacific-Basin Emerging Markets</i>	Test on the stock markets of five countries: Malaysia, Taiwan (both July 1981–June 1993), Korea (July 1982–June 1993), Hong Kong (July 1984–June 1993) and Thailand (July 1988–June 1993)	<p>The regressions on portfolios showed that the relationship between beta and return was flat for all markets. The negative size effect was significant only for Korea. Book/market ratio and return had a significantly positive relationship only for Hong Kong, Korea and Malaysia. The regressions on individual stocks showed the same conclusion regarding beta and book/market ratio. The negative size effect was significant for Hong Kong, Korea, Malaysia and Thailand. The tests showed opposite seasonal patterns of the size effect in Hong Kong and Korea: While in Hong Kong the size premium in January was positive (i.e., larger firms earned higher returns) and negative in the other months, the negative size effect in Korea was concentrated in January.</p>
Heston, Rouwenhorst, and Wessels (1999): <i>The Role of Beta and Size in the Cross-Section of European Stock Returns</i>	January 1978–December 1995: 2100 stocks from 12 European countries: Austria, Belgium, Denmark, France, Germany, Italy, Netherlands, Norway, Spain, Sweden, Switzerland and U.K., covering 60–90 % of each country's market capitalization	<p>Average return is positively related to beta, but this is partly due to the fact that high-beta countries overperform low-beta countries. Within countries, this positive relation vanishes and can only be observed in January. The size effect can be observed in the Europe-wide test and in 5 (out of 12) countries. The value-weighted MSCI index of the 12 sample countries was used as a proxy for the market portfolio.</p>
Fletcher (2000): <i>On the Conditional Relationship between Beta and Return in International Stock Returns</i>	January 1970–July 1998: Morgan Stanley Capital International (MSCI) World Equity Index and 18 MSCI developed markets country indices	<p>The CAPM was tested on the 18 country portfolios of the developed markets,^b represented by the respective MSCI country portfolios. The MSCI World Index was used as a proxy for the market portfolio. The test shows no significant relationship between beta and average return in January 1970–July 1998. But when testing up market and down market months only, beta shows a significant positive relationship with average returns in up markets and negative relationship in down markets.</p>

(continued)

Table 2.11 (continued)

Study	Data used	Results
Hodoshima, Garza-Gómez, and Kunimura (2000): <i>Cross-Sectional Regression Analysis of Return and Beta in Japan</i>	January 1956–December 1995: Japanese stocks from the first section (large-cap) of the Tokyo Stock Exchange	Empirical tests were run for the entire period 1956–1995 and for the subperiods 1956–1965, 1966–1975, 1976–1985 and 1986–1995. In the entire period and all subperiods, the relation between beta and return is flat. But in up markets/down markets only, there is a positive/negative relation between beta and return, for 1956–1995 and for all subperiods. When considering beta together with size and book/market ratio in the period July 1962–December 1995, then size is the only significant variable with negative premium. In up markets, beta has a positive, size has a negative effect on return, book/market ratio is insignificant. In down markets, the size effect becomes insignificant while beta has a negative effect on return, book/market ratio is insignificant. In January, all three explanatory variables size (negative), beta and book/market (positive) are significant in explaining returns, while in non-January months, none of these are significant.
Faff (2001): <i>A Multivariate Test of a Dual-Beta CAPM: Australian Evidence</i>	January 1974–January 1995: Australian stocks from the Price Relatives File of the Centre for Research in Finance (CRIF) at the Australian Graduate School of Management	The relation between beta and return was flat. In up markets/down markets only, the relation between beta and returns was positive/negative. There is minimal evidence of a difference between up market beta and down market beta.

Elsas, El-Shaer, and Theissen (2003): <i>Beta and Returns Revisited: Evidence from the German Stock Market</i>	January 1968–December 1995: Stocks from Germany. Sample size ranges from 211 to 316 stocks.	Over the period 1968–1995, the relation between beta and return was flat. When considering up markets/down markets only, the relation between beta and return was positive/negative.
Wang and Xu (2004): <i>What Determines Chinese Stock Returns?</i>	July 1996–June 2002: A-shares ^c (which are available only to domestic investors, as opposed to B-shares which are for international investors) from the Chinese stock market, traded on the Shanghai Stock Exchange or the Shenzhen Stock Exchange, from the China Stock Market and Accounting Research Database	The relationship between beta and return was flat. Size had a significant negative effect on return. Book/market ratio was found to be insignificant. The paper also tested free-float (percentage of shares which are tradable in the stock markets) as a proxy for good governance which is an important aspect in a country where companies are mostly government-controlled. The study found a positive relationship between free-float and return, when taking into account the other variables beta, size and book/market ratio.

^a Lintner (1965b, p. 611) for the data used in the study originally done by J. Lintner
^b Listed on Fletcher (2000, p. 238): The 18 countries are Australia, Austria, Belgium, Canada, Denmark, France, Germany, Hong Kong, Italy, Japan, Netherlands, Norway, Singapore, Spain, Sweden, Switzerland, U.K. and USA
^c See also Wang and Xu (2004, pp. 65–66) for the definition of A-shares and B-shares

returns equal the risk-free rate. This led to the introduction of a modified version of the CAPM, the zero-beta CAPM (see Black 1972).

Fama and MacBeth (1973) tested the linearity between return and beta and non-beta risk as explanatory variable for returns. Unsystematic risk was provided as a proxy for non-beta risk. The Fama–MacBeth regression, which provided an important framework for future tests, supported the linearity (C1) and beta as the only variable with explanatory power for returns (C2). The linearity assumption was also supported in other tests, like Blume and Friend (1973) and Stambaugh (1982).

Early studies declared a positive beta effect on expected returns (C3) in the U.S. stock market, as shown in Black et al. (1972) for the period 1931–1965 and in Fama and MacBeth (1973) for the period 1935–1968. But Black et al. (1972) also revealed that the relation between average return and beta flattened over time. Doubts about the role of beta in explaining returns were articulated in Reinganum (1981), when no significant relationship between beta and the returns of NYSE/AMEX stocks was found in the period 1964–1979. The same result was obtained for NYSE stocks in the period 1962–1981 in Lakonishok and Shapiro (1986). Instead of beta, other variables like P/E ratio (Basu 1977), size (Banz 1981), leverage (Bhandari 1988), and book-to-market ratio (Rosenberg et al. 1985) were found to play significant roles in explaining returns.

Fama and French (1992) raised the question “*Can β be Saved?*” when the authors discovered that beta played no role in explaining average returns for the 50-year period 1941–1990. They showed that even in the 1941–1965 period, where authors of the early CAPM tests had agreed that the beta premium was significant, the relationship between beta and return vanished when controlling for size.

Roll (1977) argued that the empirical tests were not a test of the CAPM which requires the true market portfolio. The hypotheses (C1)–(C3) are true for any mean-efficient portfolio used as a market proxy, because of a mathematical tautology and without any model assumptions.

The conclusion of the tests is that the CAPM “*never has been an empirical success*”.¹⁰¹ The zero-beta CAPM had some success until other variables like size and price ratios were discovered which explained average returns. Fama and French (2004, p. 43), states: “*The problems are serious enough to invalidate most applications of the CAPM*”.¹⁰² CAPM estimates for high-beta stocks are too high, and estimates for low-beta stocks turned out to be too low. The risk-adjusted performance measure α , introduced in Jensen (1967), which is the return in excess of the CAPM-implied return, turns out to be larger for small-beta portfolios and smaller for large-beta portfolios. Funds could simply concentrate on stocks with low beta, small size and high B/M and earn positive alpha without special stock picking skills.¹⁰³

¹⁰¹Fama and French (2004, p. 43).

¹⁰²Fama and French (2004, pp. 43–44).

¹⁰³Fama and French (2004, p. 44).

Pettengill et al. (1995) revived the importance of beta, suggesting that previous tests of the relation between beta and return have been biased due to the aggregation of up markets and down markets. By testing up markets and down markets separately, they found significant relationships (i.e., positive for up markets and negative for down markets) between beta and return.

Table 2.11 shows a summary of the empirical tests.

2.3.6 A Critical View of the CAPM Assumptions

The CAPM has been criticized for its unrealistic assumptions (listed in Sect. 2.3.1). We will now discuss some relaxations of these assumptions which lead to slight modifications of the model. This section is based on Reilly and Brown (1997, pp. 305–309).

2.3.6.1 Zero-Beta CAPM

The zero-beta CAPM was derived in Black (1972). It was postulated in Black et al. (1972) after empirical tests rejected the hypothesis that the return of a zero-beta asset is the risk-free rate.

The zero-beta CAPM makes all the assumptions from the original CAPM except (A3), i.e., that all investors can lend and borrow any amount of money at the risk-free rate.

By combining different assets (since shorting is allowed), several portfolios can be created with zero beta. Let Z be the zero-beta portfolio with minimal variance. Then all portfolios plot on a security market line (SML) which connects the portfolios Z and Mkt (market portfolio), as shown in Fig. 2.23.

The equation for the zero-beta CAPM is

$$\mathbb{E}[R_{Pf}] = \mathbb{E}[R_Z] + \beta_{Pf} \cdot (\mathbb{E}[R_{Mkt}] - \mathbb{E}[R_Z]). \quad (2.45)$$

2.3.6.2 Different Borrowing and Lending Rates

The CAPM assumption (A3) that all investors can borrow and lend any amount for the risk-free rate is unrealistic. While investors can lend any amount at the risk-free rate r_{rf} by buying Treasury bills, most investors usually have to pay a higher rate r_b for borrowing. The effect of the different borrowing and lending rates is illustrated in Fig. 2.24. It shows the mean-variance diagram with the efficient frontier, together with the risk-free rate r_{rf} and the borrowing rate r_b . The line segment between r_{rf} – Mkt represents all portfolios which are combinations of the market portfolio and the risk-free asset (i.e., lending at r_{rf}). If it were possible to borrow at r_{rf} , then this line segment would extend beyond the point Mkt . The point K is the tangency point from r_b to the efficient frontier, and this tangency line ends at G . The line segment K – G represents all investment opportunities where the investors borrow at the rate r_b and invest in the portfolio K . The segment Mkt – K on the efficient frontier does not involve any borrowing or lending. The CML, the set of all optimal

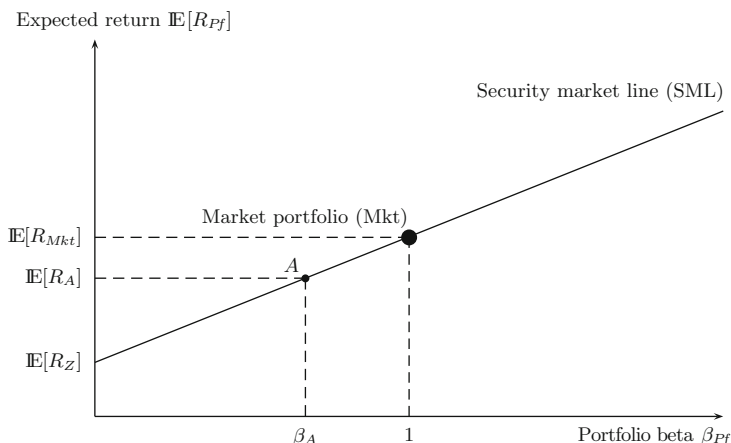


Fig. 2.23 The diagram shows the security market line (SML) without the risk-free asset. The SML connects the market portfolio with the zero-beta portfolio Z. In equilibrium, all assets plot on the SML. *Source:* Own, for illustrative purposes only

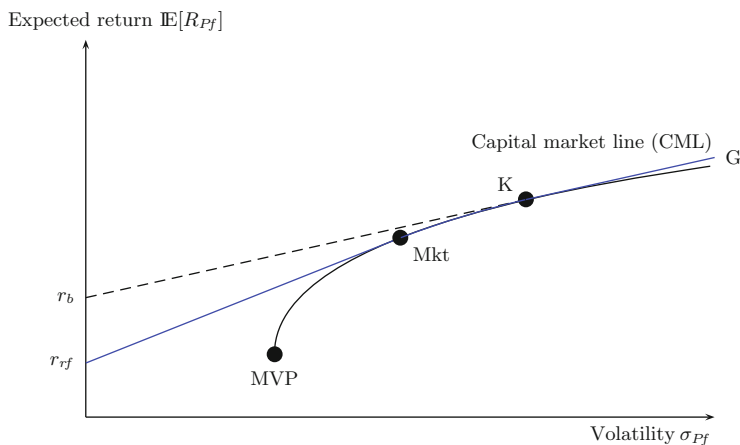


Fig. 2.24 The mean-variance diagram shows the CML when borrowing cost (r_b) is greater than the risk-free rate (r_{rf}). The CML (marked in blue) is made up of $r_{rf} - Mkt - K - G$, i.e., of a line segment $r_{rf} - Mkt$, a curve segment $Mkt - K$ and a line segment $K - G$. *Source:* Own, for illustrative purposes only

investment opportunities, is made of $r_{rf} - Mkt - K - G$, i.e., of a line segment $r_{rf} - Mkt$, a curve segment $Mkt - K$ and a line segment $K - G$.

2.3.6.3 Transaction Costs

The CAPM assumes no transaction costs (A6), which means that investors will even buy or sell securities when they are only slightly mispriced. If a security plots above the SML, then its expected return is higher than its theoretical return implied by the

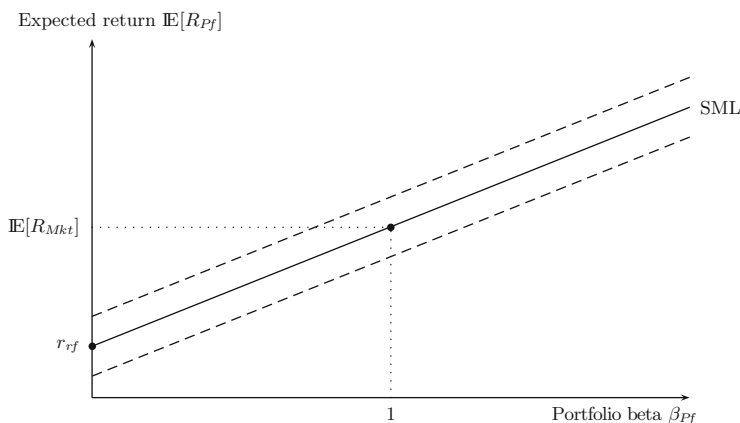


Fig. 2.25 Security market line (SML) with transaction costs. *Source:* Own, for illustrative purposes only

CAPM, i.e., the security price is underpriced. Investors will buy this security and bid up the price until it is fairly valued, i.e., it plots on the SML. If a security plots below the SML, then investors short it until it plots on the SML. With transaction costs, investors will not correct small mispricings when the costs of buying and selling eat up the potential gains. So the SML will rather be a band, as illustrated in Fig. 2.25, and the greater the transaction costs, the wider the band becomes.

2.3.6.4 Heterogeneous Expectations, Investment Horizons and Taxes

The CAPM assumes homogeneous expectations (A1), while in reality, investors have different expectations about risk and returns. If we assume that every investor has his own beliefs, then each one would have a unique CML and/or SML. The composite graph would be a band, and its breadth would reflect the divergence of opinions.

The result is similar when we allow for different investment horizons. The CAPM presupposes the same investment horizon (A4) for all investors, but in reality you have short-time investors who need their money in 1 month and long-term investors saving for their retirement in 30 years. The CAPM is a one-period model, but the investor with a 1-month investment horizon has a different CML/SML from the investor with a 30-year planning period.

The CAPM does not account for taxes (A6), but investors pay different taxes on capital gains and dividends, and rational investors will consider their after-tax returns. Since taxes have an impact on the after-tax return, different taxes will cause different CML/SML among investors.¹⁰⁴

¹⁰⁴For a detailed discussion on taxes, see Black and Scholes (1974) and Litzenberger and Ramaswamy (1979).

2.4 The Fama–French Three-Factor Model

2.4.1 Introduction

After the development of the capital asset pricing model (CAPM) in the 1960s (Treyner in 1961 (see French 2002), Sharpe 1964, Lintner 1965a and Mossin 1966), many empirical tests were developed. Early tests (Black et al. 1972; Douglas 1969; Miller and Scholes 1972) rejected the CAPM and led to a modification of the model, the zero-beta CAPM (Black 1972). Empirical tests in the 1970s (Black et al. 1972; Fama and MacBeth 1973) validated that model, but later on significant doubts were raised about the beta premium (Lakonishok and Shapiro 1986; Reinganum 1981). On the other hand, other factors like P/E ratio (Basu 1977), size (Banz 1981), leverage (Bhandari 1988), and book-to-market ratio (Rosenberg et al. 1985) were found to play significant roles in explaining returns.

In Fama and French (1992), various factors were tested (as single explanatory variables and in combinations). The size and book-to-market ratio were found to be the most significant ones for describing returns. These variables were incorporated into the *Fama–French three-factor model (FF3M)* which is a modification of the CAPM. The big difference between the two is that the CAPM was derived from market portfolio theory with a huge list of idealized assumptions, whereas FF3M is a model developed as a modification of the CAPM to better fit the empirical data.

2.4.2 The Model

The Fama–French three-factor model was introduced in Fama and French (1993) as a modification of the CAPM and included the size and book-to-market ratio as additional factors describing returns. These were found in Fama and French (1992) to be the most significant ones: Small-caps outperformed large-caps and high-B/M stocks outperformed low-B/M stocks.

For the model, we first have to specify a certain stock market. For a stock A , according to the Fama–French three-factor model,¹⁰⁵ the monthly return $R_A^{monthly}$ is

$$\begin{aligned} R_A^{monthly} = & \alpha + r_{rf}^{monthly} + \beta_{1,A} \cdot (R_{Mkt}^{monthly} - r_{rf}^{monthly}) \\ & + \beta_{2,A} \cdot SMB + \beta_{3,A} \cdot HML, \end{aligned} \quad (2.46)$$

where

- $r_{rf}^{monthly}$ is the monthly risk-free rate.
- $R_{Mkt}^{monthly}$ is the monthly return on the portfolio of all stocks (in the specified stock market).

¹⁰⁵For the equation, see Fama and French (1993, p. 24, Table 6). It is also on p. 37, Table 9a, as one of the regression equations studied in Fama and French (1993).

- *SMB* (“small minus big”) is the difference between small-cap and large-cap returns (defined below).
- *HML* (“high minus low”) is the difference between high-B/M and low-B/M returns (defined below).
- α is the component of the return not described by the factors and should be insignificant.

The model would also apply to annual, quarterly, weekly, daily returns, etc., but Fama and French (1993) originally used monthly returns when FF3M was formulated and studied.

In the Fama–French model from Fama and French (1993), the method to calculate *SMB* and *HML* is as follows¹⁰⁶:

All stocks are ranked according to size, and the largest 50 % are put into the *big* group, the smaller 50 % into the *small* group. Independent from that, the stocks are also ranked according to B/M ratio. The highest 30 % end up in the *high*, the middle 40 % in the *medium*, and the lowest 30 % in the *low* group.

Six portfolios are formed from the combinations of these groups (*small/high*, *small/medium*, *small/low*, *big/high*, *big/medium*, *big/low*).

SMB is the difference between the arithmetic average of the monthly returns on the three small-stock portfolios (*small/high*, *small/medium*, *small/low*) and the arithmetic average of the monthly returns on the three big-stock portfolios (*big/high*, *big/medium*, *big/low*). *SMB* describes the difference between a large-cap portfolio and a small-cap portfolio with similar book-to-market equity.

HML is defined similarly: the difference between the arithmetic average of the monthly returns on the high-B/M portfolios (*small/high*, *big/high*) and the arithmetic average of the monthly returns on the low-B/M portfolios (*small/low*, *big/low*). *HML* describes the difference between a high-B/M and a low-B/M portfolio with similar size.

SMB and *HML* are usually positive, since small-caps tend to outperform large-caps and high-B/M portfolios tend to outperform low-B/M portfolios.

The variables β_1 , β_2 and β_3 are determined by a time-series regression¹⁰⁷:

- β_1 is the market sensitivity of the stock, controlling for size and B/M. Note that this is usually different from the CAPM- β which accounts only for the market sensitivity of the stock.
- β_2 is the size coefficient. Smaller companies tend to have larger β_2 s than larger companies.
- β_3 is the B/M coefficient. High-B/M companies tend to have larger β_3 s than low-B/M companies.

¹⁰⁶Fama and French (1993, pp. 8–9). Although the original method in Fama and French (1993) was slightly more complicated, the method we present here will more easily convey the idea behind it.

¹⁰⁷The time-series regression was introduced on page 124 for one independent variable, but can easily be extended to multiple variables.

2.4.3 Theoretical Explanations of the Fama–French Three-Factor Model

The main difference between the capital asset pricing model and the Fama–French three-factor model is that the former has a theoretical foundation while the latter is an ad hoc model which was just introduced because it better fits the empirical data. The reason why Fama and French (1993) introduced FF3M is because a higher return for small-size and high-B/M firms was observed. The theoretical justification from Fama and French (1993) of size and B/M as factors to drive returns was that firms with high B/M or low size tend to have higher earnings, therefore describing risk factors.¹⁰⁸

Other explanations of the value premium (i.e., the effect that high-B/M firms outperform low-B/M firms) stem from behavioral finance¹⁰⁹:

Lakonishok, Shleifer, and Vishny (1994) offers the view that the value premium is due to the overreaction of the markets.¹¹⁰ They overreact to good news and therefore overbuy *glamour* stocks which have performed well in the past. These stocks are overpriced and therefore have low B/M. On the other hand, investors also overreact to bad news and oversell badly performing stocks. These *value* stocks are underpriced and have high B/M. When the market corrects the overreaction, value stocks outperform glamour stocks. De Bondt and Thaler (1985) validates the overreaction hypothesis by pointing out that “losers” overperform “winners”.

One other behavioral view is that investors simply like growth stocks (low B/M, strong firms) and dislike value stocks (high B/M, weak firms).¹¹¹ In this case, the value premium would not be due to risk but to characteristics of value stocks which have the effect of turning investors away. Daniel and Titman (1997) formulates and presents the theory and evidence that the similar characteristics of high-B/M firms rather than the factor loadings from the FF3M drive their returns and explain the high correlation of returns among those.¹¹²

We summarize the theoretical arguments for the size effect from van Dijk (2011, pp. 11–23):

The argument from Daniel and Titman (1997) that characteristics and not factor loadings describe returns also applies to size, contrary to the view in Fama and French (1993) that size describes a risk factor related to financial distress.

Another reason for the size effect is that it includes transaction costs and liquidity risks: Stoll and Whaley (1983) argues that dealers require bigger bid-ask spreads for trading in small firms because of the infrequent trading activity and higher risk.¹¹³

¹⁰⁸Fama and French (1993, pp. 7–8).

¹⁰⁹These explanations can be found in Davis, Fama, and French (2000, pp. 389–390).

¹¹⁰Lakonishok et al. (1994, p. 1542).

¹¹¹Davis et al. (2000, p. 390).

¹¹²Daniel and Titman (1997, pp. 3–4).

¹¹³Stoll and Whaley (1983, p. 58).

For NYSE stocks from 1960–1978, the abnormal returns are shown to be eliminated for investment horizons of less than 1 year.¹¹⁴ Amihud (2002) finds an *illiquidity premium*, i.e., that expected market illiquidity positively affects stock returns.¹¹⁵ Since small firms are more affected by illiquidity, this explains part of the size effect.

Pástor and Stambaugh (2003) explains that sensitivity to market liquidity is priced¹¹⁶ as a risk factor and that the smallest firms tend to be most affected by market illiquidity because their shares are illiquid.¹¹⁷ But on the other hand, the paper contends that the relation between the liquidity of stocks and market liquidity is not straightforward. A drop in market liquidity causes many investors to move from stocks to bonds, therefore selling the most liquid (large-cap) stocks to save transaction costs.¹¹⁸

Last, but not least, there are also explanations of the size effect from behavioral finance.

The overreaction hypothesis which explains the value premium might also be applicable to size, and Chan and Chen (1991) indicates that many small-size firms have performed poorly and lost market value in the past,¹¹⁹ but it seems to be unexplored whether overreaction is a driving factor of the size effect.¹²⁰

One behavioral view of the value premium is that investors like growth stocks and dislike value stocks. The same argument also applies to large-cap stocks vs. small-cap stocks. Gompers and Metrick (2001) found that institutional investors increased the demand for liquid, large-cap stocks in the U.S equity markets in 1980–1996. The shift from individual to institutional ownership led to a disappearance of the size effect in that period.¹²¹ This is an example for the theory of Daniel and Titman (1997) that size as a characteristic and not as a risk factor drives returns.

Market frictions are also found to have impact on returns: Hou and Moskowitz (2005) holds that the delay, i.e., the time needed until the price reflects information, requires a premium. Small firms are most affected by the delay which captures part of the size premium.¹²²

After we have explored the theoretical explanations of the FF3M, we turn to the empirical tests.

¹¹⁴Stoll and Whaley (1983, p. 58).

¹¹⁵Amihud (2002, p. 31).

¹¹⁶Pástor and Stambaugh (2003)

¹¹⁷Pástor and Stambaugh (2003, p. 677).

¹¹⁸Pástor and Stambaugh (2003, p. 677).

¹¹⁹Chan and Chen (1991, pp. 1467–1468).

¹²⁰van Dijk (2011, p. 16).

¹²¹Gompers and Metrick (2001, p. 17).

¹²²Hou and Moskowitz (2005, p. 981).

2.4.4 Empirical Tests

2.4.4.1 Fama and French (1993): Common Risk Factors in the Returns on Stocks and Bonds

After Fama and French (1992) found that size and book-to-market equity were the most significant drivers of returns, the Fama–French three-factor model was developed in Fama and French (1993) which included these factors besides beta. An empirical study was also done in the same paper.

The study uses the same data as Fama and French (1992)¹²³: The market data were from non-financial stocks which were traded on the NYSE, AMEX and NASDAQ and taken from the Center for Research in Security Prices (CRSP).¹²⁴ The accounting data was taken from COMPUSTAT, also maintained by the CRSP. The 1962 start date reflects the fact that book values were not generally available before. The market portfolio is the value-weighted portfolio of all NYSE, AMEX and NASDAQ stocks.¹²⁵ The risk-free rate is the 1-month treasury bill rate.¹²⁶

Calculation of SMB and HML

SMB and *HML* are calculated by dividing the U.S. stocks in *small/big* and *high/medium/low*.¹²⁷ We do these calculations year after year, from July 1963 to June 1964:

At the end of June 1963, all NYSE stocks on CRSP are ranked on size. The median NYSE size splits the NYSE/AMEX/NASDAQ stocks into the groups *small* and *big*. To form the B/M-groups, we determine the breakpoints for top 30 % (*high*), middle 40 % (*medium*) and bottom 30 % (*low*) based on the B/M ratios of the NYSE stocks. We use the accounting data from January 1962 to December 1962 from COMPUSTAT for book equity. We divide it by the market equity at the end of December 1963 to get B/M. Firms with negative book value are excluded from the study.

SMB (small minus big) is the difference between the arithmetic average of the monthly returns on the three small-stock portfolios (*small/high*, *small/medium*, *small/low*) and the arithmetic average of the monthly returns on the three big-stock portfolios (*big/high*, *big/medium*, *big/low*).

HML (high minus low) is the difference between the arithmetic average of the monthly returns on the high-B/M portfolios (*small/high*, *big/high*) and the arithmetic average of the monthly returns on the low-B/M portfolios (*small/low*, *big/low*).

¹²³Fama and French (1993, p. 4) which mentions that it extends the study of Fama and French (1992, using p. 429 for the data).

¹²⁴Financial stocks were excluded because the high leverage that is normal for these firms probably does not have the same meaning for non-financial firms, where high leverage more likely indicates distress.

¹²⁵Fama and French (1992, p. 431).

¹²⁶Fama and French (1993, p. 10).

¹²⁷Methodology from Fama and French (1993, pp. 8–9).

For July 1964–June 1965, we repeat the same method: At the end of June 1964, we form the groups *big/small* based on size and the groups *high/medium/low* based on B/M. To calculate B/M, the book equity data at the end of December 1964 is used. Then *SMB* and *HML* are calculated as above. The whole process is repeated for every year until December 1991.

Construction of Size-B/M Portfolios

The construction of the 25 size-B/M portfolios is similar to the construction of the *small/big* and *high/medium/low* portfolios above, but we use five different size groups and B/M groups (based on the NYSE size and B/M quintiles). By intersecting them we form the 25 size-B/M combinations.¹²⁸ In Table 2.12, you can see the average monthly excess returns (i.e., return minus risk-free rate) of the 25 size-B/M portfolios. They range from 0.32 to 1.05 %. The table shows the negative effect of size and the positive effect of B/M on returns. For all sizes, the returns rise with B/M. For all but the lowest B/M, the returns tend to decrease with size.

Time-Series Regression

Different time-series regressions were done on each of the 25 size-B/M portfolios to compare the explanatory power of the regression of the CAPM against the FF3M. Let

$$\tilde{r}_{Pf}^{monthly} = r_{Pf}^{monthly} - r_f^{monthly} \quad (2.47)$$

be the monthly excess return of the portfolio *Pf* over the risk-free rate $r_f^{monthly}$. The regression which is shown in Table 2.13 explains a big part of the variation of returns, although there is still some room for improvement¹²⁹: The R^2 value is around 0.9 for the big-stock and low-B/M portfolios, but most of the portfolios have an R^2 of about 0.7 to 0.8.

Table 2.12 Average monthly excess returns of the 25 size-B/M portfolios in the period July 1963–December 1991

	<i>B/M</i> -1 (low)	<i>B/M</i> -2	<i>B/M</i> -3	<i>B/M</i> -4	<i>B/M</i> -5 (high)
	Average monthly return (in %)				
Size-1 (small)	0.39	0.70	0.79	0.88	1.01
Size-2	0.44	0.71	0.85	0.84	1.02
Size-3	0.43	0.66	0.68	0.81	0.97
Size-4	0.48	0.35	0.57	0.77	1.05
Size-5 (big)	0.40	0.36	0.32	0.56	0.59

Source: Fama and French (1993, Table 2, pp. 14–15)

¹²⁸Fama and French (1993, p. 8).

¹²⁹Fama and French (1993, p. 19).

Table 2.13 Regression of excess stock returns of the 25 size-B/M portfolios on excess market return in the period July 1963–December 1991

	Regression $\tilde{r}_{Pf}^{monthly} = \alpha + \beta \cdot \tilde{r}_{Mkt}^{monthly}$				
	$B/M-1$				$B/M-5$
	(low)	$B/M-2$	$B/M-3$	$B/M-4$	(high)
	β				
Size-1 (small)	1.40	1.26	1.14	1.06	1.08
Size-2	1.42	1.25	1.12	1.02	1.13
Size-3	1.36	1.15	1.04	0.96	1.08
Size-4	1.24	1.14	1.03	0.98	1.10
Size-5 (big)	1.03	0.99	0.89	0.84	0.89
	R^2				
Size-1 (small)	0.67	0.70	0.68	0.65	0.61
Size-2	0.79	0.79	0.76	0.76	0.71
Size-3	0.84	0.84	0.80	0.79	0.74
Size-4	0.89	0.90	0.87	0.80	0.76
Size-5 (big)	0.89	0.92	0.84	0.79	0.69

Source: Fama and French (1993, Table 4, p. 20)

Table 2.14 shows the results of a linear regression of monthly excess stock returns over monthly excess market return, *SMB* and *HML* over the time period July 1963–December 1991. Compared to the CAPM- β in Table 2.13, the market sensitivities (β_1) change once other factors like *SMB* and *HML* are included in the regression. The β_1 s are much closer to 1 than the β s from Table 2.13. In Table 2.14, we can see that β_2 increases for decreasing size and β_3 tends to increase for increasing B/M. This is exactly how the coefficients are supposed to behave according to the FF3M: Small-caps earn higher returns than large-caps, high-B/M firms earn higher returns than low-B/M firms. Most of the R^2 values from Table 2.14 are greater than 90 %, which means that the data fits the model very well. The R^2 values are much higher than those from the CAPM regression in Table 2.13, indicating a significant improvement.

There were studies which have questioned the significance of the CAPM- β ,¹³⁰ and when looking at Table 2.14, one may think that β_1 plays an insignificant role. The variation is small, having barely any effect in describing returns. However, the market sensitivity β_1 in the Fama–French three-factor model does play a significant role. Fama and French (1993) ran a regression of monthly excess returns on size and B/M alone, and the results can be seen in Table 2.15. The R^2 values range from 0.04 to 0.65. For small-caps, the R^2 value is around 0.6, but it decreases for larger sizes.

Compared to the CAPM regression in Table 2.13, the R^2 values of the regression on only size and B/M are low. So although the values of β_1 differ very little across

¹³⁰See Sect. 2.3.3 about empirical tests.

Table 2.14 Regression of excess stock returns of the 25 size-B/M portfolios on excess market return, *SMB* and *HML*, in the period July 1963–December 1991

	Regression $\tilde{r}_{Pf}^{monthly} = \alpha + \beta_1 \cdot \tilde{r}_{Mkt}^{monthly} + \beta_2 \cdot SMB + \beta_3 \cdot HML$				
	<i>B/M</i> -1 (low)	<i>B/M</i> -2	<i>B/M</i> -3	<i>B/M</i> -4	<i>B/M</i> -5 (high)
β_1 (coefficient of market excess return)					
Size-1 (small)	1.04	1.02	0.95	0.91	0.96
Size-2	1.11	1.06	1.00	0.97	1.09
Size-3	1.12	1.02	0.98	0.97	1.09
Size-4	1.07	1.08	1.04	1.05	1.18
Size-5 (big)	0.96	1.02	0.98	0.99	1.06
β_2 (coefficient of <i>SMB</i>)					
Size-1 (small)	1.46	1.26	1.19	1.17	1.23
Size-2	1.00	0.98	0.88	0.73	0.89
Size-3	0.76	0.65	0.60	0.48	0.66
Size-4	0.37	0.33	0.29	0.24	0.41
Size-5 (big)	−0.17	−0.12	−0.23	−0.17	−0.05
β_3 (coefficient of <i>HML</i>)					
Size-1 (small)	−0.29	0.08	0.26	0.40	0.62
Size-2	−0.52	0.01	0.26	0.46	0.70
Size-3	−0.38	0.00	0.32	0.51	0.68
Size-4	−0.42	0.04	0.30	0.56	0.74
Size-5 (big)	−0.46	0.00	0.21	0.57	0.76
R^2					
Size-1 (small)	0.94	0.96	0.97	0.97	0.96
Size-2	0.95	0.96	0.95	0.95	0.96
Size-3	0.95	0.94	0.93	0.93	0.93
Size-4	0.94	0.93	0.91	0.89	0.89
Size-5 (big)	0.94	0.92	0.88	0.90	0.83

Source: Fama and French (1993, Table 6, pp. 24–25)

portfolios in the three-factor regression, the market sensitivity is needed as a factor together with size and B/M to make the data fit very well.¹³¹

After we have analyzed the sensitivities β_1 , β_2 , β_3 and the coefficient of determination R^2 , we will look at the intercept α and test if it is close to zero. Table 2.16 shows the results for the CAPM, the FF3M and the two-factor model with size and B/M only. The usual CAPM regression (1) shows mainly positive intercepts. This was usually observed in CAPM tests and led to an early rejection of the original CAPM. The intercept decreases with increasing size (except for the lowest B/M quintile) and increases with increasing B/M.

Regression (2) with size and B/M as factors shows positive large intercepts. Nineteen of the portfolios have an intercept greater than 0.5 %, and four have

¹³¹Fama and French (1993, p. 21).

Table 2.15 Regression of excess stock returns of the 25 size-B/M portfolios on *SMB* and *HML* in period July 1963–December 1991

	Regression $\tilde{r}_{pf}^{monthly} = \alpha + \beta_2 \cdot SMB + \beta_3 \cdot HML$				
	<i>B/M</i> -1				<i>B/M</i> -5
	(low)	<i>B/M</i> -2	<i>B/M</i> -3	<i>B/M</i> -4	(high)
	β_2 (coefficient of <i>SMB</i>)				
Size-1 (small)	1.93	1.73	1.63	1.59	1.67
Size-2	1.52	1.46	1.35	1.18	1.40
Size-3	1.28	1.12	1.05	0.93	1.16
Size-4	0.86	0.82	0.77	0.72	0.95
Size-5 (big)	0.28	0.35	0.22	0.29	0.44
	β_3 (coefficient of <i>HML</i>)				
Size-1 (small)	−0.95	−0.57	−0.35	−0.18	0.01
Size-2	−1.23	−0.66	−0.38	−0.16	0.00
Size-3	−1.09	−0.65	−0.31	−0.11	−0.01
Size-4	−1.11	−0.65	−0.36	−0.11	−0.01
Size-5 (big)	−1.07	−0.65	−0.42	−0.06	0.08
	R^2				
Size-1 (small)	0.65	0.60	0.60	0.60	0.59
Size-2	0.59	0.53	0.49	0.42	0.44
Size-3	0.51	0.43	0.37	0.31	0.35
Size-4	0.43	0.30	0.24	0.18	0.23
Size-5 (big)	0.34	0.18	0.08	0.04	0.06

Source: Fama and French (1993, Table 5, p. 22)

an intercept between 0.4 and 0.5 %. This reveals that size and B/M alone do not “explain the average premium of stock returns over 1-month bill returns.”¹³²

When adding the excess market return to the previous regression, the intercepts get closer to zero. In absolute terms, the intercepts of 16 portfolios are smaller than 0.1 %, 22 are smaller than 0.2 %. “Intercepts close to 0 say that the regressions that use [monthly excess returns], *SMB* and *HML* to absorb common time-series variation in returns do a good job explaining the cross-section of average stock returns.”¹³³

The conclusion is that *SMB* and *HML* explain the variation of returns across stocks, whereas the market factor explains why stock returns are on average higher than the risk-free rate.¹³⁴

By using the Gibbons, Ross, and Shanken (1989) test,¹³⁵ Fama and French (1993) rejected the assertion that all intercepts in the regression (1), (2) and (3) from

¹³²Fama and French (1993, p. 35).

¹³³Fama and French (1993, p. 38).

¹³⁴Fama and French (1993, p. 38).

¹³⁵Fama and French (1993, p. 40).

Table 2.16 Intercepts α from the regression of excess stock returns on excess market return, *SMB* and *HML*, in the period July 1963–December 1991

	Intercept α				
	<i>B/M</i> -1 (low)	<i>B/M</i> -2	<i>B/M</i> -3	<i>B/M</i> -4	<i>B/M</i> -5 (high)
	(1) Regression $\tilde{r}_{Pf}^{monthly} = \alpha + \beta_1 \cdot \tilde{r}_{Mkt}^{monthly}$				
Size-1 (small)	−0.22	0.15	0.30	0.42	0.54
Size-2	−0.18	0.17	0.36	0.39	0.53
Size-3	−0.16	0.15	0.23	0.39	0.50
Size-4	−0.05	−0.14	0.12	0.35	0.57
Size-5 (big)	−0.04	−0.07	−0.07	0.20	0.21
	(2) Regression $\tilde{r}_{Pf}^{monthly} = \alpha + \beta_2 \cdot SMB + \beta_3 \cdot HML$				
Size-1 (small)	0.24	0.46	0.49	0.53	0.55
Size-2	0.52	0.58	0.64	0.58	0.64
Size-3	0.52	0.61	0.52	0.60	0.66
Size-4	0.69	0.39	0.50	0.62	0.79
Size-5 (big)	0.76	0.52	0.43	0.51	0.44
	(3) Regression $\tilde{r}_{Pf}^{monthly} = \alpha + \beta_1 \cdot \tilde{r}_{Mkt}^{monthly} + \beta_2 \cdot SMB + \beta_3 \cdot HML$				
Size-1 (small)	−0.34	−0.12	−0.05	0.01	0.00
Size-2	−0.11	−0.01	0.08	0.03	0.02
Size-3	−0.11	0.04	−0.04	0.05	0.05
Size-4	0.09	−0.22	−0.08	0.03	0.13
Size-5 (big)	0.21	−0.05	−0.13	−0.05	−0.16

Source: Fama and French (1993, Table 9a, pp. 36–37)

Table 2.16 are zero.¹³⁶ The Fama–French three-factor model performed best in the test and just failed it slightly. The reason for the rejection of FF3M is the data for the low-*B/M* portfolios: The return on the small-size portfolio was too low (intercept −0.34) and the return on the big-size portfolio was too high (intercept 0.21).¹³⁷ In other words, the size effect was missing in the lowest-*B/M* quintile. But despite the marginal rejection of the FF3M based on the Gibbons et al. (1989) test, Fama and French (1993) finds that the FF3M “does a good job on the cross-section of average stock returns.”¹³⁸ The R^2 values are very high (see Table 2.14) and intercepts are all close to zero except for the smallest-size lowest-*B/M* portfolio.¹³⁹

2.4.4.2 Other Empirical Tests

In Sect. 2.4.4.1 we looked as an example at an empirical test conducted by Fama and French in 1993. However, many other tests of the Fama–French three-factor model exist. Table 2.17 shows a summary of these empirical tests.

¹³⁶Fama and French (1993, p. 39).

¹³⁷Fama and French (1993, pp. 40–41).

¹³⁸Fama and French (1993, p. 41).

¹³⁹Fama and French (1993, p. 41).

Table 2.17 Empirical tests of the Fama–French three-factor model

Study	Data used	Results
Fama and French (1993): <i>Common Risk Factors in the Returns on Stocks and Bonds</i>	July 1963–December 1991: Non-financial stocks which were traded on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and NASDAQ	Marginal rejection of the FF3M by the GRS test ^a because the size effect did not occur for the low-book/market portfolios.
Griffin (2002): <i>Are the Fama and French Factors Global or Country Specific?</i>	January 1981–December 1995: Stocks from USA, Canada, Japan and U.K.	For the countries USA, Canada, Japan and U.K., the domestic Fama–French three-factor model was compared to a world version of the FF3M. The domestic versions fit the data much better than the world Fama–French model. But the GRS test rejected the (domestic/world) FF3M for all countries except Japan.
Faff (2004): <i>A Simple Test of the Fama and French Model Using Daily Data: Australian evidence</i>	May 1996–April 1999: Stocks from the Australian Stock Exchange (ASX)	Supports the FF3M equation using the MacKinlay–Richardson test ^b but finds a reversed size effect, i.e., negative <i>SMB</i> , which means that big firms outperform small firms.
Bahl (2006): <i>Testing the Fama and French Three-Factor Model and its Variants for the Indian Stock Returns</i>	July 2001–June 2006: 79 stocks from the BSE-100 Index (Bombay Stock Exchange)	GRS test supported FF3M
Rogers and Securato (2007): <i>Comparative Study of CAPM, Fama and French Model and Reward Beta Approach in the Brazilian Market</i>	July 1995–June 2006: Non-financial stocks from the Sao Paulo Stock Exchange (BOVESPA)	Wald test supported FF3M, but the book/market factor was not found to be significant.

(continued)

Table 2.17 (continued)

Chen and Fang (2009): <i>Uniform Testing and Portfolio Strategies for Single and Multifactor Asset Pricing Models in the Pacific Basin Markets</i>	Test on seven different stock markets: Japan (1976–2000), Hong Kong (1981–1999), South Korea (1982–1999), Malaysia (1979–1999), Thailand (1983–1999), Indonesia (1990–1998) and Singapore (1976–1997)	GRS test supported FF3M in all seven stock markets. Compared to CAPM, the average adjusted R^2 improves by 29 %. Size matters more than book/market.
Artmann, Finter, Kempf, Koch, and Theissen (2012): <i>The Cross-Section of German Stock Returns: New Data and New Evidence</i>	July 1962–December 2006: Over 900 non-financial stocks from the Frankfurt Stock Exchange	The FF3M was rejected by the GRS test. The highest R^2 value for a test portfolio was 81.6 %, but most of the R^2 values were below 80 %.
Bauer, Cosemans, and Schotman (2010): <i>Conditional Asset Pricing and Stock Market Anomalies in Europe</i>	February 1986–June 2002: Over 2000 stocks from exchanges of Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the U.K., covering about 80 % of the European market capitalization	GRS test rejected FF3M: The small-cap low-book/market portfolio had big abnormal returns because it contained many start-ups which soared during the tech bubble. For the test portfolios, the adjusted R^2 ranged from 73 to 89 %. Book/market is insignificant when explaining returns. For European stocks, a significant size effect was observed, while for a sample of U.S. stocks, the size effect was insignificant.

^a GRS test stands for the test statistic developed in Gibbons et al. (1989)
^b The test used here is the generalized method of moments (GMM) developed by MacKinlay and Richardson (1991)

2.5 Summary

In this chapter, we have presented the basics of modern portfolio theory as introduced by Markowitz in the 1950s. The ideas of diversification and the efficient frontier are key when investing today. The capital asset pricing model allows investors to detect over- and undervalued securities, but it shows weaknesses when it is subjected to empirical tests. Its extension, the Fama–French three-factor model, uses more input parameters than the CAPM when determining investment returns, but also exhibits shortcomings in practice. Both models are theoretically plausible, but there is a discrepancy with reality which, in times of crisis, is significant. Moreover, traditional finance theory cannot explain market situations like crashes and stock market anomalies. The latter will be the topic of the next chapter.

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