

2 Model of the Turbine Aerodynamics

2.1 Introduction

The primary time domain of interest for dynamic grid studies is up to 10 s .. 30 s, until the key disturbances have either settled or lead to a disconnection of parts of the system. Within the time frame given, wind speed is often assumed to be constant [4]. Short term changes of wind speed in the range of seconds like gusts and turbulent wind generally do not have an impact on stability for variable speed wind turbines as they have local effects only that average out even within one wind plant.

Using a single, aggregated model of the wind power plant instead of a detailed representation of each wind turbine is usually acceptable for stability studies [5], [6]. However, the analysis of large blackouts and system disturbances like 2003 in Italy [7] or 2006 in the UCTE System [8] on the other hand shows that the time frame of interest could be up to several minutes. In these cases an aerodynamic model is required that allows simulation of the excursion of aerodynamic quantities like blade pitch angle, wind speed and wind turbine rotor speed in a simple form.

More detailed aerodynamic models are used for the modeling of fixed speed turbines [9], [10]. An overview of the aerodynamic model within the turbine model structure is shown in Fig. 1.4.

2.2 Energy Capture from the Wind

The theory of extracting energy from wind dates back to Betz [11]. The kinetic energy of wind E_W and the resulting power P_W are defined by

$$E_W = \frac{1}{2} m_W v_W^2 \quad (2.1)$$

$$P_W = \dot{E}_W = \frac{1}{2} \dot{m}_W v_W^2 = \frac{1}{2} \rho A v_W^3 \quad (2.2)$$

with the air flow $\dot{m}_W = \rho A_{WR} v_W$, ρ as air density and A as area through which the air flows. Assuming a homogenous tube, with wind speed v_{W1} at the entry and wind speed v_{W3} at the exit, and a wind speed v_{W2} in between (see Fig. 2.1), the following equation

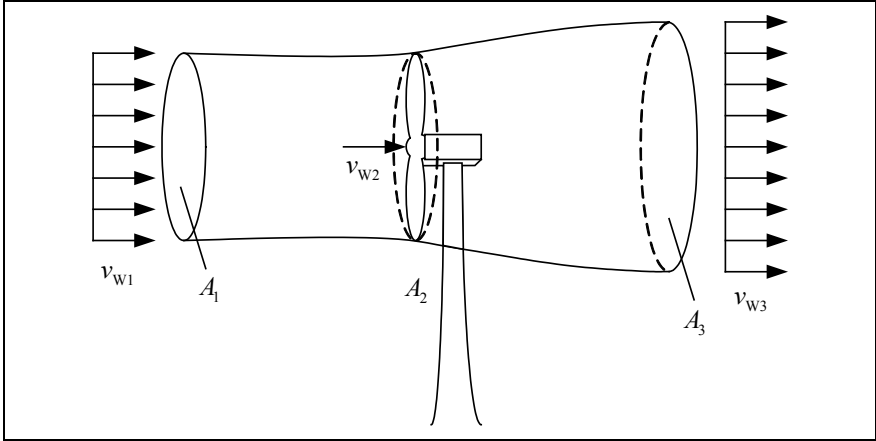


Fig. 2.1 Air flow at a wind turbine.

$$\rho A_1 v_{W1}^2 = \rho A_2 v_{W2}^2 = \rho A_3 v_{W3}^2 \quad (2.3)$$

is true if the air density ρ is assumed to be constant before and after the tube. The energy and the power that can be extracted from the wind then are

$$E_w = \frac{1}{2} m_w (v_{W1}^2 - v_{W3}^2) \quad (2.4)$$

$$P_w = \dot{E}_w = \frac{1}{2} \dot{m}_w (v_{W1}^2 - v_{W3}^2) \quad (2.5)$$

with the flow in the rotor plane as

$$\dot{m}_w = \rho A_2 v_{W2} \quad (2.6)$$

Applying the Froud-Rankine theorem ([12], S.184) it can be proven, that the wind speed in the rotor plane is the average of the wind speed in front and behind the rotor plane, that means

$$v_{W2} = \frac{v_{W1} + v_{W3}}{2} \quad (2.7)$$

By inserting (2.6) with v_{W2} according to (2.7) into (2.5) the energy extracted by the wind for a given wind turbine rotor plane $A_{WR} = A_2$ (see Fig. 2.5) can then be described as

$$P_{WR} = \underbrace{\frac{1}{2} \rho \pi R_{WR}^2 v_{W1}^3}_{\text{wind power}} \underbrace{\left[\frac{1}{2} \left(1 + \frac{v_{W3}}{v_{W1}} \right) \left(1 - \left(\frac{v_{W3}}{v_{W1}} \right)^2 \right) \right]}_{\text{power coefficient } c_p} \quad (2.8)$$

with R_{WR} as rotor radius. By substituting $x = v_{W3}/v_{W1}$ and setting the first derivative with respect to x of the resulting function

$$c_p(x) = \frac{1}{2}(1+x)(1-x^2) \quad (2.9)$$

to zero, we find a maximum for $x = 1/3$. From (2.7) we see that the maximum power extraction can be achieved by a wind speed in the rotor plane of $v_{W2} = 2/3 v_{W1}$. By inserting this result into (2.8) the maximum power coefficient c_p according to Betz is calculated as

$$c_{p, \text{Betz}} = \frac{16}{27} = 0.593 \quad (2.10)$$

This constant describes the maximum energy that can be extracted from the wind. The power factor describes a theoretical limit that is independent of factors like number of blades or blade design. With modern blades, a c_p of about 0.5 can be achieved at the optimum operation point.

2.3 Aerodynamics of Rotor Blades

For calculating the aerodynamic behavior of a turbine rotor, the blades are divided into different segments Δr . For each segment, lift and drag forces F_L and F_D can be described as functions of the angle of attack α_A between the induced wind speed v_{Wi} , and the blade chord. The induced wind speed v_{Wi} results from the reduced wind speed in the rotor plane $v_{W2} = 2/3 v_{W1}$ and the wind component caused by the rotor speed $v_{Wr} = \Omega_{WR} \cdot R_j$ with R_j as the distance from the hub center to the segment with the index j . For an easier description, the index is omitted in the following text. In Fig. 2.2 the angle of attack α_A is shown as a result of wind speed v_{W1} in front of the rotor, the wind speed induced by the rotor speed v_{Wr} and the angle Θ of the blade chord with respect to the rotor plane.

Differential lift dF_L and drag dF_D forces and can be calculated as

$$dF_L = \frac{\rho}{2} v_{Wi}^2 c \cdot dr \cdot c_L(\alpha_A) \quad (2.11)$$

$$dF_D = \frac{\rho}{2} v_{Wi}^2 \cdot c \cdot dr \cdot c_D(\alpha_A) \quad (2.12)$$

with dr as the length of the profile in radial direction and c as the depth of the blade segment. Lift and drag coefficients c_L and c_D depend on the profiles selected and can be described as functions of the angle of attack α_A .

The induced wind speed v_{Wi} increases along the rotor from the hub to the blade tip. As a result, in order to achieve a constant angle of attack, the blade segments are twisted along the rotor axis. The profiles selected also change with increasing induced wind speed along the blade. The resulting in-plane force for each segment is the basis for the calculation of the captured energy, it can be written as

$$dF = \frac{\rho}{2} v_{Wi}^2 \cdot c \cdot dr [c_L(\alpha_A) \sin \alpha - c_D(\alpha_A) \cos \alpha] \quad (2.13)$$

with α as the angle between induced wind speed and the rotor plane (see Fig. 2.2).

For detailed simulations, lift and drag as function of the angle of attack are calculated for each profile used within the rotor blade. In addition, losses resulting from the spin induced into the wind by the rotor blades [13], losses due to the blade tips [12] and losses resulting from the chosen blade profiles have to be included in the calculation. A more detailed analysis of aerodynamic theory and blade design can be found in [12], [14] and [15]. Using detailed models, the input data for simplified representations as shown in the next section can be calculated.

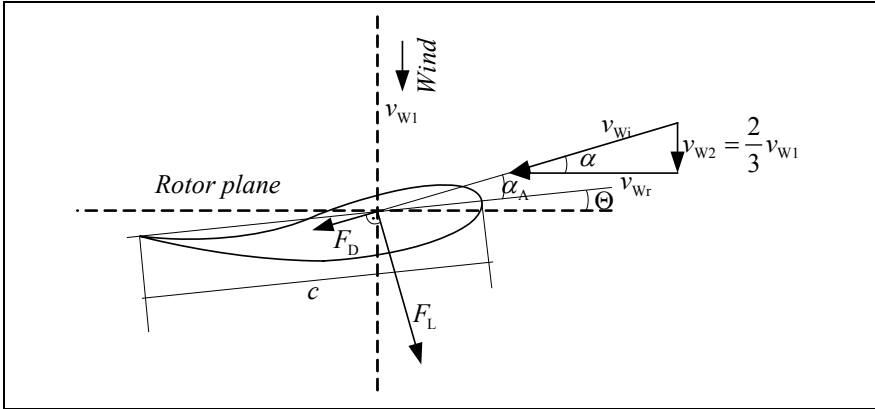


Fig. 2.2 Lift and drag of a blade in the rotor plane.

2.3.1 Simplified Representation

The energy captured by a rotor is calculated as the sum of the energy capture of each profile segment. Under steady state operating conditions, the definition of the dimensionless tip-speed ratio

$$\lambda = \frac{\Omega_{\text{WR}} R_{\text{WR}}}{v_{\text{W}}} \quad (2.14)$$

with R_{WR} as rotor radius and Ω_{WR} as rotor speed allows a simplified representation of the resulting power coefficient of a wind turbine rotor as function of blade angle Θ and tip-speed ratio λ . The power generated by the wind turbine rotor is then calculated as

$$P_{\text{WR}} = \frac{1}{2} \rho \pi R_{\text{WR}}^2 v_{\text{W}}^3 \cdot c_{\text{p}}(\lambda, \Theta) \quad (2.15)$$

where $c_{\text{p}}(\lambda, \Theta)$ is usually given as a table calculated by detailed aerodynamic simulation based on the blade element theory for a given blade design.

Consider, that the wind speed v_{W} corresponds with v_{W1} used in (2.3) - (2.8). In Fig. 2.3 the power coefficient is shown as a function of tip-speed-ratio λ for different blade angles. It can be seen that there is an optimum tip-speed-ratio λ_{opt} which results in a maximum power generation for a given wind speed at about 9 for a blade angle of 0 deg. The value λ_{n} corresponds to the operation at rated wind speed.

For practical reasons, there is an upper limit of the tip speed in order to limit noise and mechanical stress. There is also often a lower speed limit due to a limited operating range of the electrical system and in order to avoid resonances between the tower eigenfrequency and the blades. An ideal steady state operation trajectory that guarantees quasi optimal exploitation for wind is shown as black curve in Fig. 2.3. λ_{max} is the operating point at the lower rotor speed limit and the lowest wind speed used for wind turbine operation.

The same ideal steady state operation trajectory is shown in Fig. 2.4 for different wind speeds. Below rated wind speed (12 m/s in this case) the blade angle remains at zero. At higher wind speeds the blade angle is increased and thus the power is limited to the turbine rating. An operation with a blade angle of zero at higher wind speeds would allow the extraction of more power from the wind, but at the expense of a higher rating of the electrical system and the need for a stronger mechanical design. But such high wind speeds may only occur very seldom throughout a year. For economic reasons, the rated wind speed of a turbine is therefore always limited.

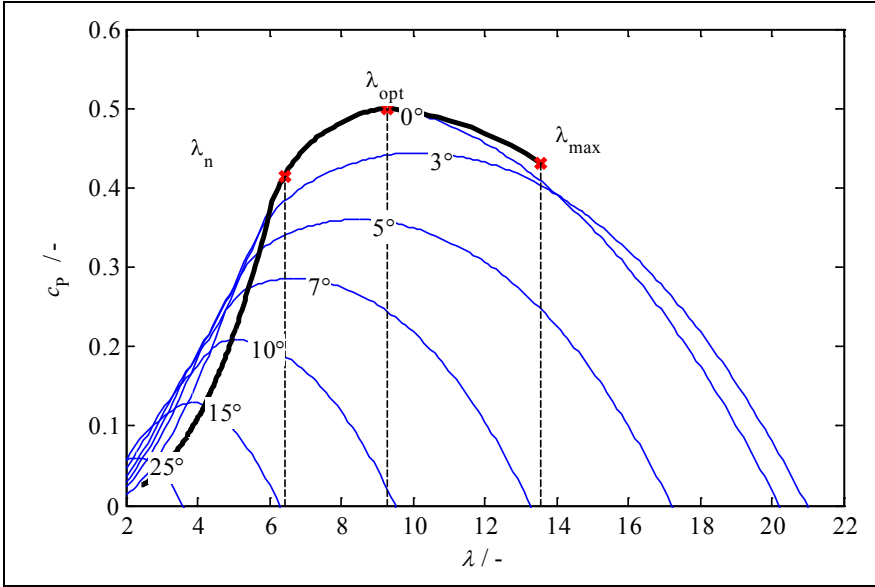


Fig. 2.3 Power coefficient c_p for different blade angles (thin) and steady state operation trajectory (thick) of the wind turbine.

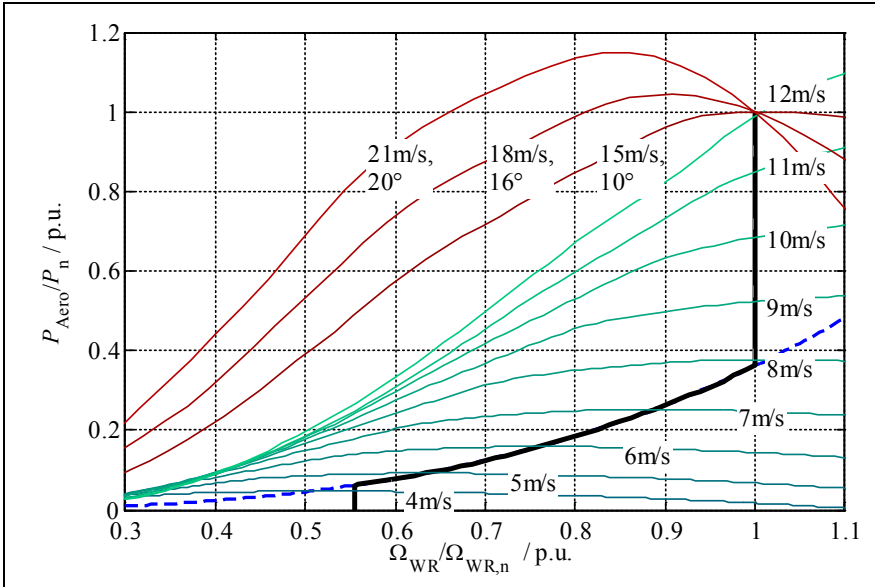


Fig. 2.4 Wind turbine rotor power as a function of rotor speed for different wind speeds (thin), optimal trajectory (dashed) and steady state operation trajectory (thick).

2.4 Simulation using c_P - λ Tables

The power coefficient c_P can be represented as a function of tip-speed-ratio λ and blade angle θ as shown in Fig. 2.5. The thick line shows the operating trajectory for different wind speeds, starting from low wind speeds at high tip-speed-ratios to high wind speeds at low tip-speed-ratios. This is the same trajectory that can be seen in a different representation in Fig. 2.3 and Fig. 2.4. At very high wind speeds the power is limited to rated power by increasing the blade angle. At very low wind speeds and if the lower rotor speed is limited, an increase of the blade angle can increase the c_P -value and thereby the power captured from the wind to a certain degree. Describing c_P as function of λ and θ (this is commonly referred to as c_P - λ table) allows calculating the power of the turbine in a wide operation range.

However, the c_P - λ table is usually large and therefore the method is not very efficient in simulations especially if many different wind turbines have to be represented. Besides, searching the right operating point in a two-dimensional table requires an interpolation in every simulation time step, this can also be time consuming. Therefore, a simplified representation of c_P is favorable if the accuracy requirements are not very challenging as it is usually the case in grid studies.

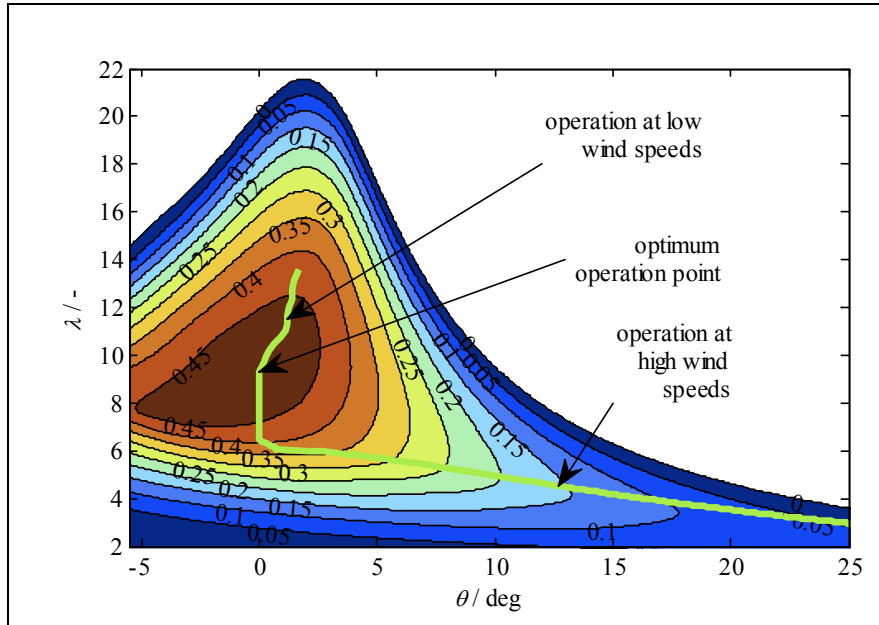


Fig. 2.5 Power coefficient c_P as function of blade angle θ and tip-speed ratio λ .

2.4.1 Functional Representations for c_p - λ Tables

Some efforts have been published to reduce the number of parameters needed to represent the content of c_p - λ tables. A functional representation and a modification of this approach are presented here and analyzed later in section 2.5.9.

Functional Representation by Anderson and Bose

An often cited functional representation first published by [16] in 1983 and modified for the development of the GROWIAN by [17] in the form presented by [18] is

$$c_p(\Theta, \lambda) = c_1 \left(\frac{c_2}{\lambda_1} - c_3 \Theta - c_4 \Theta^{c_5} - c_6 \right) e^{c_7 / \lambda_1} \quad (2.16)$$

with

$$\frac{1}{\lambda_1} = \frac{1}{\lambda + c_8 \Theta} - \frac{c_9}{\Theta^3 + 1} \quad (2.17)$$

and the coefficients

$$c_{1-9} = [0.5 \quad 116 \quad 0.4 \quad 0 \quad 2 \quad 5 \quad 21 \quad 0.08 \quad 0.035] \quad (2.18)$$

Modified Functional Representation

A comparison with blade data used in modern turbines shows that the accuracy of (2.16) is not acceptable for modern turbines. The parameter sets given by the authors does not fit with modern blades good enough (see Fig. 2.6). A comparison of the representation with real turbine data of a 2 MW turbine shows that a rescaling of the blade angle allows an improved representation. This can be achieved with

$$\Theta' = c_{10} \Theta \quad (2.19)$$

by adding and additional coefficient c_{10} and substituting Θ by Θ' in (2.16) and (2.17). In a second step, the coefficients need to be updated. Using a nonlinear parameter fit [19], the following coefficients were calculated for a 2 MW turbine:

$$c_{1-10} = [0.297 \quad 118 \quad -0.50 \quad 0.922 \quad 1.12 \quad 3.33 \quad 15.6 \quad 0.102 \quad 0.017 \quad 0.751] \quad (2.20)$$

This modification leads to a significantly improved correlation of the c_p - λ data generated by (2.16) with real turbines (see Fig. 2.7) The results of these functions are further analyzed in section 2.5.9.

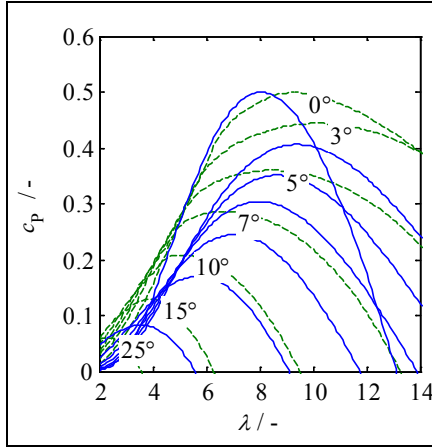


Fig. 2.6 c_p values for the original functional representation according to (2.16) (solid) and reference (dashed).

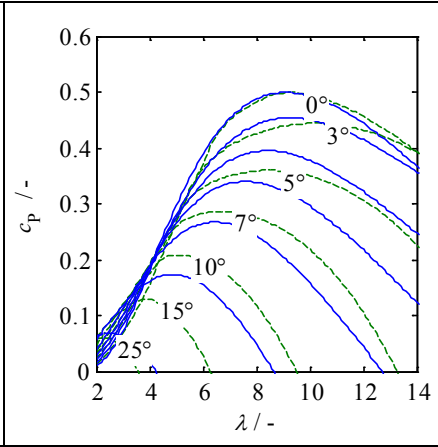


Fig. 2.7 c_p values for the modified functional representation according to (2.17) (solid) and reference (dashed).

2.4.2 Polynomial Fit Representation for c_p - λ Tables

A different approach for reducing the number of parameters compared to a c_p - λ table consists in using a polynomial fit representation [20]

$$c_p(\Theta, \lambda) = \sum_{i=1}^n \sum_{j=1}^m \alpha_{i,j} \Theta^i \lambda^j \quad (2.21)$$

with the coefficients $\alpha_{i,j}$ and the order of the polynomial given by n and m .

It can be shown that an even number parameters leads to an improved representation at the outer limits of the function. A good representation within a limited range from $\lambda = [2; 13]$ and $\Theta = [0; 25]$ deg can be reached with a 4×4 representation using 16 coefficients.

$$[\alpha_{i,j}] = \begin{bmatrix} 8.4018\text{e-}008 & -8.2359\text{e-}007 & 9.8303\text{e-}006 & -1.8864\text{e-}005 \\ -8.4245\text{e-}006 & 7.5152\text{e-}005 & -5.8513\text{e-}004 & 7.3378\text{e-}004 \\ 7.2467\text{e-}005 & -1.0032\text{e-}003 & 1.5504\text{e-}003 & 1.1218\text{e-}002 \\ 5.8657\text{e-}005 & -9.0477\text{e-}003 & 1.6776\text{e-}001 & -3.3920\text{e-}001 \end{bmatrix} \quad (2.22)$$

A comparison of the polynomial representation with the c_p - λ curves of a 2 MW turbine is shown in Fig. 2.8. The results of this function are further analyzed in section 2.5.9.

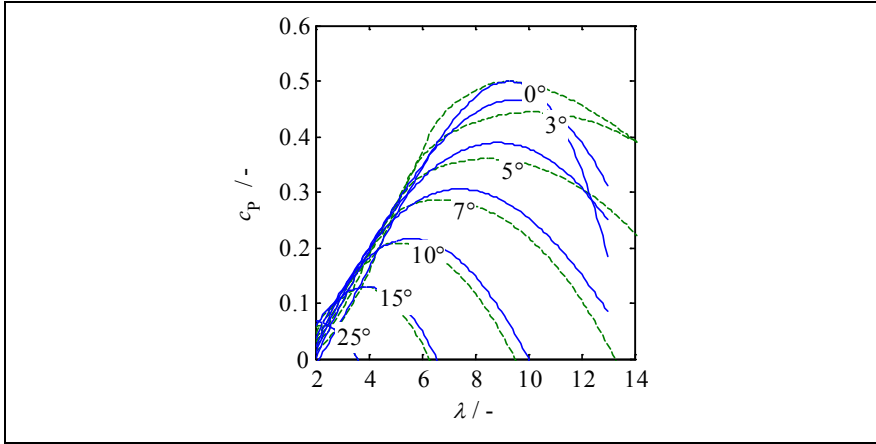


Fig. 2.8 c_p values for the 3rd order polynomial aerodynamic model (solid) and reference (dashed).

2.5 Linearized Aerodynamic Models

The aerodynamic models presented have in common that

- a large number of parameters is required. These parameters may to some extent be confidential and not available for users of the model
- the parameters give no insight into the physical relations of the model
- the model is highly nonlinear, the calculation of initial values for the simulation is therefore possible only through iteration

The possible advantages of a linearized aerodynamic model are

- a reduction of the number of parameters required
- the use of parameters that show clear physical relations
- a direct calculation of the initial values
- reduction of the simulation effort

An approach for a linearized aerodynamic model had been proposed in [4], the results had been compared to simulations with a more detailed model in [21]. But applicability of this model is limited to transmission system faults cleared within 150 ms to 200 ms and the assumption of constant wind speed [22].

An improved model is proposed that includes the impact of changes of rotor speed and changes of wind speed. As a result, the accuracy of the model for the

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