2 Description of the Model

The following model mainly follows Galí and Monacelli (2005) and Monacelli (2005). It is the standard DSGE model of an open economy with microfoundations. The model is extended by external habit formation, labor market imperfections, incomplete asset markets, indexation of prices and various stochastic shocks. The habit formation is based on an argument called 'keeping up with the Joneses', which describes social comparisons in consumption choices. The concept was introduced to the monetary policy literature by Fuhrer (2000).[1] In the goods and in the labor market, prices and wages are set according to a price-setting à la Calvo (1983) where non-optimized prices and wages are adapted to past inflation as suggested by Christiano et al. (2005). Compared to Galí and Monacelli (2005) and Monacelli (2005), the modified set-up contains preference, labor supply and cost-push shocks. Shocks are directly introduced in the following derivations. A small open DSGE model with analogous modifications was given by Justiniano and Preston (2008). A complete set of the model's equations is provided in chapter 1 of the appendix.

The model consists of two economies: a domestic and a foreign economy. The domestic economy is assumed to be small compared to the foreign economy. Variables describing the foreign economy are indicated with the superscript '*'.

The monetary policy rule of the domestic economy is distinguished in two cases: inflation targeting and price-level targeting. Because of their importance to the topic, the two rules are discussed in a separate chapter, 2.4.

[1] The history of the concept of habit formation is long. For example, in the finance literature it was applied by Constantinides (1990).

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2.1 Domestic Economy

The domestic economy consists of households, producers, retail firms and a monetary authority. The following sections derive the optimal behavior of each of these agents.

2.1.1 Households

The domestic economy has access to foreign goods through imports and to foreign bonds through financial markets. The domestic households undertake final consumption of domestic and foreign goods. They provide working hours as a production factor to domestic producers. The optimal mixture of consumption and work is derived by maximizing the household's preferences. There exists a continuum of households in the domestic economy. However, for the goal of deriving the optimality conditions, the focus stays on a single representative household. The derived optimality conditions hold analogously for the whole continuum.

Utility

The domestic household chooses consumption (C_t), labor input (N_t, working hours), domestic bonds (D_t) and foreign bonds (B_t) such as to maximize its utility function.

$$\max_{\{C_t,N_t,D_t,B_t\}_{t=0}^{\infty}} E_0 \sum_{s=0}^{\infty} \beta^s \epsilon_{g,t} \left[\frac{(C_{t+s} - H_{t+s})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{\epsilon_{n,t}(N_{t+s})^{1+\varphi}}{1+\varphi} \right] \tag{2.1}$$

H_t describes the habit formation. It is not regarded as a choice variable, since the habit formation is assumed to be external. $H_t = hC_{t-1}$ holds in all periods with $0 < h < 1$. $\epsilon_{g,t}$ and $\epsilon_{n,t}$ denote the preference and the labor supply shock. The parameters σ^{-1} and φ are assumed to be positive: $\sigma^{-1} > 0$ and $\varphi > 0$. β is the discount factor.

The maximization is limited by the following budget constraint that has to be satisfied in each period:

$$
\begin{aligned}
P_tC_t + D_t + X_tB_t =& R_{t-1}D_{t-1} + X_{t-1}R^*_{t-1}\phi_t(A_t)B_{t-1} + W_tN_t + T_t \\
&\text{with} \quad \phi_t(A_t) = exp\left[-\chi(A_t + \epsilon_{rp,t})\right] \\
&\text{and} \quad A_t = \frac{S_{t-1}B_{t-1}}{\bar{C}_F P_{t-1}}
\end{aligned} \tag{2.2}
$$

The price index (P_t) corresponds to the domestic consumer price index (CPI). W_t is the domestic wage level. R_t and R_t^* are the domestic and the foreign interest rate, respectively. T_t is assumed to be a lump-sum transfer including profits of the firms as well as taxes. X_t is the nominal exchange rate. Following Justiniano and Preston (2008) and Kollmann (2002), the function $\phi(.)$ is a debt elastic interest rate and A_t is the real quantity of outstanding foreign debt in terms of domestic currency. $\epsilon_{rp,t}$ denotes the risk premium shock. Finally, $\bar{C}_F$ is the steady state consumption of imported goods. The debt elastic interest rate is crucial for the stationarity of foreign debt in the log-linearized model.

The described maximization problem can be solved using the Lagrangian method with a multiplier λ_t.

$$
\begin{aligned}
\mathcal{L} = \quad & E_0 \sum_{s=0}^{\infty} \beta^s \epsilon_{g,t} [\left(\frac{(C_{t+s} - H_{t+s})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{\epsilon_{n,t}(N_{t+s})^{1+\varphi}}{1+\varphi} \right) \\
& - \lambda_{t+s}(P_{t+s}C_{t+s} + D_{t+s+1} + X_{t+s}B_{t+s} - R_{t+s-1}D_{t+s-1} \\
& - X_{t+s}R^*_{t+s-1}\phi_{t+s}B_{t+s-1} - W_{t+s}N_{t+s} - T_{t+s})]
\end{aligned} \tag{2.3}
$$

Taking the derivatives with respect to C_t, N_t, B_t and D_t yields to the following optimality conditions.

$$
\frac{\partial \mathcal{L}}{\partial C_t} : \epsilon_{g,t} \left(C_t - H_t\right)^{-\frac{1}{\sigma}} - \lambda_t P_t = 0 \quad \text{with} \quad H_t = hC_{t-1} \tag{2.4}
$$

$$
\frac{\partial \mathcal{L}}{\partial N_t} : - \epsilon_{n,t} N_t^{\varphi} + \lambda_t W_t = 0 \tag{2.5}
$$

$$
\frac{\partial \mathcal{L}}{\partial B_t} : - \lambda_t + \beta R_t E_t \left[\lambda_{t+1}\right] = 0 \tag{2.6}
$$

$$
\frac{\partial \mathcal{L}}{\partial D_t} : - \lambda_t X_t + \beta R_t^* E_t \left[\lambda_{t+1} X_{t+1} \phi_{t+1}\right] = 0 \tag{2.7}
$$

Furthermore, the budget constraint (derivative with respect to λ_t) and the transversality condition have to be satisfied.

(2.4) can be used to substitute for the multiplier λ_t in the conditions (2.5) to (2.7).

$$\epsilon_{n,t} N_t^{\varphi} \left(C_t - hC_{t-1}\right)^{1/\sigma} = \frac{W_t}{P_t} \tag{2.8}$$

$$\epsilon_{g,t} \left(C_t - hC_{t-1}\right)^{-1/\sigma} = \beta R_t E_t \left[\left(\frac{P_t}{P_{t+1}} \right) \epsilon_{g,t+1} \left(C_{t+1} - hC_t\right)^{-1/\sigma} \right] \tag{2.9}$$

$$\epsilon_{g,t} \left(C_t - hC_{t-1}\right)^{-1/\sigma} = \beta R_t^* E_t \left[\phi_{t+1} \left(\frac{X_{t+1} P_t}{X_t P_{t+1}} \right) \epsilon_{g,t+1} \left(C_{t+1} - hC_t\right)^{-1/\sigma} \right] \tag{2.10}$$

Consumption Allocation

Maximizing its utility function, the household has chosen an optimal consumption bundle C_t. The household's consumption bundle is a combination of a domestic consumption bundle ($C_{H,t}$) and foreign consumption bundle ($C_{F,t}$).

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2.11}$$

The parameter η is assumed to be positive: $\eta > 0$. α indicates the openness of the domestic economy and lies between 0 and 1: $0 < \alpha < 1$. Furthermore, the domestic and foreign consumption bundles are themselves combinations of domestic produced goods, $C_{H,t}(i)$, and foreign produced goods, $C_{F,t}(i)$, according to a Dixit-Stiglitz aggregator.

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\theta-1}{\theta}} \, di \right]^{\frac{\theta}{\theta-1}} \tag{2.12}$$

$$C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\theta-1}{\theta}} \, di \right]^{\frac{\theta}{\theta-1}} \tag{2.13}$$

θ is the elasticity of substitution and assumed to be positive: $\theta > 0$. The household allocates the given expenditures for consumption ($P_t C_t$) from the utility maximization regarding the aggregation index for C_t.

$$\min_{\{C_{H,t},C_{F,t}\}} P_t C_t - P_{H,t}C_{H,t} - P_{F,t}C_{F,t}$$

$$\text{s.t.} \quad C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2.14}$$

Replacing C_t by its aggregation index, it becomes an unconstrained maximization problem.

$$\mathcal{L} = P_t \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} - P_{H,t}C_{H,t} - P_{F,t}C_{F,t} \tag{2.15}$$

The corresponding optimality conditions are given by equations (2.16) and (2.17).

$$\frac{\partial \mathcal{L}}{\partial C_{H,t}} : P_t \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\left(\frac{\eta}{\eta-1}\right)\left(\frac{1}{\eta}\right)} (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{-\frac{1}{\eta}} - P_{H,t} = 0 \tag{2.16}$$

$$\frac{\partial \mathcal{L}}{\partial C_{F,t}} : P_t \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\left(\frac{\eta}{\eta-1}\right)\left(\frac{1}{\eta}\right)} \alpha^{\frac{1}{\eta}} C_{F,t}^{-\frac{1}{\eta}} - P_{F,t} = 0 \tag{2.17}$$

Solving for $C_{H,t}$ and $C_{F,t}$ delivers the following demand functions.

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{2.18}$$

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{2.19}$$

Secondly, the same optimization procedure is followed for domestic goods ($C_{H,t}$) and foreign goods ($C_{F,t}$). The households allocate their expenditures for domestic and foreign goods across a continuum of domestic and foreign produced goods.

$$\min_{\{C_{j,t}(i)\}} P_{j,t}C_{j,t} - \int_0^1 P_{j,t}(i)C_{j,t}(i)\, di$$

$$\text{s.t.} \quad C_{j,t} = \left[\int_0^1 C_{j,t}(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \quad \text{with} \quad j = \{H, F\} \tag{2.20}$$

The Lagrangian for the unconstrained problem is given by equation (2.21).

$$\mathcal{L} = P_{j,t}\left[\int_0^1 C_{j,t}(i)^{\frac{\theta-1}{\theta}}\, di\right]^{\frac{\theta}{\theta-1}} - \int_0^1 P_{j,t}(i) C_{j,t}(i)\, di \quad \text{with} \quad j = \{H, F\} \tag{2.21}$$

Taking the derivative with respect to $C_{j,t}(i)$ delivers the following first order condition (FOC).

$$\frac{\partial \mathcal{L}}{\partial C_{j,t}(i)} : P_{j,t}\left[\int_0^1 C_{j,t}(i)^{\frac{\theta-1}{\theta}}\, di\right]^{\left(\frac{\theta}{\theta-1}\right)\left(\frac{1}{\theta}\right)} C_{j,t}(i)^{-\frac{1}{\theta}} - P_{j,t}(i) = 0$$
$$\text{with} \quad j = \{H, F\} \tag{2.22}$$

Solving for $C_{H,t}(i)$ and $C_{F,t}(i)$ delivers the two demand functions (2.23) and (2.24).

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\theta} C_{H,t} \tag{2.23}$$

$$C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\theta} C_{F,t} \tag{2.24}$$

Price-Level

The following expression holds because of the price-taking and non-satiation argument: $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$. Using the demand functions (2.18) and (2.19) to replace $C_{H,t}$ and $C_{F,t}$ gives the following definition of the price-level:

$$P_t = \left[(1-\alpha)\, P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2.25}$$

Using the same methodology, the price-level for domestic goods ($P_{H,t}$) and foreign goods ($P_{F,t}$) can be calculated. In this case, the price-taker and non-satiation assumption yields: $P_{H,t} C_{H,t} = \int_0^1 P_{H,t}(i) C_{H,t}(i)\, di$ and $P_{F,t} C_{F,t} = \int_0^1 P_{F,t}(i) C_{F,t}(i)\, di$. Replacing $C_{H,t}(i)$ and $C_{F,t}(i)$ by its demand functions, (2.23) and (2.24), defines the price-levels $P_{H,t}$ and $P_{F,t}$.

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{2.26}$$

$$P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{2.27}$$

Labor Supply

Finally, the households decide how many working hours they provide to the domestic producers. There is a single domestic labor market where domestic producers hire labor inputs at a common wage. For the derivation of the optimal labor supply, the representative household makes use of the producer's labor demand function (see chapter 2.1.2).

The households are assumed to have monopolistic power in the labor market. They are allowed to re-optimize their wage in a given period with a probability of $1 - \xi_w$ where $0 < \xi_w < 1$. On the contrary, the probability of staying with the same wage as in the previous period is simply ξ_w. If this is the case, the wage is adjusted according to the following indexation rule:

$$log\ W_t(k) = log\ W_{t-1}(k) + \gamma_w \pi_{t-1} \tag{2.28}$$

$W_t(k)$ is the wage of household k. γ_w denotes the degree of indexation to the previous period's inflation and is limited by: $0 \leq \gamma_w \leq 1$. The inflation rate is defined as follows: $\pi_{t-1} = log\ P_t - log\ P_{t-1}$. This methodology was introduced by Calvo (1983) for the price-setting behavior in a monopolistic good market, whereas Erceg et al. (2000) applied the concept to the labor market.

Under this assumption, the wage-setting problem becomes dynamic. The representative household solves the following maximization problem by its choice of $\bar{W}_t(k)$, that is constrained by the producer's demand function. A derivation is provided in chapter 2.1.2.

$$\max_{\{\bar{W}_t(k)\}} E_t \sum_{s=0}^{\infty} (\xi_w \beta)^s \left[\lambda_s \bar{W}_t(k) \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\gamma_w} N_{t+s}(k) - \frac{\epsilon_{n,t} N_{t+s}(k)^{1+\varphi}}{1+\varphi} \right]$$
$$\text{s.t.} \quad N_{t+s}(k) = \left(\frac{\bar{W}_t(k)}{W_{t+s}} \right)^{-\theta_w} N_{t+s}(j) \tag{2.29}$$

As the following Lagrangian shows, the optimization problem become unconstrained by plugging the demand function for $N_{t+s}(k)$.

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} (\xi_w \beta)^s \left[\lambda_s \bar{W}_t(k) \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\gamma_w} \left(\frac{\bar{W}_t(k)}{W_{t+s}} \right)^{-\theta_w} N_{t+s}(j) - \frac{\epsilon_{n,t}}{1+\varphi} \left(\left(\frac{\bar{W}_t(k)}{W_{t+s}} \right)^{-\theta_w} N_{t+s}(j) \right)^{1+\varphi} \right] \tag{2.30}$$

This yields to the following optimality condition.

$$\frac{\partial \mathcal{L}}{\partial \bar{W}_t(k)} : E_t \sum_{s=0}^{\infty} (\xi_w \beta)^s N_{t+s}(k) \left[\lambda_s \left(\frac{P_{t+s-1}}{P_{t-1}} \right) \bar{W}_t(k) - \left(\frac{\theta_w}{\theta_w - 1} \right) \epsilon_{n,t} N_{t+s}(k)^{\varphi} \right] = 0 \tag{2.31}$$

2.1.2 Producers

There is a continuum of producers in the domestic economy. The producers hire labor input from the households in the labor market and sell their outputs to domestic and foreign households. Therefore, the producers have to decide about their optimal labor demand and their optimal price-setting behavior. For the purpose of getting the optimality conditions, the focus stays on a single representative firm. The conditions hold analogously for the whole continuum of firms.

Labor Demand

The domestic goods from firm j, $Y_t(j)$, are produced according to the given production function.

$$Y_t(j) = \epsilon_{a,t} f(N_t(j)) \tag{2.32}$$

The function $f(.)$ is assumed to satisfy the Inada conditions. $\epsilon_{a,t}$ is a domestic technology shock that is independent of j and therefore the same for all domestic producers. $N_t(j)$ is the aggregated labor input for firm j that is given by the following constant elasticity of substitution (CES) aggregator.

$$N_t(j) = \left[\int_0^1 N_t(k)^{\frac{\theta_w - 1}{\theta_w}}\right]^{\frac{\theta_w}{\theta_w - 1}} \tag{2.33}$$

θ_w is assumed to be greater than 1: $\theta_w > 1$. Similar to the households, producers allocate their expenditures for labor input optimally across all households. Given the expenditure for labor input, producer j solves the following problem to allocate its expenditures.

$$\max_{\{N_t(k)\}} W_t N_t(j) - \int_0^1 N_t(k) W_t(k)\, dk \quad \text{s.t.} \quad N_t(j) = \left[\int_0^1 N_t(k)^{\frac{\theta_w - 1}{\theta_w}}\right]^{\frac{\theta_w}{\theta_w - 1}} \tag{2.34}$$

Replace $N_t(j)$ by the CES aggregator. The unconstrained problem is given by the following Lagrangian.

$$\mathcal{L} = \quad W_t \left[\int_0^1 N_t(k)^{\frac{\theta_w - 1}{\theta_w}}\right]^{\frac{\theta_w}{\theta_w - 1}} - \int_0^1 N_t(k) W_t(k)\, dk \tag{2.35}$$

The FOC is derived by setting the derivative of the Lagrangian with respect to $N_t(k)$ equal to 0.

$$\frac{\partial \mathcal{L}}{\partial N_t(k)} : W_t \left[\int_0^1 N_t(k)^{\frac{\theta_w - 1}{\theta_w}}\right]^{\left(\frac{\theta_w}{\theta_w - 1}\right)\left(\frac{1}{\theta_w}\right)} N_t(k)^{-\frac{1}{\theta_w}} - W_t(k) = 0 \tag{2.36}$$

Solving this expression for $N_t(k)$ yields to the producer's demand function for labor input of type k.

$$N_t(k) = \left(\frac{W_t(k)}{W_t}\right)^{-\theta_w} N_t(j) \tag{2.37}$$

Price-Setting

The representative producer j has monopolistic power in its good market. The price-setting method is adopted from Calvo (1983). With probability $1 - \xi_H$ the producer is allowed to change its price in a given period. ξ_H is limited by: $0 < \xi_H < 1$. Prices that are not re-optimized are indexed to previous period's inflation rate. The indexation rule is given by the following expression.

$$log\ P_{H,t}(j) = log\ P_{H,t-1}(j) + \gamma_H \pi_{H,t-1} \tag{2.38}$$

γ_H defines the degree of indexation and is limited as follows: $0 \leq \gamma_H \leq 1$. $\pi_{H,t}$ is the inflation rate of the domestic goods price. It is defined as $\pi_{H,t} = log\ P_{H,t} - log\ P_{H,t-1}$. As with every firm that has monopolistic power, producer j wants to maximize the expected discounted value of its profits by setting an optimal price, $\bar{P}_{H,t}(j)$. The maximization problem is constrained by the demand function for its goods.

$$\begin{aligned} \max_{\{\bar{P}_{H,t}(j)\}} E_t \sum_{s=0}^{\infty} \xi_H^s Q_{t,t+s} [\bar{P}_{H,t}(j) \left(\frac{P_{H,t+s-1}}{P_{H,t-1}}\right)^{\gamma_H} Y_{H,t+s}(j) \\ - P_{H,t+s} MC_{t+s}^n Y_{H,t+s}(j)] \\ \text{s.t.} \quad Y_{H,t+s}(j) = \left(\frac{\bar{P}_{H,t}(j)}{P_{H,t}} \left(\frac{P_{H,t+s-1}}{P_{H,t-1}}\right)^{\gamma_H}\right)^{-\theta} \left(C_{H,t+s} + C_{H,t+s}^*\right) \end{aligned} \tag{2.39}$$

$Q_{t,t+1}$ is the firm's discount factor and MC_t^n is the nominal marginal cost. Again, the demand function can be replaced to derive an unconstrained optimization problem. The Lagrangian is given by the following equation (2.40).

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