

Chapter 2

Methods and Mathematical Models of Budget Management

2.1 Current Trends in Budgeting

Budget is a centralized monetary state fund the designated for financial support of implementation of its tasks and functions. The state put forward the task of creation of a program-based targeted control of budget funds with account for priorities of socio-economic development of the country. Taking into account specific features of the development of the budgeting process in the Republic of Kazakhstan and provisions of the new Law on Budget Planning it is expedient to consider the budgeting process as a management system based on the principles of budget programming.

In the conditions of socio-economic development of the state the problem of allocation of budgetary funds is one of the main tasks of public administration and state regulation of financial and fiscal processes. In the annual program of Government of the Republic of Kazakhstan the first priority task in the budget policy is implementation of long-term planning and construction of trajectories of program actions aimed at budget performance as a system of allocation of budget funds. A proper budget planning must take into account budget targets and expenses needed for their realization and must be focused on solution of key tasks and problems for the planned system as well as planned concentration of funds. It generated a necessity of implementation of a new method called *budget programming* as a means of development of a “clever” economy. This method justifies interrelation between economic forecasts and long-term targets in the development of the country/region; it forms the base for various scenarios of economic development, thus, implementing the program-targeted approach to planning.

The concept of budget programming is in compliance with one of the modern directions of the result-oriented budgeting (ROB), its aim is to interrelate the decisions on expenditures with the expected return of the expenditures, their effectiveness and efficiency. Budget programming is based on the principles of mathematical methods of the control theory, namely, program control which is oriented on the final result and may be considered as an optimization task of a system transfer from the initial state to the required one.

Budget forecasting is a methodological approach to budget control, in western countries such a methodology was called *budgeting oriented on the result or a concept* (model) *of result-oriented budgeting in the framework of the middle-term financial planning* (ROB). Its essence is the distribution of budget resources among administrators of budget funds and/or budget programs realized by the administrators with account for or in direct dependence on the achievement of concrete results (rendering of services) according to the middle-term priorities of the socio-economic policy and within the budget funds forecasted for the middle-term period.

The pioneers of the result-oriented budgeting are the USA, New Zealand, Australia, the Netherlands, Great Britain and Sweden. France and Germany started to implement the model of result-oriented budget planning later.

The term *result-oriented budgeting* (ROB) means such an approach to the budget process where spending of financial resources is related to expected important social results. Unlike the traditional system of cost-in-no-object approach answering the question, “How much must we spend?” the ROB system enables us to answer the question, “What social result will be achieved at the expense of spent funds?”

The present-day concepts of the result-oriented budgeting are based on the concept of the Program-Targeted Planning developed in the 1960–1970s in the USSR and the Planning-Programming-Budgeting System (PPBS) developed in the USA in the late 1950s–early 1960s.

The key unit in the ROB system is monitoring of the results. Based on its results the officials make decisions on the expediency of financing of a budget program in the sums specified in the previous fiscal year.

The international experience of ROB implementation [1–5] shows that the system of indicators of budget efficiency is used as an instrument of accounting of public authorities to the society.

In [2–4] the author presents a description of the interrelation between priorities at national, regional and municipal levels. The upper horizon of planning is presented by a five-year strategic plan of the territory development. The middle-term level is presented by a two-year plan-schedule of measures taken to realize the strategy (it is updated and specified by the results of each fiscal year) and a three-year financial plan (budget). Targets of the plan-schedule and financial plan concretize the long-term targets presented in the strategic plan. The first financial year is described in more detail than the following years. The next two years are described at a larger scale. Such a system makes it possible to maximally use the potential of the model of the result-oriented regional financial management.

According to [2–4] the experience of Great Britain demonstrates that ROB use makes it possible to develop approaches to solving the following problems:

- Allocation of budget funds not according to the types of expenditures but according to strategic plans;
- Rendering of services really needed by the population;
- Control of allocations of budget services by choosing the most economically efficient method of their rendering;
- Comparison of expenditure programs and choice of the most efficient ones by the results of efficiency of expenditures;

- Comparison of services in terms of their quality with other similar services in other countries, regions and cities;
- Higher transparency and better substantiation of budget expenditures;
- Determination of positive social effect of the service and not only its cost;
- Shift of the accent from the external control of the targeted use of financial allocations to the increase in internal responsibility and internal control of the efficiency of expenditures;
- Account for the consequences of decisions on the volume and structure of the expenditure items of the budget.

The ROB concept is a modern methodological approach to budget control, the principles of which are reflected in the program method of budget control.

2.2 Current State of Budget Control Methods and Mathematical Models

To forecast budget development there are a number of methods that can be used, including mathematical modeling, index, normative projection, expert evaluation, and balance, among others.

The first of these, the mathematical modeling method, is based on the usage of economic and/or mathematical models that allow a great number of interrelated factors influencing budget items to be taken into account. They also allow leeway in determining the technique of budget forecasting, and in choosing from among several budget variants the optimal one corresponding to the accepted strategy of social-economic development of the country and the budgetary policy being pursued. Such methods are considered in [10–27].

The index—or indicative—method is based on various indicators characterizing socio-economic development of the country/region. The indicators connect the decisions on expenditures with the expected returns from such expenditures, their efficiency, and their effectiveness. This concept is also used to assess the quality of budget control in order to improve the effectiveness of financial resource management in the area. Such methods are considered in [19].

The normative method is based on progressive norms and financial budgetary standards required to calculate budget revenue on the basis of established tax rates and a number of macroeconomic factors such as the tax burden, the budget deficit limit (percentage of GDP and budget expenditures), the maximal national debt, etc. This method was used to determine the budget paying capacity in [8, 31].

The method of expert evaluation is used when tendencies in the development of certain economic processes have not been determined, no analogues are available, and it is necessary to use special calculations performed by highly qualified experts. Various methods used to evaluate experts' estimations are outlined in [10, 20, 31, 32].

The balance method based on comparisons (assets with liabilities, the whole and its parts, etc.) enables the expenditures of any budget to be examined in relation to

its revenue. It is a useful method for ascertaining proportions in allocating funds amongst different budgets. This method is considered in detail in [29], where it is supposed that each item of budget receipts is proportionately distributed among all expenditure items.

Based on the logic of constructing balance models, the relation between the revenue and expenditure items can be presented as a matrix expression:

$$\vec{G} = \frac{1}{m} A \vec{D}, \quad (2.1)$$

where $A = \{a_{ij}\}$ is a matrix of interaction between revenue and expenditure items, consisting of n lines and m columns; $\vec{D} = (d_1 \dots d_j \dots d_m)^T$ is the vector of revenue; $\vec{G} = (g_1 \dots g_i \dots g_n)^T$ is the vector of expenditures; $d_1, d_2, \dots, d_m, g_1, g_2, \dots, g_n$ are the items of the corresponding budget parts at a certain level of the uniform budget classification.

The elements of matrix A correspond to the values:

$$a_{ij} = \frac{g_i}{d_j}, \quad (2.2)$$

where a_{ij} are the elements of the matrix of interaction of revenue and expenditure, g_i is the absolute value of the i -th item of expenditures $i = \overline{1, n}$, d_j is the absolute value of the j -th item of revenue $j = \overline{1, m}$.

This research is basic for the development of the technique of program budget control, as its structure and characteristics are able to reflect the relation between revenues and expenditures. Such an approach makes it possible not only to estimate the interaction between budget revenues and expenditures but also to show the interaction between current and capital components of the budget expenditure parts, as well as interrelation between indicators of socio-economic development and the current state of budget expenditures.

While a static model characterizes the budget state at a certain moment in time, if budget programming is focused on the program of socio-economic development, the static model will not adequately reflect the state of the budget system [6]. Therefore it is expedient to consider control, distribution, and redistribution of budget funds as a dynamic system.

General formulation of the problem of program control Let we know the initial state \tilde{g}_i^0 ($i = \overline{1, n}$) of the expenditure part of the budget. In such a case the rule $u_i^0 = w_0(g_i^0)$ enables us to find an optimal value \tilde{u}_i^0 of the controlling action of the i -th budget item at the first step: $\tilde{u}_i^0 = w_0(\tilde{g}_i^0)$. Then the next state of the item i will be unambiguously determined by the equation of the item movement: $\tilde{g}_i^1 = f(\tilde{g}_i^0, \tilde{u}_i^0)$. This property makes it possible to determine the optimal value \tilde{u}_i^1 of the control action at the second stage: $u_i^1 = w_1(g_i^1)$. Using the same procedure further we come to the conclusion: the optimal control strategy $u_i^s = w_s(g_i^s)$, $s = \overline{0, N}$ found using the principles of dynamic programming enables us for a given initial state \tilde{g}_i^0 of

the item i to consequently determine the optimal values $\tilde{u}_i^0, \tilde{u}_i^1, \dots, \tilde{u}_i^N$ of control action for the whole period of controlling of the item $g_i^{s+1} = f(g_i^s, u_i^s), s = \overline{0, N}$. The set of values of control action can be considered as a program control planned in advance for the item $g_i^{s+1} = f(g_i^s, u_i^s)$ which is in state \tilde{g}_i^0 at a given moment [14]. What structure does the control correspond? Let us suppose that there is an object of control—a budget item i and there is a control system—the department of budget planning, which sends impulses $\tilde{u}_i^s, s = \overline{0, N}$ to the input of the object calculated in advance for the initial state \tilde{g}_i^0 of the item i . It means that the control system must have an a priori information about the state \tilde{g}_i^0 . The feedback is not needed. At the same time this open control system realizes the same level of control as does the system with the feedback. If we solve the control problem, when we know in advance the behavior of the object and the conditions of its functioning, the feedback principle is not important and closed control can be replaced by an open control i.e. *program control*.

The program control is based on program movements that can transfer the system from the point $x = x_0$ at $t_0 = 0$ to the point $x = x_1$ at $t_T = T$ [15].

This approach considers a relation between socio-economic development of the country/region with budget funds. The presence of the target object as a group of indicators of socio-economic development makes it possible to develop a program control that allows us to transfer the budget system to the desired state defined by a system of indicators and to estimate a possibility of achieving a desired level of development under some limitations of budget resources and budget potential determined for the period of the middle-term planning.

2.3 General Concept of the Programmable Method of Budget Mechanism Control

2.3.1 General Statement of the Problem of Budget Mechanism Control

Problem statement. Consider a budget system as a programmable control system able to respond to change fulfill the movement program and to find the best solution to the given control task [6, 33–36].

Let us consider a macroeconomic overview of the role of the budget in the state structure.

The state as a subject of control acts as a force of unification, cooperation, and integration. The state is a regulator controlling the socio-economic sphere and regulating the economic and political processes that determine the socio-economic development of the country. Therefore a very important role in the macroeconomic process is played by the budget—a centralized monetary fund that acts as an object regulating state financial funds. Its role is to observe and control socio-economic development of the country. To provide high-quality control of the socio-economic sphere the state uses a regulating function allocating budget funds to support or

develop industries and regions; it can also purposefully intensify or hamper production rates, accelerate or decelerate growth of capital and private incomes, or change the supply and demand structure [7]. The dynamic nature of economic development makes it necessary to know or at least to have a qualitative picture of the future state of the socio-economic sphere, which depends on state financial resources accumulated in the budget. As a socio-economic sphere is a macroeconomic model, it is quite natural that the results of management are seen after some period of time, and are reflected in the budget [8, 9]. This means that the budget characterizes the level of state development. Therefore, it is possible to manipulate the budget or to develop a program algorithm of budget control, the output information of which will serve as control information for algorithm operation. And further, it is necessary therefore to formulate law regulating the “State–Budget–Socio-Economic Sphere” system, so as to provide optimal management of the object: the budget.

2.3.2 *Cybernetic Approach to the Description of Budget Mechanism*

In providing an overview of budget planning we will refer to the three principles of budget programming: determination of the program strategy for development, evaluation of budget resources, and planning of budget programs.

Definition 2.1 *The budget mechanism* is the process of management of budget resources.

Definition 2.2 *The program method* of budget management mechanism is the method based on the principles of budget programming.

Definition 2.3 *Budget programming* is the methodology of the middle-term planning and is focused on the final product, which must connect budget expenditures with the expected socio-economic results.

In terms of system analysis, the system of the budget management mechanism is determined by system objects, their properties, and their relations. Analysis of the process of budget management mechanism in terms of the system approach is based on the logical model described in the following schema (Fig. 2.1) [22].

The input in this schema is the budget dynamics, the system of indicators of socio-economic development.

The output is the forecast budget state, which can be corrected through the feedback system as the purpose of the suggested management model is sustainable budget development and achievement of a predetermined level of socio-economic development of the country/region.

The main purpose is realization of the strategy of national/regional development in terms of budget mechanism, taking into account resource limitations qualitatively

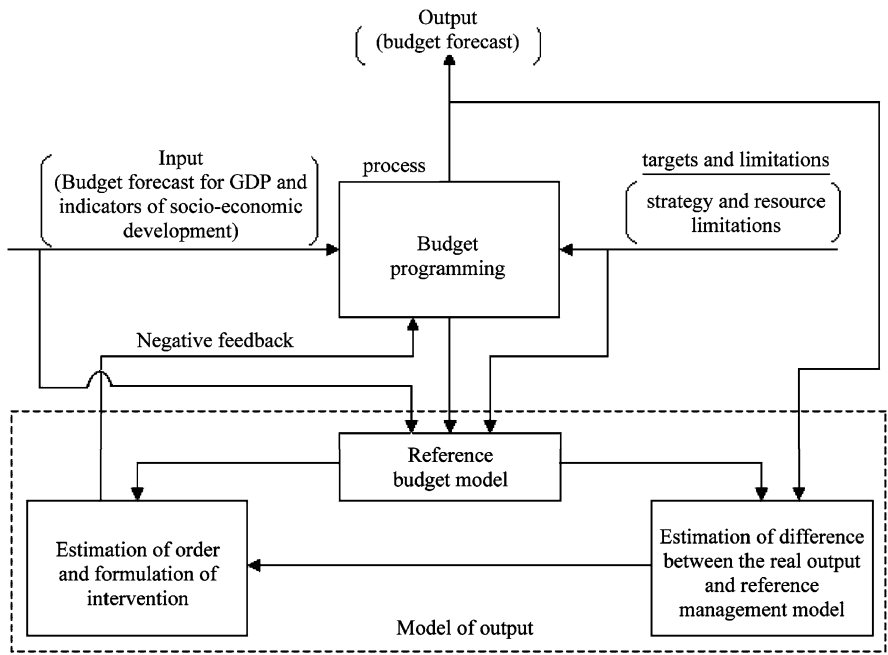


Fig. 2.1 A logical model schema

expressed in the general concept of sustainable budget development as a balance of income and expenditure budget parts.

The system is studied in terms of the process of budget mechanism management and the main principles of system operation, retention, and development.

Limitation is the amount of money allocated for development according to the strategic plan. However, in practice limitations and targets are not always coordinated in the solution of stated problems.

The main process is the process of budget programming focused on the final result through actions aimed at strategic distribution of budget resources for the main sectors of socio-economic development. The method focuses on a program-oriented control of the budget mechanism.

A block output model is made to compare the factual state of budget management with its predetermined value. This is a reference model which enables correcting actions aimed at system improvement.

The schema outlined above is a general functional one, where each system object is an independent model.

The main procedures of the program method of budget control are as follows:

Stage I: Development of a middle-term (3-year) plan of strategic development on the basis of the long-term development strategy: separation, definition, and analysis of targets and macroeconomic indicators of socio-economic development.

Stage II: Analysis of the budget system, namely data preparation as a forecast of the budget revenue using statistical forecast models; forecast of the expenditure base as a percentage of the GDP; and statistical analysis of the forecast dynamics of the main macroeconomic indicators of a basic scenario of economic development.

Stage III: Budget planning according to the targets of development. This stage solves the problem of allocation of budget resources among the expenditure items according to the concept of budget programming.

Stage IV: Evaluation of budget resource management and correction of priority directions of socio-economic development.

Budget management is transformation of the budget system to the desirable state or behavior.

The above stages of the program method may, in their turn, be subjected to further decomposition.

Let us describe the budget mechanism by means of the program method, taking into account decompositions at each stage.

The **first procedure**—development of the middle-term fiscal policy—is decomposed into the following stages:

- 1.1. Formulation of targets and resource limitations.
- 1.2. Preliminary forecast of the main macroeconomic factors according to the base scenario of the middle-term economic development formulated by the government; analysis and processing of indicators of the socio-economic and financial state of the country/region.
- 1.3. Identification and verbal (qualitative) description using scenario of system targets, their analysis and decomposition.
- 1.4. Development and analysis of qualitative criteria of target achievement.
- 1.5. Development and analysis of programs providing achievement of identified targets.

The first procedure describes operation of system objects—input, targets, and limitations which form a preparatory stage of the general model of the budget mechanism operation.

The second procedure is the stage of data generation by the mathematical analysis module and data designing for planning.

This **second procedure** of the program method for budget system analysis can be subdivided into the following stages:

- 2.1. Collection and processing of budget data (revenue and expenditures).
- 2.2. Analysis of the resource (revenue) budget potential.
- 2.3. Analysis and identification of the priority budget recipients.
- 2.4. Forecast of expenditure and revenue parts according to the basic development scenario, i.e., as percentage of GDP.

The **third procedure** describes strategic budget formation—formation of a reference budget by solving the problem of optimal control: distribution of capital investments among budget recipients.

Stages of construction of management mechanisms:

- 3.1. Formation and analysis of the forecast procedures for control systems—the task of middle-term budget distribution among recipients.
- 3.2. Formation and analysis of planning procedures inside the system—organization of program movement.
- 3.3. Formation of systems used to analyze, control, and assess functioning of elements, including research of stability and assessment of the budget system state.

The **fourth procedure** depicts development of scenarios of budget control. This procedure describes the system object—output.

The main functions of this mechanism are:

- Interpretation of obtained results.
- Formation of budget scenarios.
- Determination of budget type.
- Development of recommendations.
- Development of procedures to correct program strategy and budget state.

2.3.3 System Approach to the Mathematical Model of Budget Mechanism

Let us consider the budget system as an integrated single unit, and begin by formulating of the purpose of its functioning.

The purpose of the budget system functioning is planning of budget incomes and execution of the budget with regard to priorities established according to the tasks of socio-economic development [6].

A structural approach to the budget system enables us to determine the system elements and their interrelations. The budget structure may be presented as:

$$\vec{D} = (d_1 \quad \dots \quad d_j \quad \dots \quad d_m)^T$$

is a vector of receipts,

$$\vec{G} = \vec{X} + \vec{Y} = (g_1 \quad \dots \quad g_i \quad \dots \quad g_n)^T = (x_1 \quad \dots \quad x_i \quad \dots \quad x_n)^T + (y_1 \quad \dots \quad y_i \quad \dots \quad y_n)^T$$

is a vector of expenditures, where g_i is the absolute value of the i -th item of expenditures $i = \overline{1, n}$; x_i is the current component of the i -th item of expenditures $i = \overline{1, n}$; y_i is the capital component of the absolute value of the i -th item of expenditures $i = \overline{1, n}$; d_j is the absolute value of the j -th item of revenue $j = \overline{1, m}$.

Capital expenditures are investments allocated to the country's economy and development, i.e., investments in the development of infrastructure, urban development, creation and development of information systems, science, investments in human resources, etc. Current expenditures are expenditures for fulfillment of current needs of the state.

To accomplish the above task it is necessary to construct a mathematical model of the budget system—a model of optimal distribution of budget resources. This model refers to the category of analytical models in which the processes of operation of system elements are expressed as functional relations (algebraic, integral-differential, finite-difference, etc.) or logical conditions. The analytical model can be studied by the following methods: (a) the analytical method, applicable when explicit solutions for the unknown characteristics can be obtained in the general form; (b) the numerical method, for when the equations cannot be solved in the general form, and only numerical solutions for certain initial data can be obtained; and (c) the qualitative method, which is used when, in the absence of an explicit solution *per se*, some of its properties can be determined (for example, to estimate the solution's stability) [12, 28].

The model of the object of modeling, i.e., the budget system S , may be expressed as a set of quantities describing the process of a real system functioning, and in general forming the following subsets [12, 28, 33, 34]:

The set of input actions (revenue and expenditures) on the system

$$\begin{aligned} d_j \in D, \quad j = \overline{1, m_D}; \quad g_i \in G, \quad i = \overline{1, n_Z}; \\ x_i \in X, \quad i = \overline{1, n_X}; \quad y_i \in Y, \quad i = \overline{1, n_Y}. \end{aligned}$$

The set of external actions

$$f_l \in F, \quad l = \overline{1, n_F}.$$

The set of controlling actions of the system

$$u_k \in U, \quad k = \overline{1, n_U}.$$

The set of output (revenue and expenditures) system characteristics

$$\begin{aligned} d'_j \in D', \quad j = \overline{1, m_{D'}}; \quad g'_i \in G', \quad i = \overline{1, n_{Z'}}; \\ x'_i \in X', \quad i = \overline{1, n_{X'}}; \quad y'_i \in Y', \quad i = \overline{1, n_{Y'}}. \end{aligned}$$

In modeling of system S the input actions and external actions on the system E are independent (exogenous) variables which are expressed in vector form as follows:

$$\begin{aligned} \vec{d}(t) &= (d_1(t), d_2(t), \dots, d_{m_D}(t)); \\ \vec{g}(t) &= (g_1(t), g_2(t), \dots, g_{n_Z}(t)); \\ \vec{x}(t) &= (x_1(t), x_2(t), \dots, x_{n_X}(t)); \\ \vec{y}(t) &= (y_1(t), y_2(t), \dots, y_{n_Y}(t)); \\ \vec{f}(t) &= (f_1(t), f_2(t), \dots, f_{n_F}(t)), \end{aligned}$$

whereas output characteristics and control actions are dependant (endogenous) variables and are expressed in vector form as follows:

$$\begin{aligned}\vec{d}'(t) &= (d'_1(t), d'_2(t), \dots, d'_{nD'}(t)); \\ \vec{g}'(t) &= (g'_1(t), g'_2(t), \dots, g'_{nZ'}(t)); \\ \vec{x}'(t) &= (x'_1(t), x'_2(t), \dots, x'_{nX'}(t)); \\ \vec{y}'(t) &= (y'_1(t), y'_2(t), \dots, y'_{nY'}(t)); \\ \vec{u}'(t) &= (u'_1(t), u'_2(t), \dots, u'_{nU'}(t)).\end{aligned}$$

The process of S system functioning is described in time by the operator R_S , which in the general case transforms exogenous variables into endogenous according to the law of system functioning by the rules:

$$\vec{d}'(t) = R_S(\vec{d}, \vec{f}, \vec{u}, t); \quad \vec{g}'(t) = R_S(\vec{g}, \vec{f}, \vec{u}, t). \quad (2.3)$$

Expression (2.3) is a mathematical description of the behavior of the object (system) of modeling as a function of time t , i.e., it reflects dynamic properties of the system.

For static models the mathematical model (2.3) is an image of the properties of modeled objects D and G , $\{D, F, U\}$ and $\{G, F, U\}$, in two subsets which can be expressed in vector form as follows:

$$\vec{d} = r(\vec{D}, \vec{F}, \vec{U}); \quad \vec{g} = r(\vec{G}, \vec{F}, \vec{U}). \quad (2.4)$$

The relations (2.3) and (2.4) can be predetermined in various ways: analytically (by means of formulas), as a graph, as a table, and so on. In some cases such relations can be obtained through the properties of the budget system S at certain moments of time called states. The state of the budget system S is characterized by vectors

$$\vec{s}' = (s'_1, s'_2, \dots, s'_k) \quad \text{and} \quad \vec{s}'' = (s''_1, s''_2, \dots, s''_k), \quad (2.5)$$

where $s'_1 = s_1(t')$, $s'_2 = s_2(t')$, \dots , $s'_k = s_k(t')$ at the moment $t' \in (t_0, T)$; $s''_1 = s_1(t'')$, $s''_2 = s_2(t'')$, \dots , $s''_k = s_k(t'')$ at the moment $t'' \in (t_0, T)$, etc., $k = \overline{1, n_S}$.

If the process of functioning of budget system S is considered as a sequential change of states $s_1(t), s_2(t), \dots, s_k(t)$, these states can be interpreted as point coordinates in the k -dimensional phase space. Each realization of the budget process will correspond to some phase trajectory. The set of all possible values of the budget state $\{\vec{s}\}$ is called the space of states of the modeled object—the budget system S , where $s_k \in S$.

In studying this model we did not take into account the external impact on the system, therefore in constructing the planning model the set of external impacts $f_l \in F, l = \overline{1, n_F}$ was not taken into account.

The state of the budget system S at the moment of time $t_0 < t^* \leq T$ is fully determined by the initial conditions $\vec{s}^0 = (s_1^0, s_2^0, \dots, s_k^0)$, input conditions, and controlling actions $\vec{u}(t)$, undertaken during the time interval $t^* - t_0$, and is expressed

by two vector equations:

$$\vec{s}(t) = \Phi(\vec{s}^0, \vec{d}, \vec{g}, \vec{u}, t); \quad (2.6)$$

$$\vec{d}(t) = R(\vec{s}, t), \quad \vec{g}(t) = R(\vec{s}, t). \quad (2.7)$$

The first equation including the initial state \vec{s}^0 and exogenous variables $\vec{d}, \vec{g}, \vec{u}$ defines the vector-function $\vec{s}(t)$, and the second equation for the obtained values of states $\vec{s}(t)$ gives endogenous variables at the system output. Therefore the chain of equations of the object “input–states–output” enables us to determine characteristics of the system:

$$\vec{d}'(t) = R[\Phi(\vec{s}^0, \vec{d}, \vec{u}, t)], \quad \vec{g}'(t) = F[\Phi(\vec{s}^0, \vec{g}, \vec{u}, t)]. \quad (2.8)$$

In this general case, the time in the system S model can be considered in the modeling interval $(0, T)$ as both continuous and discrete.

Therefore the mathematical model of the object—the budget system—is a finite subset of variables $\{\vec{d}(t), \vec{g}(t), \vec{u}(t)\}$ with their mathematical interrelations and characteristics $\vec{d}(t)$ and $\vec{g}(t)$.

2.4 Mathematical Models of Budget Expenditure

2.4.1 Construction of Program Movements for Budget Expenditure

Definition 2.4 The *program control* of the budget is budgeting focused on the final product as determined by the development program.

In the schema of middle-term budgeting the regulation time considerably exceeds the planning time, which makes the process of middle-term budgeting interrelated and coordinated in time, thus allowing us to consider the process of budget planning as discrete. Such a schema enables to reconsider (correct) the plan for future planning periods as they move closer, in order to reduce disproportions in development, and points out the underlying nature of the program-targeted planning method: program control [1, 6, 24].

Let us consider how program movements are constructed in the simplest case, in linear management systems [15, 28].

Let the expenditure budget part be described by a system of n differential equations in vector form, which describes the simplest case of the linear system of budget expenditure control

$$\dot{g}(t) = Pg(t) + Qu(t), \quad (2.9)$$

where

$$g^{(1)} = (g_1^{(1)} \quad g_2^{(1)} \quad \dots \quad g_n^{(1)})^T, \\ g^{(2)} = (g_1^{(2)} \quad g_2^{(2)} \quad \dots \quad g_n^{(2)})^T, \quad \dots, \quad g^{(n)} = (g_1^{(n)} \quad g_2^{(n)} \quad \dots \quad g_n^{(n)})^T$$

is a vector characterizing the state of budget expenditure items; $P = \{\frac{y_i}{x_j}\}$ is $(n \times n)$ is a matrix describing the current state of the budget expenditure items whose elements reflect interconnection between capital (y_i) and current (x_j) expenditures; Q is a $(n \times r)$ matrix characterizing the program of achievement of the development level formulated in the strategic plan of socio-economic development for the middle-term period [28].

Let two constant vectors g_0 and g_1 define the initial and final state of the system (2.9). It is necessary to find such a vector-function $u = u(t)$ of r dimension, that solution to the system (2.9) starting at $t_0 = 0$ in the point $g = g_0$ goes to point $g = g_1$ at $t_T = T$ and the integral $\int_{t_0}^{t_T} u^T(\tau)u(\tau)d\tau$ is limited. Such controls $u = u(t)$ are called program controls [13].

Along with Eq. (2.9) let us consider a homogeneous equation

$$\dot{g} = Pg. \quad (2.10)$$

Let

$$g^{(1)} = (g_1^{(1)} \quad g_2^{(1)} \quad \dots \quad g_n^{(1)})^T, \\ g^{(2)} = (g_1^{(2)} \quad g_2^{(2)} \quad \dots \quad g_n^{(2)})^T, \quad \dots, \quad g^{(n)} = (g_1^{(n)} \quad g_2^{(n)} \quad \dots \quad g_n^{(n)})^T$$

be n linearly independent solutions of the homogeneous equation (2.10). Any such system of solutions is called a fundamental system of solutions.

Let us denote as $\Phi(t)$ a fundamental matrix of solutions to the system of Eqs. (2.10), at initial conditions $\Phi(t_0) = I_n$, $t_0 = 0$, where I_n is a unitary matrix of the order n [15, 28].

In the system (2.9) we will make a nonsingular linear transformation over vector g :

$$g = \Phi(t)z, \quad (2.11)$$

where vector z is a new vector-function. This vector-function satisfies the condition

$$\dot{z} = B(t)u, \quad (2.12)$$

where $B(t) = \Phi^{-1}(t)Q$ [15, 28].

For simplicity let us assume that $r = 1$. The integration of Eq. (2.12) from $t_0 = 0$ to $t_T = T$ gives

$$z_1 - z_0 = z(t_T) - z(t_0) = \int_{t_0}^{t_T} B(\tau)u(\tau)d\tau. \quad (2.13)$$

The system of linear integral equations (2.13) enables us to find program control $u(t)$.

A solution to Eqs. (2.13) will be sought in the form [15, 28]

$$u(t) = B^T c + v(t), \quad (2.14)$$

where c is a constant vector to be determined, $v(t)$ is a function summed up with its square in the interval $[t_0, t_T]$ such that

$$\int_{t_0}^{t_T} b_s(t)v(t)dt = 0, \quad s = 1, 2, \dots, n. \quad (2.15)$$

Here $b_s(t)$ are components of vector B , and equality (2.15) expresses the condition of orthogonality of function $v(t)$ to all components of vector B .

Substituting (2.14) into (2.13) we find [15, 28]

$$z_1 - z_0 = A(t_T)c, \quad (2.16)$$

where $A(t_T) = \int_{t_0}^{t_T} B(t)B^T(t)dt$.

Equation (2.16) has a solution if $\det A(t_T) \neq 0$ or if matrix $A(t)$ has the same rank as the extended matrix $\bar{A} = \{A(t_T); z_1 - z_0\}$.

In order to understand the existence of program controls correctly, let us formulate the analogue of the classical theorem of program control [28] existence in terms of the budget system.

Theorem 2.1 *In order to obtain a program control which transfers the budget system (2.9) from any initial state g_0 to any other state g_1 within time t_T , it is necessary and sufficient that matrix $A = \int_{t_0}^{t_T} B B^T dt$, where $B = \Phi^{-1}Q$, be nonsingular. In this case the entire subset of program controls is determined by the formula:*

$$u = B^T c + v,$$

where $v(t)$ is a function summed up with its square in the interval $[t_0, t_T]$ and

$$\int_{t_0}^{t_T} B v dt = 0, \quad c = A^{-1}(t_T)[\Phi^{-1}(t_T)g_1 - g_0].$$

2.4.2 A Model of Program Control of the Expenditure Budget Part

After introduction of the general notations of the system and general procedure of budget mechanism control, let us consider the general formulation of the mathematical model of optimal distribution of budget resources.

The optimal distribution of budget resources is realized according to the priority directions and general concept of socio-economic development for the middle-term planning period. The task in allocation of budget resources, i.e., in developing a plan for dividing these resources among expenditure items, is to enable socio-economic development for the main macroeconomic indicators up to the desired level of the basic development scenario [6, 33–36].

The expenditure budget part consists of $i = \overline{1, n}$ expenditure items whose qualitative state is denoted as z_i .

The mathematical formalization [35] of the budget mechanism control is presented as a task of optimal distribution of budget resources ΔC among the expenditure items and budget recipients ($i = \overline{1, n}$) in order to bring budget potential of budget recipients to the planned state \tilde{Z} (from the initial state Z^0), where $\tilde{Z} = (\tilde{z}_1, \dots, \tilde{z}_i, \dots, \tilde{z}_n)$ is a vector characterizing the state of an expenditure budget item in the future, i.e., at the end of the middle-term planning period. Formulating the problem in this way can be considered a task of long-term planning. Plan \tilde{Z} ($\tilde{Z} \geq 0$) is a final result of strategic development determining the final target of the budget mechanism functioning which defines the qualitative state of the reference budget model at the end of the middle-term planning period. In the plan \tilde{Z} the component \tilde{z}_i denotes the planned qualitative state of the expenditure item i ($i = \overline{1, n}$).

Vector \tilde{Z} is expressed as a sum of two vectors $\tilde{Z} = \tilde{X} + \tilde{Y}$. Let us consider these parameters in the framework of investment distribution. $\tilde{X} = (\tilde{x}_1, \dots, \tilde{x}_i, \dots, \tilde{x}_n)$, $\tilde{X} \geq 0$ is a vector of future expenditures designated to provide a guaranteed budget standard, vector $\tilde{Y} = (\tilde{y}_1, \dots, \tilde{y}_i, \dots, \tilde{y}_n)$, $\tilde{Y} \geq 0$, is a vector of capital expenditures designated to enable formation of a budget characterizing socio-economic development in the framework of the guaranteed standard state defined by the basic development scenario.

The procedure of deriving vector \tilde{Z} is an auxiliary optimization task. Let us consider how the perspective state is derived.

Suppose that every year t , $t_0 \leq t \leq T$ the development of the budget recipient (an expenditure item as a functional group of budget classification) is proportional to the invested capital, i.e., if $u_i(t)$ is a capital investment directed to the development of the budget recipient i in a year t then the following relation can be written [6, 28]:

$$\begin{aligned} z_i(t + \Delta t) &= z_i(t) + \lambda_i u_i(t), \\ t_0 \leq t \leq T, \quad i &= \overline{1, n}, \quad (z_i(t) = x_i(t) + y_i(t)), \end{aligned} \quad (2.17)$$

where λ_i is a weight coefficient, with the formula for its calculation being given below, and Δt is a period of discretization equal, for example, to 1 year, 1 quarter, etc.

Every year t for item i the following balance relations must be fulfilled for item i [6, 28]:

$$\sum_i (x_i(t) + y_i(t)) \alpha_{ij} = y_j(t), \quad j = \overline{1, n}. \quad (2.18)$$

The element α_{ij} in the relation (2.18) is an element of the balance matrix $A = \{\alpha_{ij}\}$, $\alpha_{ij} \geq 0$; in matrix form the relation (2.18) is written as $(\tilde{X} + \tilde{Y})A = \tilde{Z}A = \tilde{Y}$ [29], where

$$\alpha_{ij} = \frac{y_i}{x_j \sum_{i=1}^n \frac{z_i}{x_j}}, \quad i = j = \overline{1, n}. \quad (2.19)$$

This coefficient shows which fraction corresponds to the capital component of the i -th item from all volume of the budget expenditure fund.

To determine the perspective plan of budget resource allocation it is necessary to know the planned volume of expenditures by the budget items for each period of the middle-term planning, $\sum_{i=1}^n \hat{u}_i(t)$, $t_0 \leq t \leq T$, which is included in the development program as a rough estimation of the budget state for the planning period. Here $\hat{u}_i(t)$ is capital investments planned in the basic development scenario for a year t for the upper-level items in the budget classification functional group. Such a distribution plan is called a desirable plan of distribution of capital investments for the year t . This plan $\{\hat{u}_i(t)\}$ is used to determine the desired state of capital investments

$$\tilde{z}_i = z_i(t) + \lambda_i \left(\sum_{t=t_0}^T \sum_{i=1}^n \hat{u}_i(t) \right). \quad (2.20)$$

Thus, we have found the perspective state of the expenditure part at the end of the middle-term planning.

The coefficient λ_i , $i = \overline{1, n}$ is a weight coefficient which characterizes the desired socio-economic development. The basic scenario contains a forecast of the main macroeconomic indicators which will be denoted as I_q , $q = \overline{1, k}$. In the formulation of the task we are interested in the average growth rate (\overline{TR}_q) of the above indicators in the planned period [6, 28]:

$$\overline{TR}_q = \sqrt[r-1]{\frac{I_T}{I_1}} 100 \%, \quad (2.21)$$

where I_1 , I_T are the values of the factor (indicator) at the beginning $t = t_0$ and end of the middle-term planning $t = T$.

In order to determine the weight coefficient it is also necessary to estimate the relation between the current state of the budget expenditure part and the current revenue base. This relation is expressed as a matrix of interaction of budget items $\bar{A} = \{a_{ij}\}$, $i = \overline{1, n}$, $j = \overline{1, m}$.

The relation between the desired state of the main macroeconomic indicators I_q , $q = \overline{1, k}$ and the structure of the budget expenditure part is expressed as the weight coefficient calculated by the formula [30]:

$$\lambda_i = \frac{1}{\sum_{q=1}^k \frac{\overline{TR}_q}{\sum_{j=1}^m a_{ij}/m}}, \quad i = \overline{1, n}. \quad (2.22)$$

This coefficient characterizes the fraction of influence of the current state of the i -th budget item on the forecasted average growth rate of the main macroeconomic indicators. Such a scheme requires more precise estimation of macroeconomic indicators of economic development, which will enable us to get a more justified scenario of the budget expenditure plan for the forthcoming middle-term planning period and to coordinate the main economic forecasts with the middle-term targets and tasks of national/regional development.

Having determined the desired normative state of the expenditure item, one must develop an admissible plan of allocation of capital investments—which is called a

reference plan—transferring the budget system from state Z^0 to state \tilde{Z} simultaneously for all expenditure items. According to the plan the resources allocated for the expenditure item are proportional to the lag of this item from the values specified in the basic scenario. The reference plan is determined by the following formulae:

$$\bar{u}_i^1(t) = \frac{\tilde{x}_i - x_i(t)}{\sum_i (\tilde{z}_i - z_i(t))} \Delta C(t), \quad (2.23)$$

$$\bar{u}_i^2(t) = \frac{\tilde{y}_i - y_i(t)}{\sum_i (\tilde{z}_i - z_i(t))} \Delta C(t), \quad (2.24)$$

$$\bar{u}_i(t) = \bar{u}_i^1(t) + \bar{u}_i^2(t), \quad i = \overline{1, n}. \quad (2.25)$$

The reference plan is such that every year $t, t = 0, 1, \dots, T - 1$ the state of the budget expenditure part satisfies the balance equation (2.18) if the initial task is correctly formulated.

The state of the budget expenditure part for the period $t + \Delta t$ is determined by the formulae:

$$Z(t + \Delta t) = X(t + \Delta t) + Y(t + \Delta t) \quad (2.26)$$

where

$$\begin{aligned} x_i(t + \Delta t) &= x_i(t) + \lambda_i \bar{u}_i^1(t), \\ y_i(t + \Delta t) &= y_i(t) + \lambda_i \bar{u}_i^2(t), \quad i = \overline{1, n}. \end{aligned} \quad (2.27)$$

When the budget is transferred from an initial state $Z^0 = X^0 + Y^0$ to state \tilde{Z} , the following conditions must be satisfied:

1. \tilde{Z} level must be achieved simultaneously for all expenditure items.
2. The balance equation (2.18) must be fulfilled at each stage of development.
3. The plan of budget resource distribution among the expenditure items must be supported by the external resources needed to provide guaranteed normative funds to cover \tilde{X} in addition to the development budget. If the plan of budget resource distribution is not supported by resources, then part of the budget funds must be allocated for enhancement of the budget revenue part (i.e., attraction of investments, improvement of fiscal policy, or transfers from higher offices, which last may result in passiveness of authoritative regional bodies in forming their own incomes).

The reference plan guarantees:

- (A) Simultaneous achievement of state $\tilde{Z} = \tilde{X} + \tilde{Y}$ by all expenditure items, balance, and fulfillment of the plan of budget resource distribution with accompanying development of the revenue base.
- (B) Smoothing of disproportions in the development of expenditure items, as realization of this plan enables lags in expenditure items to be overcome proportionally to their values.

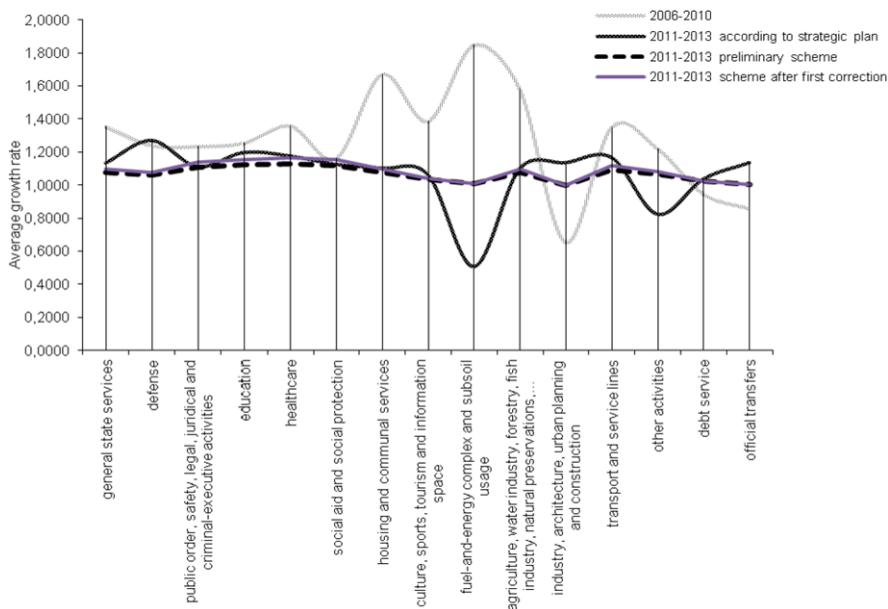


Fig. 2.2 A diagram showing variations in the average growth rates of budget expenditure items

Existence of the reference plan $\bar{u}^1(t)$, $\bar{u}^2(t)$ guarantees the existence of at least one reference plan of distribution of capital investments satisfying conditions 1, 2, and 4, which enables formulation of the problem of optimal development [6, 28].

In the computational experiment we considered the statistical data for performance of the 2008–2013 state budget of the Republic of Kazakhstan for the upper level of the budget system classification (categories of revenues and functional groups of expenditures). Therefore the elements of the revenue vector (D) and the expenditure vector (Z) include the following components: d_1 —tax earnings; d_2 —non-tax earnings; d_3 —earnings from operations with capital; z_1 —general state services; z_2 —defense; z_3 —public order, safety, legal, juridical and criminal-executive activities; z_4 —education; z_5 —healthcare; z_6 —social aid and social protection; z_7 —housing and communal services; z_8 —culture, sports, tourism and information space; z_9 —fuel-and-energy complex and subsoil usage; z_{10} —agriculture, water industry, forestry, fish industry, natural preservations, protection of environment and wildlife, land-use relations; z_{11} —industry, architecture, urban planning and construction; z_{12} —transport and service lines; z_{13} —other activities; z_{14} —debt service; z_{15} —official transfers.

According to the suggested method of budget performance it is necessary to analyze the average rate of growth in expenditure items for the retrospective period planned in the strategic plan and the average rate of increase in the normative states during the planned period (Fig. 2.2).

The results of the analysis show a stepwise trend for the previous period, which is demonstrated by the chaotic statistics. The plan of strategic development also

envisages nonuniform development of budget items, with allocation of budget funds on program control smoothing disproportions in the development of budget items, which is clearly seen in the diagram. The program movement guarantees uniform development of the expenditure part of the budget preventing unequal development of budget items.

2.4.3 Model of Management Adjustment

Having solved the problem of budget resource distribution, we get some states of budget expenditure items for the middle-term period. In modeling the process of budget resources control a very important modeling stage is estimation of correctness of the obtained solution.

A correct solution corresponds to such a normative state of budget items at the end of the middle-term period which is lower than the desired state defined by the strategic development plan.

IF the quantitative value of the obtained normative state ($z_i(T)$, $i = \overline{1, n}$) of all expenditure part of the budget system at the end of the middle-term period exceeds the quantitative value of the predetermined volume of monetary funds in the strategic plan ($u_i(T)_i$),

THEN it is necessary to correct the target reference point or weight coefficients (λ_i) in order to reduce disproportions between the strategic planning and planned state of budget performance,

OTHERWISE the solution is not correct.

In case the solution needs correction it is necessary to find the ratio:

$$\delta_i = \frac{z_i(T)}{z_i}. \quad (2.28)$$

The weight coefficient of item i , $i = \overline{1, n}$ is corrected according to the following rules:

IF $\delta_i = 1$,

THEN the value of the weight coefficient is not changed $\lambda_i^k = \lambda_i$;

otherwise IF $\delta_i \neq 1$,

THEN the value of the weight coefficient λ_i^k is corrected according to the gradient method $\lambda_i^k = (\delta_i - \lambda_i)\lambda_i(1 - \lambda_i)$.

The value of the weight coefficient is reduced in order to bring the system closer to the desired result. The algorithm acts on all the system because the budget system is an integral system and it is desirable to provide centralized control of the system in order to avoid excessive attention to one of the programs.

Then the algorithm of allocation of budget resources is called where calculations are made with the corrected target.

After getting a correct solution i.e. such a normative state of the expenditure budget items which does not exceed the desired state one can switch to estimation of the state of the revenue budget part.

Let $\Delta Z(t) = \Delta X(t) + \Delta Y(t)$ be the increment of the development of the revenue budget part per year t . To simplify the task let us assume that the development of expenditure budget items is proportional to the budget resources allocated for their development i.e. $\Delta z_i(t) = \lambda_i u_i(t)$, $i = 1, \dots, n$, where $u_i(t)$ are budget resources allocated for the development of budget item i [6, 28].

As the budget is a financial system consisting of revenue and expenditure parts, its revenue part requires replenishment in the form of input of resources, needed for its development, which are further accumulated in expenditures characterizing socio-economic and financial development of the country/region.

Let $\Delta W^1(t)$ be a vector of resources needed for socio-economic development of recipients of the expenditure budget part available at a given planning period (this year), which can be used for the development (funds formed by the income part of the budget). Let $\Delta W^1(t) = \{\Delta W^1(t)\} \geq 0$ and $\Delta W^2(t) = \{\Delta W^2(t)\} \geq 0$ be vectors of resources which require additional budgetary funds needed for full realization of the budget development plan (additional investments to the budget guaranteeing achievement of the stated purposes).

In order to determine vector $\Delta W^2(t)$ it is necessary to estimate the excess of the expenditure base over the revenue base, and to do this one must find the ratio:

$$\eta = \frac{\sum_{t=t_0}^T Z(t)}{\sum_{t=t_0}^T \Delta W^1(t)} \quad (2.29)$$

where $\sum_{t=t_0}^T Z(t)$ is the total consumption volume for the middle-term planning period determined by the program control; $\sum_{t=t_0}^T \Delta W^1(t)$ is the total forecasted receipts volume for the middle-term planning period.

For ratio η let us find such $\Delta W^{1*}(t)$ which will be a balance with respect to normative states of the budget expenditure component, i.e. η is a coefficient providing balance between the expenditure and revenue components:

$$\Delta W_j^{1*}(t) = \eta \Delta W_j^1(t), \quad j = \overline{1, m}, \quad (2.30)$$

where m is the number of budget income items.

According to $\Delta W^{1*}(t)$ the revenue budget part needs additional budget resources in amount $\Delta W_j^2(t) = v_j(t)$ where $v_j(t)$ are monetary funds (capital) needed for the development of the j -th revenue item in a year t .

The plan of budget resources allocation $\bar{u}(t) = \{\bar{u}_1(t), \dots, \bar{u}_i(t), \dots, \bar{u}_n(t)\}$ satisfies the following conditions: level \tilde{Z} must be reached simultaneously for all expenditure items; balance equation (2.18) must be satisfied at each stage of development. Then, due to linearity, the system of the type $\alpha(t)\bar{u}(t) = \{\alpha(t)\bar{u}_1(t), \dots, \alpha(t)\bar{u}_i(t), \dots, \alpha(t)\bar{u}_n(t)\}$ has the same property, where $\alpha(t) > 0$ is the growth coefficient for the development of the expenditure budget base in the year t .

IF resources $\Delta W^1(t)$ are sufficient for realization of $\alpha \bar{u}(t)$,
THEN $\Delta W^2(t) = 0$,
OTHERWISE

$$\Delta W^2(t) = \sum_{j=1}^m \mu_j v_j(t), \quad (2.31)$$

where $v_j(t)$ are monetary funds (capital) needed for the development of the j -th revenue item in the year t ; μ_j is the average rate of growth of budget revenue items in the middle-term planning period.

The budget fund $\Delta C(t)$ allocated on the development of the expenditure base consists of two parts:

$$\Delta C(t) = \Delta C_1(t) + \Delta C_2(t). \quad (2.32)$$

Here $\Delta C_1(t) = \alpha \sum_i \bar{u}_i(t)$ are resources directed directly to the development of the expenditure base of budget recipients, $\Delta C_2(t) = \sum_j v_j(t)$ are resources directed to the accompanying attraction of capital resources for the development of income base needed for the development of expenditure base of budget recipients.

The plan $\alpha(t)\bar{u}(t)$ will be provided with resources if

$$\sum_j \mu_j v_j(t) \geq \alpha(t) \sum_i \bar{u}_i(t) \lambda_i$$

$$v_k(t) \geq 0, \alpha \geq 0. \quad (2.33)$$

This condition means sufficiency of additional resources providing the predetermined level of expenditure base for the period t .

In case the expenditure base is developed according to the maximal growth scenario in the period of observations, the necessity of financing needed to support the predetermined level of development is transformed into the task of maximization of development by attracting financial resources:

$$\max \alpha \quad \text{for condition (2.33)}. \quad (2.34)$$

Thus, we get a model correcting the forecast of the budget revenue part preserving balance with the planned expenditure part and providing development of the budget expenditure part.

2.4.4 Description of Algorithms of Basic Processes

Program control is a process of planning of probable or logical future and forecasted future states. It is the process which enables to understand how to achieve the desired aims, how to use knowledge to turn the logical future into a desired one, and how to make such actions.

The basic component in the program control is a plan having three general elements—an initial state, a target (or final state) and processes connecting these states. The target of planning is to connect the elements in such a way that it will enable to reach maximal efficiency with minimal expenditures.

The first element of any plan is *the initial state*.

The second element is the *target*. The target must be properly formulated, achievable and restated or changed according to the changed circumstances.

The third element of the plan is *processes*. This element is the plan itself as it contains the description of the method providing transition from the initial state to the target.

The planning process mainly has one direction i.e. it is a time-ordered sequence of events beginning at the present time $t = 0$ and ending at a certain moment in future $t = T$. This sequence that is called a *direct process* considers current factors and suggestions which give a certain logical result. In the second sequence that is called a *reverse process* the states are considered beginning from the desired result at a certain moment of time T in the reverse time direction, to the initial state, in order to estimate factors and intermediate results needed to reach the desired final result.

The direct planning process gives assessment of the state of probable final result. The reverse planning process gives the means of control and management of the direct process on the way to the desired state.

Let us consider the algorithms of direct and reverse processes [6].

Direct process algorithm

Step 1. Acquisition and processing of input information.

At this stage the current budget state and the strategic plan are analyzed according to the basic scenario of economic development. To do this it is necessary to present the vector of interaction between the two main indicators and the revenue items as weight coefficients, expressed as vector $Q(t)$, the target indicator of performance of the strategic development plan. At this stage the volumes of financing of expenditure items in the strategic development plan for the middle-term planning period are calculated.

Step 2. Direct pass: Determination of the desired states of the budget expenditure part.

At this step the desired state is calculated by formula (2.20), this state is a transformation of the target indicator into a concrete state which is to be reached to provide a required level of development.

Step 3. Distribution of resources by the expenditure items for the middle-term period.

Knowing the desired and current state of the budget expenditure part, we can find the control which can transfer the system into the desired state by formulae (2.23)–(2.25). The obtained control is a program of system transfer from one state to the other according to the formulae (2.26) and (2.27), this procedure enables to determine normative states of expenditure items in the middle-term planning period.

Step 4. Comparative analysis of obtained solutions.

At this step works the control function which finds incorrect solutions by comparing the normative state in the last period of the middle-term planning and the desired state at the end of the planning period. At this step the decision to correct the obtained solution is taken, it includes the following operations:

IF the quantitative value of the obtained normative state of all expenditure part of the budget system at the end of the middle-term period exceeds the quantitative value of the planned state that was predetermined in the middle-term fiscal policy, THEN it is necessary to correct the target reference point or weight coefficients in order to reduce disproportions between the desired and planned states, OTHERWISE The algorithm must be stopped.

Step 5. Reverse pass: Correction of weight coefficients.

Weight coefficients are corrected using the gradient method. After correction of weight coefficients it is necessary to repeat steps 2–4.

Reverse process algorithm

Step 1. Estimation of stability margin of obtained solutions.

At this step it is necessary to determine the confidence interval for every item which will characterize the stability margin of the item.

Step 2. Estimation of stability of obtained solution.

At this step to estimate stability of obtained solutions Lyapunov's function is used.

To go to the next step it is necessary to fulfill the following rules:

IF the obtained solution is not stable,

THEN in the direct process algorithm it is necessary to correct values of forecast items,

OTHERWISE give qualitative interpretation of the obtained solution.

Step 3. Correction of the forecast of the budget revenue part.

At this step formulae (2.29) and (2.30) are used to correct the budget revenue part in such a way as to provide balance between revenue and expenditure parts.

Step 4. Determination of the development coefficients for expenditure items for the period t .

At this step according to the development coefficient the volume and financial resources required for the development are determined by formula (2.32), and possibility to achieve planned development level is estimated by formula (2.34).

The above algorithms are used to perform a computational experiment; the algorithms are the base for programming of the program control process.

2.5 Mathematical Models of Budget Revenue Part

2.5.1 Basic Provisions Describing Interactions of Budget Items

In the authors presented a mathematical model of the static budget state (2.1) which enables to answer the following questions:

1. Are there principles according to which budget resources are distributed by the items of state programs of strategic development?
2. Is it possible to assess the budget (deficiency/proficiency) state?
3. How does the budget allocation change when the budget items are changed?
4. What is the reaction of budget indicators on the change of external conditions?
5. Is it possible to use the current budget state for further planning?

Answers to the above questions will help to find optimal variants of the current and future budget development.

The constructed static model reflects the current budget state. A specific feature of the suggested model is that the conditions of operation of the budget model are well coordinated with the program control model, which enables to get correct values of the budget revenue. Let us reformulate the conditions of static model functioning for the planned period $t \in [t_0, T]$ in terms of program control of budget resources:

1. the model operates the data within one planned period $t \in [t_0, T]$;
2. the initial data are the dynamics of revenue and expenditure budget values before the moment $t = t_0$;
3. the output data are the corrected plan of budget receipts—a balance with respect to the expenditure plan;
4. the computational scheme of the static model is based on the principle of matrix budget representation as a matrix of interaction of budget items whose elements are indicators of the internal budget state.

2.5.2 Learning Elements of Budget System

The model of learning elements is based on the principle of matrix representation of the budget system in order to simplify calculations on the nature of object domain [20].

Problem formulation Consider a budget system as a matrix of interaction of revenue and expenditure items a_{ij} formed to determine the budget state. The initial elements of the system are revenue items, denoted as $D = \{d_j; j = \overline{1, m}\}$, and expenditure items $G = \{g_i; i = \overline{1, n}\}$. These elements form $m \times n$ matrix for interaction of budget items $A = \|a_{ij}\|$. The system operates in the dynamic regime $\tau = \overline{1, t_0}$. The matrix elements are indicators of the internal budget state generating learning elements of the mathematical model of budget forecast ε_{ij}^+ and ε_{ij}^- , which can reflect development and crises moments in time needed to find an indicator of the internal budget state in future.

Definition 2.5 *Learning elements of budget system* are such elements which can correct the future budget; their purpose is to bring the system to the balance as close as possible.

Essence of learning elements These elements are numbers but these numbers initially contain information about processes in national economy i.e. numbers with history. A learning element of any item gives the coefficient of future variation of this item relative to its current state as it was obtained from dynamics analysis.

As the statistics increases such elements will also be corrected i.e. they will be taught. A learning element with plus characterizes the degree of item development in future, whereas a learning element with minus shows the degree of criticality of the item in dynamics and its influence on future.

The forecast system based on such system can give more reliable information on system development and crisis. A dramatic effect on the item future is imposed by its current state; such concept enables to speak about dynamic memory of budget elements.

The above elements play a role of an auxiliary reference mark in the direction of balanced state of the item and budget as a whole achieved by coefficients of items interactions reflecting mathematical relations between budget elements.

The elements of the budget system form a matrix of interactions of revenue and expenditure budget items consisting of n lines and m columns:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2m} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{im} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nj} & \dots & a_{nm} \end{pmatrix}.$$

The elements of A matrix correspond to the values in (2.2).

The relation (2.2) is an internal indicator of the system state or, in other words, it is the coefficient of interaction of input and output system parameters.

The object of modeling will be considered in dynamics, therefore for every time period we will construct its matrix of budget items interaction. The time series is presented as $\tau = \overline{1, t_0}$, the forecasted year is denoted by t .

Every matrix element in the current period can be expressed as the value of the previous year plus the difference between the current and the previous years:

$$a_{ij}^{\tau} = a_{ij}^{\tau-1} - e_{ij}^{\tau} \quad (2.35)$$

where $a_{ij}^{\tau-1}$ is an element of the interaction matrix in the previous time period; e_{ij}^{τ} is the difference between the values of elements in the interaction matrix in the current and previous time periods.

Learning elements of this model contain all past information, thus, they predetermine the future internal indicator of the system state ($a_{ij}^{\tau+1}$) retaining balance budget state. If it is not the case, it means that the budget has deficiency or proficiency.

This model has two types of learning elements: one of them contains information about development ε_{ij}^{+} (a tendency of the budget item to increase which is the result of development of the state economy), the other element contains information about

crisis situations in the internal system state ε_{ij}^+ (when there is a tendency to decline i.e. there are sharp jumps from rise to fall and vice versa), such information is needed to determine the internal state of the system in future.

After careful analysis of the budget dynamics the authors heuristically got expressions for calculation of learning elements.

The learning element ε_{ij}^+ for each budget matrix element can be expressed as:

$$\varepsilon_{ij}^+ = \frac{\sum_{\tau=1}^{t_0} a_{ij}^2}{\sum_{\tau=1}^{t_0} \tau \cdot \sum_{\tau=1}^{t_0} \tau a_{ij}}; \quad i = \overline{1, m}; j = \overline{1, n}, \quad (2.36)$$

where $\tau = \overline{1, t_0}$ is the period of observations; a_{ij} are budget matrix elements.

The learning element ε_{ij}^+ can be expressed as:

$$\varepsilon_{ij}^- = \frac{\sum_{\tau=2}^{t_0} e_{ij}}{\sum_{\tau=1}^{t_0} \tau}; \quad i = \overline{1, m}; j = \overline{1, n}, \quad (2.37)$$

where $\tau = \overline{1, t_0}$ is the period of observations; ε_{ij} is the difference between the values of budget matrix elements.

Matrix representation of learning elements is expressed as $\|E_{ij}^+\|$ and $\|E_{ij}^-\|$, respectively.

The obtained learning elements give the forecast indicator of the internal budget state i.e. the elements of the budget forecast matrix:

$$a_{ij}^t = a_{ij}^{t_0} + \varepsilon_{ij}^+ + \varepsilon_{ij}^- \quad (2.38)$$

where $a_{ij}^{t_0}$ is the value of the budget matrix for the current period (t_0); ε_{ij}^+ and ε_{ij}^- are the elements of the learning model.

The expression (2.38) gives a forecast matrix of budget items interactions:

$$A^{\tau+1} = \begin{pmatrix} a_{11}^{\tau+1} & a_{12}^{\tau+1} & \dots & a_{1j}^{\tau+1} & \dots & a_{1m}^{\tau+1} \\ a_{21}^{\tau+1} & a_{22}^{\tau+1} & \dots & a_{2j}^{\tau+1} & \dots & a_{2m}^{\tau+1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1}^{\tau+1} & a_{i2}^{\tau+1} & \dots & a_{ij}^{\tau+1} & \dots & a_{im}^{\tau+1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1}^{\tau+1} & a_{n2}^{\tau+1} & \dots & a_{nj}^{\tau+1} & \dots & a_{nm}^{\tau+1} \end{pmatrix}$$

According to the main principle of the budget system the budget state must be balanced i.e. the condition $\sum_{i=1}^n g_i = \sum_{j=1}^m d_j$ must be fulfilled for any level of budget classification. From the above-stated it follows that the budget matrix must give the balance state of the budget item and retain it under possible current corrections.

From the condition of balance budget state it follows that the following relation must be fulfilled:

$$\sum_{i=1}^n a_{ij}^t = \frac{\sum_{i=1}^n g_i^t}{d_j^t} = \frac{\sum_{j=1}^m d_j^t}{d_j^t}. \quad (2.39)$$

Hence, the expression (2.39) is a sufficient condition of the balance state of the budget matrix: the sum of the j -th column of the budget matrix elements is equal to the fraction of the budget revenue per the j -th revenue item.

Let us check the balance of the forecast matrix:

$$E(A) = \sum_{j=1}^m \left(\frac{1}{\sum_{i=1}^n a_{ij}^t} \right) = 1. \quad (2.40)$$

The expression (2.40) is a sufficient condition of the budget state analysis. If the above condition exceeds 1, then fractions of the expenditure items exceed the budget, which corresponds to the proficiency state. Similarly, if the expression is less than 1, the state of the budget is called deficiency [29].

Fulfillment of the condition of balance enables to make a conclusion about the balanced budget state for the forecasted period. Now we can switch from the elements to finding forecasted values of budget revenue items.

2.5.3 Model of Correction of Budget Revenue Forecast

After solving the problem of budget resource distribution among budget recipients for the middle-term planning period, it is possible to determine the volumes of budget resources (corrected) $\sum_{i=1}^n z_i(t)$ for the planned periods $t \in [t_0; T]$ used to calculate the volumes of budget receipts for certain time periods based on the principle of balance of revenue and expenditure expressed in the matrix of interaction of budget items.

Taking the forecast matrix of item interaction and the corrected budget $\sum_{i=1}^n z_i(t)$ for the planned periods $t \in [t_0; T]$ as the basic calculated parameter, it is then possible to calculate vector elements by the formula:

$$d_j = \frac{\sum_{i=1}^n z_i(t)}{\sum_{i=1}^n a_{ij}^{t+1}}. \quad (2.41)$$

Thus, the connection between the static model and the program control model is based on the natural connection between budget components—revenue and expenditure.

2.6 Model of Information System for Program Budget Control

As an instrument controlling budget process we propose to use the information system of budget control based on the principles of budget programming, i.e. a budget process modified in the framework of the program-targeted approach. The information system must enable us to solve the following problems: support of decision making, organizational control, control of the budget system, system diagnostics, complex informational search and other operations. All these tasks have a high degree of information complexity.

The information system of budget control must be able to fulfill the following functions:

- automation of solution of current and standard tasks of the budget performance: collection, processing and storage of information, making reports, compiling statistics, etc.;
- development of budget programs and projects;
- definition of targets and resource limitations in budget formulation;
- program control of allocation of budget funds among budget recipients according to the strategic plan of socio-economic development.

The information system of budget control is a complicated multi-level information system guaranteeing automatic control of all subprocesses of budget system and types of budget activities. The model of budget control information system can be presented as an interaction of 6 subsystems (Fig. 2.3) reflecting the system of criteria used for description of a scheme of multi-level (middle-term) budgeting. The program realization of such a system will become an important instrument in such an approach to budgeting, which makes it possible to solve complicated information problems related to organization and control of result-oriented budgeting.

As it is seen in Fig. 2.3 the structure consists of 6 major subsystems: interface, structure, targets and target formulation, mechanism of program control, automation and data bases. Let us consider each subsystem:

Interface is a dialogue regime of contact with the user which provides flexible control of the system.

The *structure* consists of three modules. Each module is an independent element of the budget process that forms commands for users in the form specific for the given area.

The subsystem “*targets and target formulation*” is an addition to the *structure* as each element of the budget process must have a purpose and a final product in order to take into account state purposes with the volumes of revenues needed for their achievement, and must be oriented on solution of key targets and tasks for the planning system as well as planned allocation of budget funds.

Automation is a subsystem designated for solution of current and standard problems of budget process: collection, processing and storage of information, making reports, compiling statistics and, besides, this subsystem has a module for input of

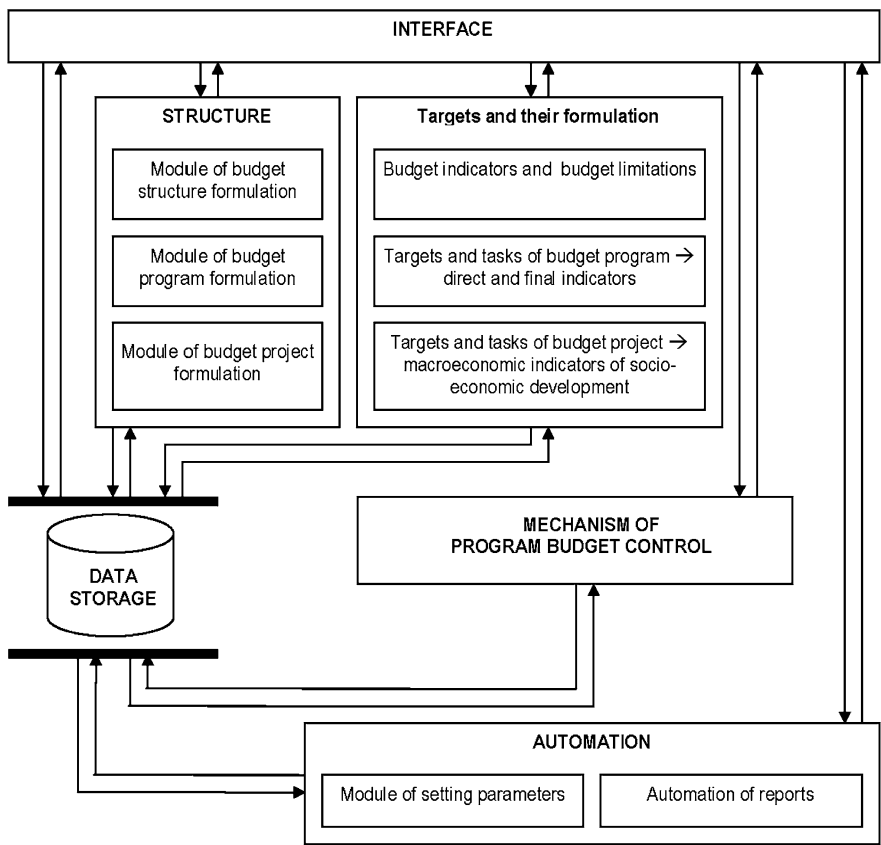


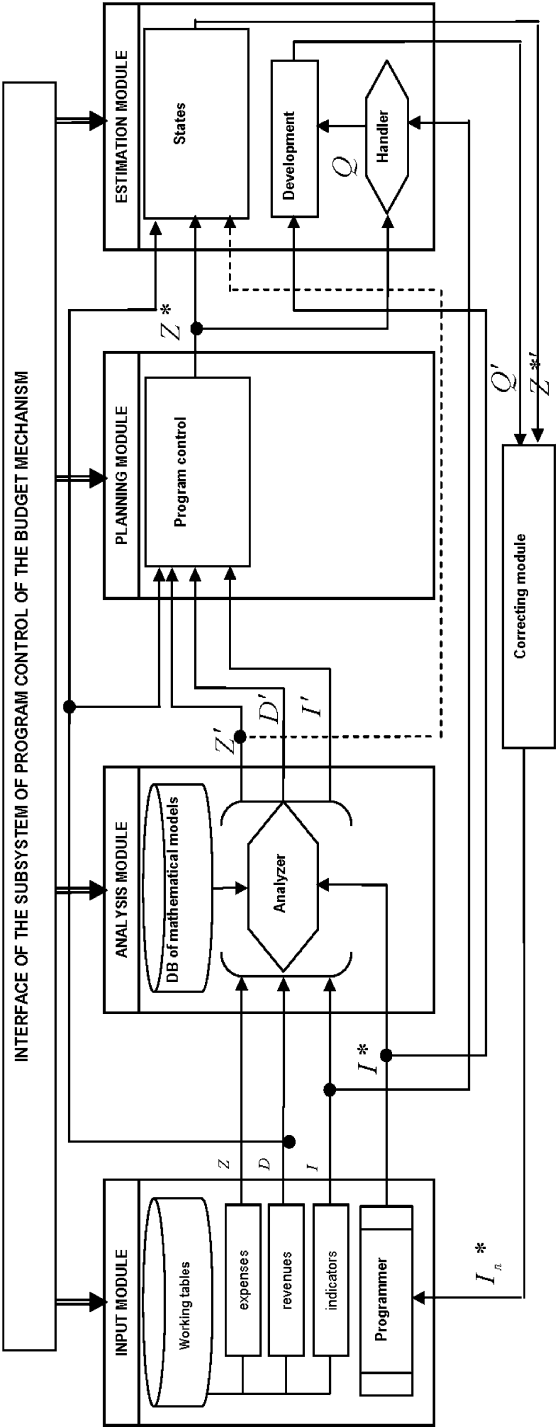
Fig. 2.3 A model of IS budget control of the result-oriented program-targeted budgeting approach

parameters that can set parameters needed for budget modeling and making managerial decisions.

Data storage is a database the organization of which satisfies the requirements of emergency use of the budget system and result-oriented budgeting technology.

The mechanism of program control of the budget is a modeling block of program control of budget funds in the form of an optimal allocation scheme. This block realizes the following criterion from the system of criteria used to describe the middle-term budgeting scheme—procedures and methods of estimation of demands (budget funds) for achieving purposes in the long-term time periods.

In terms of cognitive approach the information model is a set of the following subsystems: a complex of distributed technical means; a complex of mathematical models; a complex analyzing states and making decisions; data bases; a system of information processing and mapping. The information system includes the three main components: database, base of mathematical models and program subsystem.



Notes: Z is the dynamics of expenditure items; D is the dynamics of revenue items; I is the dynamics of the main indicators of socio-economic development; I^* is the forecast of the indicators for the planned middle-term period; Z' is the forecast of the expenditure items for the middle-term period; D' the forecast of the revenue items for the middle-term; I' is transformation of indicators into dimensionless form; Z^* is a forecasted state of expenditure items obtained as a result of optimal expenditure of budget funds; Z^{**} is a comparative estimation of the state of the expenditure base; Q is calculation of new values of the main indicators of socio-economic development; Q' is a comparative estimation of the state of socio-economic development; I_n^* is a correcting value of the forecasts of indicators for the planned middle-term period; DB is the database with the frame data organization; $Analyzer$ is a mathematical module transforming information from one type into the other by mathematical models stored in the database; $Programmer$ is a module for input of target indicators; $Handler$ is a gateway providing calculation of new values of indicators; $State$ is a block for analysis of the state of the budget system in the current and planned periods; $Development$ is a block for analysis of the state of socio-economic development of the country/region; $Correcting$ module is a block correcting the strategy of development in the form of adjustment of indicators.

Fig. 2.4 Architecture of the subsystem of program control of budget mechanism

The main purpose of development of the IS program control is a complex information support of the processes of budget control with the use of possibilities of information and telecommunication technologies.

In order to achieve the main purposes it is necessary to solve the following problems:

- to form a common data bank reflecting the state of the budget system and providing timely and operative allocation of full, objective, reliable and unambiguous information on the budget process;
- to provide an information environment common for all users, to provide common standards for preparation of information and normative-reference materials;
- to provide efficient two-way communications and feedback channels;
- to develop a common style of paper work, to provide a centralized access to reports and user-friendly navigation in the information system.

Characteristics of the “portrait” of the IS for program budget control:

- methodology of control aimed at achieving strategic purposes of the administration of a state institution expressed in the IS as a system of controlling actions regulating user’s activities;
- possibility of access to data for many users connected in a local network of the organization;
- availability of communication means;
- advanced, user-friendly graphic interface of the terminal user;
- regimes of processing of operative information close to the real-time regime;
- means of access authentication and differentiation, which make it possible to dose information according to job responsibilities of users, high level of protection from unauthorized access;
- a common server for databases, possibility of processing of thousands and millions of records for compiling reports;
- use of standard languages and protocols for data presentation and manipulation.

The program control module of the information system (Fig. 2.4) simulates program control of budget funds as an optimal distribution scheme providing a possibility of corrections by means of adjustment of the system of indicators. This module clearly demonstrates the algorithm for budget planning and control formulated on the base of mathematical methods and models.

References

1. Implementation of the middle-term financial planning in the budget process of the Russian Federation. Report of the Fiscal Policy Center. Contract OUT-PER-I-00-99-00003-00 (2002)
2. Serpilin: The Great Britain experience in regional management and result-oriented budgeting: materials of the “International Cooperation” program. <http://www.unicom-ms>. 10 March 2006
3. Melkers, J., Willoughby, K.: The state of the states: performance-based budgeting requirements in 47 out of 50. *Public Adm. Rev.* **58**(1), 66–73 (1998). doi:[10.2307/976891](https://doi.org/10.2307/976891)

4. Lee, J.Y.-J., Wang, X.: Assessing the impact of performance-based budgeting: a comparative analysis across the United States, Taiwan, and China. *Public Adm. Rev.* **69**, 60–66 (2009)
5. Result-oriented budgeting: international experience and possibilities of its use in Russia. Report of the Fiscal Policy. Contract OUT-PER-I-00-99-00003-00 (2002)
6. Mutanov, G.M., Mamykova, Zh.D.: *Methods and Mathematical Models of the Software Budget Management*. University, Almaty (2008), p. 144. ISBN 978-601-208-014-8
7. Bruins, R.J.F., Munns, W.R. Jr., et al.: A new process for organizing assessments of social, economic, and environmental outcomes: case study of wildland fire management in the USA. *Integr. Environ. Assess. Manag.* **6**, 469–483 (2010)
8. Narver, J.C., Slater, S.F.: The effect of a market orientation on business profitability. *J. Mark.* **54**, 20–35 (1990)
9. Hurley, R.F., Hult, G.T.M.: Innovation, market orientation and organizational learning: an integration and empirical examination. *J. Mark.* **62**, 42–54 (1998)
10. Mutanov, G.M., Kulikova, V.P.: *Mathematical Modeling of Economic Processes*. Economics, Almaty (1999), p. 356
11. Intriligator M. *Mathematical Methods of Optimization and Economic Theory*. Progress (1975), p. 607. Translation from English
12. Sovetov, B.Ya., Yakovlev, S.Ya.: *Modeling of Systems*. Moscow (1985), p. 239
13. *The Theory of Forecasting and Decision Making: A Teaching Aid*. Higher School (1977)
14. Neimark, Yu.I., Kogan, N., Savelyev, V.: *Dynamic Models of Management Theory*. Nauka, Moscow (1985), p. 400. Main editorial of physico-mathematical literature
15. Zubov, V.I.: *Lectures on Control Theory*. Nauka, Moscow (1975)
16. Eliseyeva, I.I.: *Econometrics*. Moscow (2003)
17. Dougerti, K.: *Introduction to Econometrics*. INFRA-M, Moscow (1999), p. 402. Translated from English
18. Chernyak, Y.A.: *System Analysis in Economic Control*. Economics, Moscow (1975), p. 191
19. Yudin, D.B., Yudin, A.D.: *Extreme Models in Economics*. Economics, Moscow (1979), p. 288
20. Shelobayev, S.I.: *Mathematical Methods and Models*. Moscow (2001)
21. Kholod, N.I., Kuznetsov, A.V., Zhikhar, Y.N., et al.: *Economic and Mathematical Methods and Models*. BGEU (2000), p. 412
22. Sulitsky, V.N.: *Methods of Statistical Analysis in Management*. Moscow (2002)
23. Lewis, K.D.: *Medium-Term Forecasting*. Moscow (1993)
24. Teece, D.J., Pisano, G.P., Shuen, A.: Dynamic capabilities and strategic management. *Strateg. Manag. J.* **18**, 509–533 (1997)
25. Yefimova, M.P., Petrova, E.V., Rumyantsev, V.N.: *General Theory of Statistics*. Moscow (2004)
26. Zhdanov, S.A.: *Economic Models and Methods in Management*. Business and Service, Moscow (1988), p. 176
27. Dubrova, T.A.: *Statistical Methods in Forecasting*. UNITY, Moscow (2003)
28. Mutanov, G.M., Bykova, I.Yu., Mamykova, Zh.D.: Software control of budget funds. *Bull. Natl. Acad. Eng. Repub. Kaz.* **4**(26), 33–38 (2007)
29. Mutanov, G.M., Shintemirova, A.U.: *Mathematical Budget Models*. Astana (2003), p. 176
30. Afanasyev, V.N., Kolmanovskii, V.B., Nosov, V.R.: *Mathematical Theory of Constructing Control Systems*. “High School” (1998), p. 574
31. Odintsov, B.E.: *Designing Economic Expert Systems: Teaching Aid for Higher Education Institutions*. Computer, UNITY (1996), p. 166
32. Heerdt, J.A., Coutinho, D.F., Mussa, S.A., Heldwein, M.L.: Control strategy for current harmonic programmed AC active electronic power loads. *IEEE Trans. Ind. Electron.* **61**(8), 3810–3822 (2014)
33. Adi, V.S.K., Chang, C.-T.: A mathematical programming formulation for temporal flexibility analysis. *Comput. Chem. Eng.* **57**, 151–158 (2013)

34. Grigoroudis, E., Phillis, Y.A.: Modeling healthcare system-of-systems: a mathematical programming approach. *IEEE Syst. J.* **7**(4), 571–580 (2013)
35. Takagi, T., Sugeno, M.: Fuzzy identification of systems and its applications to modeling and control. *IEEE Trans. Syst. Man Cybern.* **15**(1), 116–132 (1985)
36. Rachel, F.M., Cugnasca, P.S.: The multi-agent programming paradigm use for railway applications. In: Allan, J., Arias, E., Brebbia, C.A., Goodman, C., Rumsey, A.F., Sciutto, G., Tom, N. II (eds.) *Computers in Railways XI: Computer System Design and Operation in the Railway and Other Transit Systems*, vol. 103, pp. 641–650 (2008)

Mathematical Methods and Models in Economic
Planning, Management and Budgeting

Mutanov, G.

2015, XIV, 356 p. 99 illus., 19 illus. in color., Hardcover

ISBN: 978-3-662-45141-0