

2 Vehicle Routing

In Chapter 2 and Chapter 3, the basic decision problems in the operational transportation planning of freight forwarding companies are introduced. If own disposable vehicles are used for the request fulfillment, the forwarders have to solve the vehicle routing and scheduling problems. The requests have to be assigned to different trucks, and for each truck, the order of the visits at the customer nodes assigned to this truck has to be specified. This topic is discussed in this chapter. The focus of the discussion lies in the PDP, which is considered as the basic routing problem for the study of the transportation planning in the context of IOTP and CTP in later chapters.

The problems of vehicle routing belong to the most studied combinatorial optimization problems. A great number of specific problems in this problem family have been discussed to which numerous publications are dedicated. Besides transportation of commercial goods, these research efforts embrace a broad spectrum of real-world applications including, for instance, solid waste collection, street cleaning, school bus routing, dial-a-ride systems, and transportation of handicapped persons.

This chapter focuses on some basic static routing problems. We begin with a brief introduction on this topic with some illustrative examples. Then, a mathematical formulation is presented to model the PDP with time windows (PDTPW). Next, a short overview of the algorithms proposed for the PDPTW is given. Finally, the ALNS heuristic for the PDPTW proposed by Ropke and Pisinger (2006) is described. This heuristic will be used and further developed in the forthcoming chapters to solve the PDPTW and some related extended routing problems in the context of IOTP and CTP.

2.1 Introduction

The simplest problem of vehicle routing is the traveling salesman problem (TSP), which deals with the following question: Given a set of cities a salesman has to visit and the distances between each pair of these cities, what is the shortest route that visits each city exactly once and returns to the origin city? This problem is one of the most famous NP-hard problems in combinatorial optimization. According to Müller-Merbach (1983), the research on the TSP can be traced back to 1832, when the problem was formulated for the first time in a German manual, in which five tours through Germany are suggested and

one of them also through some Swiss cities. Figure 2.1 shows one of these five tours along 45 German cities, which is 1285 km long¹. Schrijver (2005) studies briefly the history of the research on TSP till 1960. Detailed discussions about later research can be found in Lawler et al. (1985), Laporte (1992), Gutin and Punnen (2002), and Applegate et al. (2006).

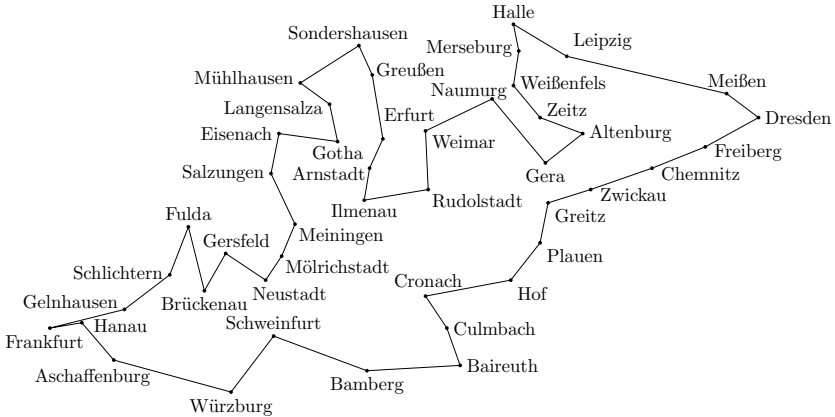


Figure 2.1: A tour along 45 German cities suggested in 1832, cf. Schrijver (2005)

Another very important and intensively studied problem is the vehicle routing problem (VRP) which was introduced by Dantzig and Ramser in 1959 (Dantzig and Ramser, 1959). Given a set of geographically scattered customers, to whom goods are to be delivered from the depot of a vehicle fleet, the VRP can be defined as the problem of constructing vehicle routes to serve all customers while the total distances of all routes are minimized. It must be assured that each customer is served exactly once by one vehicle that starts and ends its route at the depot and the total demands of customers served in a route do not exceed the capacity of the vehicle. The VRP can also be used to model the case when goods are to be gathered and transported to the depot, and the vehicles are empty when they start their routes. Figure 2.2 shows an example of the VRP with 4 vehicles and 14 customer nodes. The capacity of the vehicles is limited to 20 and the numbers around the customer nodes represent the demand of the customers.

Two closely related subproblems have to be considered by solving the VRP. The first one is to assign each customer to a specific vehicle and the second one is to construct a route for each vehicle that serves all customers assigned to it. Thus, the VRP presents

¹The shortest tour with a total length of 1248 km can be found for this instance. However, if the local conditions are taken into account, this 1832 tour might be optimum. (Schrijver, 2005)

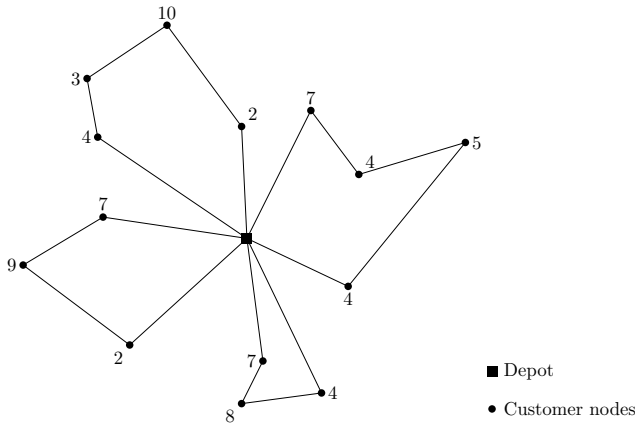


Figure 2.2: An example of the VRP

a generalization of the TSP and is therefore an NP-hard problem. The TSP can be seen as a special case of the VRP while only one vehicle exists in the fleet and its capacity is large enough to serve the total demands of all customers. Since its introduction, a great number of scientific articles have been published. Recent reviews of the literature on the VRP can be found in Cordeau et al. (2007), Parragh et al. (2008a), and Laporte (2009). The two books edited by Toth and Vigo (2002a) and by Golden et al. (2008) offer a comprehensive introduction on the VRP and its most common variants as well as an extensive discussion about the solution methods.

In the classical VRP, all customer requests are of the same type, i.e., either delivery requests or pickup requests. A nature extension of this problem is the VRP with Backhauls (VRPB) which considers both request types. In the VRPB, the customer set is partitioned into two subsets. The first subset contains the *linehaul customers* (delivery requests) and each of them requires a given load to be transported from the depot to the customer node. The second one contains the *backhaul requests* (pickup requests), where a given quantity of goods have to be picked up at the customer node and transported back to the depot. A practical restriction considered additionally is that on each route, the backhaul customers, if any, are visited after all linehaul customers, for the fact that vehicles are often rear-loaded and the on-board load rearrangement required by a mixed service is difficult or even impossible at customer locations (Toth and Vigo, 2002b).

In the VRP and the VRPB, the transportation of goods between customer locations and a depot is studied. A further extension is the PDP, which considers the transportation of goods or persons between pickup and delivery locations without transshipment at

intermediate locations². Since both the pickup and delivery locations related to a request need not be the depot, the PDP is a generalization of the VRP and the VRPB. If the pickup location (or the delivery location) of all requests is the depot, the PDP returns to the VRP. If either the pickup or the delivery location of each single request is the depot, the PDP goes back to the VRPB. Some recent surveys on the PDP can be found in Desaulniers et al. (2002), Berbeglia et al. (2007), Parragh et al. (2008b), and Cordeau et al. (2008). Figure 2.3 shows an example of the PDP with 7 homogeneous LTL requests (presented by the arrow with dotted lines in Figure 2.3a) and one of its possible solutions with 3 vehicles (depicted by the arrows with solid lines in Figure 2.3b).

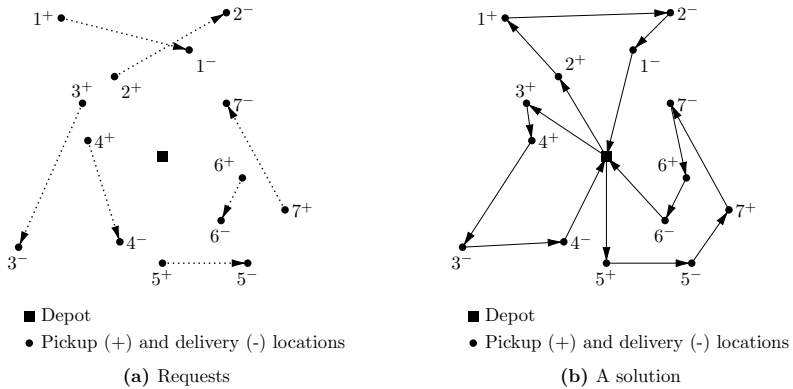


Figure 2.3: An example of the PDP

The routing problems can be further extended by introducing the time window at the locations, which is defined as a time interval, in which the needed operation at the location must be started. Time windows can be either soft or hard. Soft time windows can be violated at a penalty cost, while hard time windows must be strictly held. In case of hard windows, vehicles can only start the services within the associated time windows. In other words, they have to wait at the customer locations if they arrive before the customers are ready for the service. The extensions of the TSP, the VRP, and the PDP by time windows can be referred to as the TSP with time windows (TSPTW), the VRP with time windows (VRPTW), and the PDPTW, respectively. These problems can be further extended by multiple depots. In the most general case of the PDP, each vehicle can have its own start and end depots. In the following section, a mathematical model is presented for this case.

²The PDP is also referred to as the VRP with pickup and delivery (VRPPD) in, for instance, Desaulniers et al. (2002) and Parragh et al. (2008b). In this thesis, the abbreviation “PDP” is used to denote this problem.

2.2 Mathematical model for the PDPTW

In the PDPTW, a set of transportation requests have to be fulfilled using a vehicle fleet. Each request is specified by a pickup location, a delivery location, and a load to be transported between these two locations. Moreover, a customer payment will be paid for the fulfillment of a request. The objects of transportation related to the requests can be either goods or persons. Specifically, the problem of transporting persons is referred to as the dial-a-ride problem (DARP) in which the convenience of the people is usually considered in the objective function. The service at each location must be started within a time window defined by the customer and is associated with a service time. If a vehicle arrives at a location prior to the beginning of the time window, it has to wait. The service time indicates how long it will take to finish the service. Furthermore, some requests may require special equipment and thus can be only served by specific vehicles in the fleet. The vehicles for the request fulfillment can be heterogeneous. They can have different start and end depots, different capacities, and can be differently equipped. Also the costs related to the vehicles can vary from each other.

The task of the PDPTW is to construct a set of valid vehicle routes to serve all the requests. A route for a vehicle is a sequence of locations, which begins at the vehicle's start depot and terminates at its end depot. It is valid when:

1. each customer location in this route is visited exactly once,
2. both the pickup and the corresponding delivery of a request assigned to this vehicle are served in this route,
3. the pickup location of a request must be visited before its corresponding delivery location,
4. the capacity of the vehicle is not exceeded, and
5. the compatibility constraints are satisfied.

The objective of the PDPTW is to minimize the total fulfillment costs. As all customers must be served and the customer payments are fixed, minimizing the total costs is equivalent to maximizing the total revenue, determined as the difference between the total customer payments and the total fulfillment costs.

The PDPTW can be mathematically formulated based on Desaulniers et al. (2002) and Ropke and Pisinger (2006) in the following way. Let $R = \{1, 2, \dots, n\}$ be the set of all requests. Define the set of pickup nodes as $P = R = \{1, 2, \dots, n\}$ and the set of delivery nodes as $D = \{n+1, n+2, \dots, 2n\}$. Since $P = R$, the origin-destination (O-D) pair of a request $r \in R$, (r^+, r^-) , can be specified as $(u, u+n)$, $u = r \in P$. It is possible that

different nodes represent the same geographical location. Denote the transportation load required by request r as ℓ_r . The capacity requirement at the pickup node of a request $r \in R$ can be defined as $\ell_u = \ell_r$, $u = r$, and at its corresponding delivery node as $\ell_{u+n} = -\ell_r$, $u = r$. The time window at node $u \in P \cup D$ is given by $[b_u, e_u]$ and the service time is defined as s_u .

Let K be the vehicle set including ϱ vehicles. As not all nodes can be served by all vehicles in K , two specific sets of pickup nodes P_k and delivery nodes D_k corresponding to the requests that can be served by a vehicle $k \in K$ can be defined, while $P_k \subseteq P$ and $D_k \subseteq D$. The start and end depots of a vehicle k are denoted as o_k and o'_k , respectively. A graph of the entire problem can be defined as $G = (V, A)$, where $V = P \cup D \cup \{o_1, o_2, \dots, o_\varrho\} \cup \{o'_1, o'_2, \dots, o'_\varrho\}$ is the node set and $A = V \times V$ is the arc set. The distance and the travel time of an arc $(u, v) \in A$ are given by d_{uv} and d'_{uv} , respectively. For each vehicle k , a graph $G_k = (V_k, A_k)$ can be defined in the same way as G with $V_k = P_k \cup D_k \cup \{o_k, o'_k\}$ and $A_k = V_k \times V_k$. Each vehicle $k \in K$ has a limited capacity Q_k and is associated with a fixed cost α_k , a cost rate per distance unit (DU) β_k , and a cost rate per time unit β'_k . It is further assumed that vehicle $k \in K$ leaves its start depot o_k without any load when its time window opens at b_{o_k} and $e_{o_k} = b_{o_k}$. The service time at the depots s_{o_k} and $s_{o'_k}$, $k \in K$ is defined as 0.

Three decision variables are needed in the formulation. The binary variable x_{uvk} , $u, v \in V_k$, $k \in K$, equals to 1 if vehicle k travels from node u to node v , and 0 otherwise. Variable t_{uk} defines the time when the service at node $u \in V_k$ starts by using vehicle $k \in K$. Variable l_{uk} gives the load of vehicle $k \in K$ after the service at node $u \in V_k$ is completed. Both variables t_{uk} and l_{uk} are only well-defined when node u is actually served by vehicle k . The PDPTW can be modeled as follows:

$$\min \sum_{k \in K} \alpha_k + \sum_{k \in K} \sum_{(u,v) \in A_k} \beta_k d_{uv} x_{uvk} + \sum_{k \in K} \beta'_k (t_{o'_k k} - b_{o_k}) \quad (2.1)$$

subject to:

$$\sum_{k \in K} \sum_{v \in V_k \setminus \{o_k\}} x_{uvk} = 1 \quad \forall u \in P \quad (2.2)$$

$$\sum_{v \in P_k \cup D_k} x_{uvk} - \sum_{v \in P_k \cup D_k} x_{v, n+u, k} = 0 \quad \forall k \in K, u \in P_k \quad (2.3)$$

$$\sum_{v \in P_k \cup \{o'_k\}} x_{o_k, v, k} = 1 \quad \forall k \in K \quad (2.4)$$

$$\sum_{u \in D_k \cup \{o_k\}} x_{u, o'_k, k} = 1 \quad \forall k \in K \quad (2.5)$$

$$\sum_{u \in V_k \setminus \{o'_k\}} x_{uvk} - \sum_{u \in V_k \setminus \{o_k\}} x_{vuk} = 0 \quad \forall k \in K, v \in P_k \cup D_k \quad (2.6)$$

$$x_{uvk}(t_{uk} + s_u + d'_{uv} - t_{vk}) \leq 0 \quad \forall k \in K, (u, v) \in A_k \quad (2.7)$$

$$b_u \leq t_{uk} \leq e_u \quad \forall k \in K, u \in V_k \quad (2.8)$$

$$t_{uk} + d'_{u,n+u,k} \leq t_{n+u,k} \quad \forall k \in K, u \in P_k \quad (2.9)$$

$$x_{uvk}(l_{uk} + \ell_v - l_{vk}) = 0 \quad \forall k \in K, (u, v) \in A_k \quad (2.10)$$

$$\ell_u \leq l_{uk} \leq Q_k \quad \forall k \in K, u \in P_k \quad (2.11)$$

$$0 \leq l_{n+u,k} \leq Q_k - \ell_u \quad \forall k \in K, u \in P_k \quad (2.12)$$

$$l_{o_k k} = 0 \quad k \in K \quad (2.13)$$

$$x_{uvk} \in \{0, 1\} \quad \forall k \in K, (u, v) \in A_k \quad (2.14)$$

$$t_{uk} \geq 0 \quad \forall k \in K, u \in V_k \quad (2.15)$$

$$l_{uk} \geq 0 \quad \forall k \in K, u \in V_k \quad (2.16)$$

The objective function (2.1) minimizes the total fulfillment costs including both the fixed and variable costs, while the time dependent variable cost is calculated based on the total operating time of the vehicles. Because the fixed costs of vehicles are constant, the corresponding term in (2.1) can be omitted and an equivalent formulation of this objective function is given by:

$$\min \sum_{k \in K} \sum_{(u,v) \in A_k} \beta_k d_{uv} x_{uvk} + \sum_{k \in K} \beta'_k (t_{o'_k k} - b_{o_k}) \quad (2.17)$$

Constraint (2.2) imposes that each request is served by exactly one vehicle. Constraint (2.3) ensures that the corresponding pickup and delivery nodes are served by the same vehicle. Constraints (2.4) and (2.5) guarantee that each vehicle begins its route at its start depot and terminates it at its end depot. Constraint (2.6) is the flow balancing constraint and makes sure that if a vehicle serves a customer node, it has to leave it either. Constraints (2.7) and (2.8) determine the start time of the service at customer nodes, which must lie in the time window, while constraint (2.7) also eliminates any subtours. (2.9) ensures that the pickup is performed before its corresponding delivery. Constraints (2.10)-(2.13) make sure that the load variable is set correctly along the routes and the capacity of the vehicles is not exceeded. Moreover, constraints (2.7) and (2.10) in this model can be linearized by introducing a *big M* that is a large number:

$$t_{uk} + s_u + d'_{uv} - t_{vk} \leq (1 - x_{uvk})M \quad \forall k \in K, (u, v) \in A_k \quad (2.18)$$

$$l_{uk} + \ell_v - l_{vk} \leq (1 - x_{uvk})M \quad \forall k \in K, (u, v) \in A_k \quad (2.19)$$

For the homogeneous PDPTW, to minimize the number of total used vehicles is often considered as the primary objective, while to minimize the total variable costs of the routes as the objective function (2.17) does or just to minimize the total distances as the secondary one does. This means that a solution is better than another one when in the first solution, i.e., the better one, less vehicles are used to serve all requests or both solutions use the same number of vehicles but the total costs of the first one, especially the variable costs, are less than that of the second one. The usage of the lexicographic objectives is motivated by the fact that the fixed costs of a vehicle are usually much higher than the variable costs. Actually, these two objectives can be united into one single objective function by adding a fixed term into the objective function (2.17) by introducing a binary variable z_k , $k \in K$, which will be one if vehicle k is used or zero if not, and by setting the fixed costs related to the vehicles α_k very high. The united objective function for the homogeneous PDPTW can be formulated as:

$$\min \sum_{k \in K} \alpha_k z_k + \sum_{k \in K} \sum_{(u,v) \in A_k} \beta_k d_{uv} x_{uvk} + \sum_{k \in K} \beta'_k (t_{o'_k k} - b_{o_k}) \quad (2.20)$$

The following constraint can help decide whether a vehicle is used:

$$\sum_{v \in P_k} x_{o_k, v, k} \leq M \cdot z_k \quad \forall k \in K \quad (2.21)$$

In case that all vehicles are homogeneous, we have $\alpha_k = \alpha$, $P_k = P$, $o_k = o$, $o'_k = o'$, for all $k \in K$ and the index k can be removed from these notations.

2.3 Solution approaches for the PDPTW

The PDPTW is an NP-hard problem since it is a generalization of the VRP. Great efforts have been done in developing efficient algorithms to solve this problem and its variants in the last three decades. In this section, the algorithms proposed for the multi-vehicle PDP are briefly reviewed. The review is limited to the important contributions since 1990. An overview of the early stage of the research on the PDP from the early 1980s to the mid-1990s can be found in Savelsbergh and Sol (1995). For comprehensive reviews on the solution methods for a wider spectrum of PDP variants, the reader is referred to Berbeglia et al. (2007); Parragh et al. (2008b); Cordeau et al. (2008).

2.3.1 Exact algorithms

The very first exact algorithm proposed for the multi-vehicle PDP may be the algorithm of Dumas et al. (1991). The authors use the column generation scheme by applying a set partitioning formulation of the PDPTW. The subproblem of generating valid routes is modeled as a constrained shortest path problem and solved by a forward dynamic programming (DP) approach. Some small instances with up to 4 vehicles and 14 customers and 3 vehicles and 15 customers with tight time windows are solved to optimality. Sigurd et al. (2004) also use the column generation technique and develop an exact algorithm to solve a practical variant of the PDP considering additionally a precedence restriction associated with the requests. This additional restriction enables the authors to strongly reduce the computation complexity of the shortest path problem and to solve instances up to 580 nodes.

Another methodology that has proved successful for solving the PDP to optimality is the branch-and-cut which does not necessarily require tight constraints that can lead to a significant reduction of the solution space (Lu and Dessouky, 2004). Different branch-and-cut algorithms are proposed in Lu and Dessouky (2004) and Ropke et al. (2007) for the PDP and in Cordeau (2006) for the DARP. The algorithm of Lu and Dessouky (2004) is developed based on a formulation of the PDP with two-index flow variables. The authors also introduce a precedence variable in their formulation and identify several valid inequalities based on it. Instances with up to 5 vehicles and 25 customers were solved to optimality. Cordeau (2006) uses a three-index formulation of the DARP. Besides some inequalities adapted from existing algorithms for the TSP and the VRP, the author also proposes some new inequalities taking the specific structure of the problem into account. The algorithm was able to solve instances with up to 4 vehicles and 32 requests. In Ropke et al. (2007), two-index formulations of the PDPTW are used. However, the new formulations contain exponential number of constraints but fewer variables. The authors report better bounds than those obtained by previous algorithms. Instances with up to 8 vehicles and 96 requests were solved to optimality.

Motivated by the fact that set partitioning formulations of the VRP tend to provide stronger lower bounds than formulations based on flow variables (Bramel and Simchi-Levi, 2002) and by the success of the combined approaches of branch-and-price, i.e., the column generation scheme, and branch-and-cut in solving the VRP, Ropke and Cordeau (2009) propose a branch-and-cut-and-price algorithm for the PDPTW. Through perturbing the cost matrix of the pricing problem, fast algorithm can be used to solve the pricing problem with valid inequalities. Some large instances with up to 500 requests with tight time windows were solved using this algorithm. The latest exact algorithm for the PDPTW is

proposed by Baldacci et al. (2011) based on a set-partitioning-like integer formulation. A bounding procedure is used to find a near-optimal dual solution to the linear programming (LP) relaxation of the formulation. The final dual solution is used to generate a reduced problem containing only selected routes, which is solved either by an integer programming solver or a branch-and-cut-and-price algorithm, depending on the size of this resulted problem. Computational results indicate a great improvement of the performance of this algorithm compared with the one of Ropke and Cordeau (2009).

2.3.2 Heuristic algorithms

Although the research on exact solution methods for the PDPTW has been put forward greatly in the last several years, some limitations still have to be recognized. For instance, only two of the algorithms described in Section 2.3.1, i.e., the algorithms of Lu and Dessouky (2004) and Sigurd et al. (2004) deal explicitly with the heterogeneity of vehicle fleet in terms of capacity and depots. In addition, no exact algorithm has been proposed to solve PDPTW instances when vehicles have different cost structures. Thus, efficient heuristics are still indispensable for solving more complex instances of the PDPTW.

Nanry and Barnes (2000) propose a reactive tabu search (TS) heuristic to solve the PDPTW. This heuristic begins with a greedy procedure for obtaining a feasible initial solution. Three distinct move neighborhoods that capitalize on the dominance of the precedence and coupling constraints are used in the following search process, which is directed by a hierarchical multi-neighborhood search strategy based on the average time window length of the instance.

A tabu-embedded simulated annealing (SA) heuristic (Dowsland, 1995) is developed by Li and Lim (2001) for the PDPTW. Each time a neighbor solution of the current one is accepted, it will be further improved through a descent local search (LS) algorithm. After some consecutive iterations without any improvement of the current solution, the heuristic restarts from the current best solution. A further contribution of this work is the generation of benchmark instance sets based on the VRP instance of Solomon (1987). The instance data and the best-known solutions can be found on the special website operated by SINTEF³.

Similar neighborhood structures as defined in Nanry and Barnes (2000) are used in the TS heuristic proposed by Lau and Liang (2002). The authors use a partitioned insertion heuristic as the construction algorithm to generate initial feasible solutions, which combines the ideas of a simple insertion heuristic and an adapted version of the swap heuristic widely used for the VRP.

³ <http://www.sintef.no/Projectweb/TOP/PDPTW/Li--Lim-benchmark>

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