

# Preface

When no samples are available to estimate a probability distribution, we have to invite some domain experts to evaluate the belief degree that each event will happen. Perhaps some people think that the belief degree should be modeled by subjective probability or fuzzy set theory. However, it is usually inappropriate because both of them may lead to counterintuitive results in this case. In order to rationally deal with belief degrees, uncertainty theory was founded in 2007 and subsequently studied by many researchers. Nowadays, uncertainty theory has become a branch of axiomatic mathematics for modeling belief degrees.

## **Uncertain Measure**

The most fundamental concept is uncertain measure that is a type of set function satisfying the axioms of uncertainty theory. It is used to indicate the belief degree that an uncertain event may happen. Chapter 1 will introduce normality, duality, subadditivity and product axioms. From those four axioms, this chapter will also present uncertain measure, product uncertain measure, and conditional uncertain measure.

## **Uncertain Variable**

Uncertain variable is a measurable function from an uncertainty space to the set of real numbers. It is used to represent quantities with uncertainty. Chapter 2 is devoted to uncertain variable, uncertainty distribution, independence, operational law, expected value, variance, moments, entropy, distance, conditional uncertainty distribution, uncertain sequence, and uncertain vector.

## **Uncertain Programming**

Uncertain programming is a type of mathematical programming involving uncertain variables. Chapter 3 will provide a type of uncertain programming model with applications to machine scheduling problem, vehicle routing problem, and project scheduling problem. In addition, uncertain multiobjective programming, uncertain goal programming and uncertain multilevel programming are also documented.

## **Uncertain Statistics**

Uncertain statistics is a methodology for collecting and interpreting expert's experimental data by uncertainty theory. Chapter 4 will present a questionnaire survey for collecting expert's experimental data. In order to determine uncertainty distributions from those expert's experimental data, Chapter 4 will also introduce empirical uncertainty distribution, principle of least squares, method of moments, and Delphi method.

## **Uncertain Risk Analysis**

The term risk has been used in different ways in literature. In this book the risk is defined as the accidental loss plus the uncertain measure of such loss, and a risk index is defined as the uncertain measure that some specified loss occurs. Chapter 5 will introduce uncertain risk analysis that is a tool to quantify risk via uncertainty theory. As applications of uncertain risk analysis, Chapter 5 will also discuss structural risk analysis and investment risk analysis.

## **Uncertain Reliability Analysis**

Reliability index is defined as the uncertain measure that some system is working. Chapter 6 will introduce uncertain reliability analysis that is a tool to deal with system reliability via uncertainty theory.

## **Uncertain Propositional Logic**

Uncertain propositional logic is a generalization of propositional logic in which every proposition is abstracted into a Boolean uncertain variable and the truth value is defined as the uncertain measure that the proposition is true. Chapter 7 will present uncertain propositional logic and uncertain predicate logic. In addition, uncertain entailment is a methodology for determining the truth value of an uncertain proposition via the maximum uncertainty principle when the truth values of other uncertain propositions are given. Chapter 8 will discuss an uncertain entailment model from which uncertain modus ponens, uncertain modus tollens and uncertain hypothetical syllogism are deduced.

## **Uncertain Set**

Uncertain set is a set-valued function on an uncertainty space, and attempts to model "unsharp concepts". The main difference between uncertain set and uncertain variable is that the former takes values of set and the latter takes values of point. Uncertain set theory will be introduced in Chapter 9. In order to determine membership functions, Chapter 9 will also provide some methods of uncertain statistics.

## **Uncertain Logic**

Some knowledge in human brain is actually an uncertain set. This fact encourages us to design an uncertain logic that is a methodology for calculating the truth values of uncertain propositions via uncertain set theory. Uncertain logic may provide a flexible means for extracting linguistic summary from a collection of raw data. Chapter 10 will be devoted to uncertain logic and linguistic summarizer.

## **Uncertain Inference**

Uncertain inference is a process of deriving consequences from human knowledge via uncertain set theory. Chapter 11 will present a set of uncertain inference rules, uncertain system, and uncertain control with application to an inverted pendulum system.

## **Uncertain Process**

An uncertain process is essentially a sequence of uncertain variables indexed by time. Thus an uncertain process is usually used to model uncertain phenomena that vary with time. Chapter 12 is devoted to basic concepts of uncertain process and uncertainty distribution. In addition, extreme value theorem, first hitting time and time integral of uncertain processes are also introduced. Chapter 13 deals with uncertain renewal process, renewal reward process, and alternating renewal process. Chapter 13 also provides block replacement policy, age replacement policy, and an uncertain insurance model.

## **Uncertain Calculus**

Uncertain calculus is a branch of mathematics that deals with differentiation and integration of uncertain processes. Chapter 14 will introduce Liu process that is a stationary independent increment process whose increments are normal uncertain variables, and discuss Liu integral that is a type of uncertain integral with respect to Liu process. In addition, the fundamental theorem of uncertain calculus will be proved in this chapter from which the techniques of chain rule, change of variables, and integration by parts are also derived.

## **Uncertain Differential Equation**

Uncertain differential equation is a type of differential equation involving uncertain processes. Chapter 15 will discuss the existence, uniqueness and stability of solutions of uncertain differential equations, and will introduce Yao-Chen formula that represents the solution of an uncertain differential equation by a family of solutions of ordinary differential equations. On the basis of this formula, some formulas to calculate extreme value, first hitting

time, and time integral of solution are provided. Furthermore, some numerical methods for solving general uncertain differential equations are designed.

### Uncertain Finance

As applications of uncertain differential equation, Chapter 16 will discuss uncertain stock model, uncertain interest rate model, and uncertain currency model.

### Law of Truth Conservation

The law of excluded middle tells us that a proposition is either true or false, and the law of contradiction tells us that a proposition cannot be both true and false. In the state of indeterminacy, some people said, the law of excluded middle and the law of contradiction are no longer valid because the truth degree of a proposition is no longer 0 or 1. I cannot gainsay this viewpoint to a certain extent. But it does not mean that you might “go as you please”. *The truth values of a proposition and its negation should sum to unity.* This is the law of truth conservation that is weaker than the law of excluded middle and the law of contradiction. Furthermore, the law of truth conservation agrees with the law of excluded middle and the law of contradiction when the uncertainty vanishes.

### Maximum Uncertainty Principle

An event has no uncertainty if its uncertain measure is 1 because we may believe that the event happens. An event has no uncertainty too if its uncertain measure is 0 because we may believe that the event does not happen. An event is the most uncertain if its uncertain measure is 0.5 because the event and its complement may be regarded as “equally likely”. In practice, if there is no information about the uncertain measure of an event, we should assign 0.5 to it. Sometimes, only partial information is available. In this case, the value of uncertain measure may be specified in some range. What value does the uncertain measure take? *For any event, if there are multiple reasonable values that an uncertain measure may take, then the value as close to 0.5 as possible is assigned to the event.* This is the maximum uncertainty principle.

### Matlab Uncertainty Toolbox

Matlab Uncertainty Toolbox (<http://orsc.edu.cn/liu/resources.htm>) is a collection of functions built on Matlab for many methods of uncertainty theory, including uncertain programming, uncertain statistics, uncertain risk analysis, uncertain reliability analysis, uncertain logic, uncertain inference, uncertain differential equation, scheduling, logistics, data mining, control, and finance.

## **Lecture Slides**

If you need lecture slides for uncertainty theory, please download them from the website at <http://orsc.edu.cn/liu/resources.htm>.

## **Uncertainty Theory Online**

If you want to read more papers related to uncertainty theory and applications, please visit the website at <http://orsc.edu.cn/online>.

## **Purpose**

The purpose is to equip the readers with a branch of axiomatic mathematics to deal with belief degrees. The textbook is suitable for researchers, engineers, and students in the field of mathematics, information science, operations research, industrial engineering, computer science, artificial intelligence, automation, economics, and management science.

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Baoding Liu  
Tsinghua University  
<http://orsc.edu.cn/liu>

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Liu, B.

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