

Chapter 2

Principle of INS/CNS/GNSS Navigation System

2.1 Introduction

Inertia/celestial/satellite navigation can provide high-precision, highly reliable position, velocity, and attitude information with strong autonomous ability, thus it is the most effective means of high-performance navigating for new generation aircraft. To achieve high accuracy navigation for vehicles, i.e., to determine the position, velocity and attitude information, we must first understand the motion information of the vehicles to which the reference coordinate system relates; meanwhile, for mastering the inertia/celestial/satellite navigation system technology, the basic principle and knowledge of inertial, satellite, and celestial navigation also need to be understood. For this reason, coordinate systems commonly used in navigation and Earth reference model will be briefly introduced firstly in the following paragraphs, and then, the working principle and error characteristics of inertial, satellite, and celestial navigation system (CNS) will be further described.

2.2 Coordinate Frames and Earth Reference Model Commonly Used in Navigation

2.2.1 *The Coordinate Frames Used in Navigation*

For the purpose of navigation, the vehicles need to be provided with the position, velocity, and attitude information relative to space or an object. Due to the relativity of movement, an appropriate coordinate system needs to be established to describe the parameters of a vehicle. Coordinate systems used in navigation can be divided into inertial (absolute) and noninertial (relative) coordinate system.

2.2.1.1 Inertial Coordinate Frame of Reference

Inertial coordinate system complies with Newton's laws of mechanics, in which any free motion has a constant magnitude and direction. Everything in the universe is in motion, thus selection of the inertial coordinate system will be based on different navigation needs of objects. The following paragraphs will describe two common inertial coordinate systems.

Geocentric Inertial Coordinate System (Referred to as the i)

Geocentric inertial coordinate system is also known as celestial coordinate system. It does not consider the orbital motion of the Earth around the Sun or the movement of the Sun relative to space. The origin of the coordinate system is the center of the Earth and axis x_i , y_i are in the Earth's equatorial plane. The axis x_i points to the equinox, the axis z_i points to the Earth's polar, and the right-hand rule determines the direction of axis y_i .

Launch Point Inertial Coordinate System (Referred to as the li)

Some vehicles (such as missiles) use launch point inertial coordinate system as the reference for measuring flying position. The origin of the coordinate system is the emission point; axis x_{li} points to the launching direction, axis y_{li} points to the sky, and the direction of axis z_{li} is determined by the right-hand rule. After launching, the launch point inertial coordinate system will be constantly set in the inertial space.

2.2.1.2 Noninertial Coordinate System

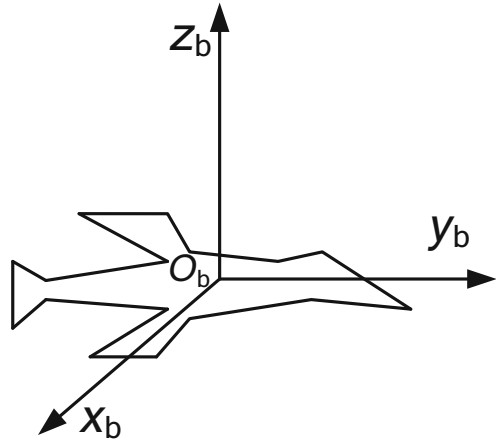
Earth Coordinate System (Referred to as the e)

Earth coordinate system is fixed with the Earth and rotates with the Earth. It is approximated that the coordinate system revolves at the Earth's rotation rate $\omega_{ie}(\omega_{ie} \approx 15.04107^\circ/h)$ relative to the inertial coordinate system. The origin of the coordinate system is the center of the Earth; axis z_e points to the Earth's polar, axis x_e extends to zero meridian line, and direction of axis y_e is determined by the right-hand rule (pointing to 90° E).

Geographic Coordinate System (Referred to as the t)

The geographic coordinate system commonly used is the east-north-up coordinate system and the north-east-down coordinate system. The origin of east-north-up coordinate system is the centroid of the vehicle; axis x_t points toward the east along the

Fig. 2.1 Body coordinate system for airplane



prime vertical of reference ellipsoid direction, axis y_t points to the north along the meridian of reference ellipsoid, and axis z_t is determined by the right hand rule. The origin of the north-east-down coordinate system is also the centroid of the vehicle; axis x_t points to the north along the meridian of reference ellipsoid, axis y_t points to the east along prime vertical of the reference ellipsoid, and axis z_t is determined by the right-hand rule.

Body Coordinate System (Referred to as the b)

This section will introduce the body coordinate system for aircraft, missiles, and satellites.

① *Body Coordinate System for Airplane*

The plane body coordinate system is a coordinate system fixed on the body of the plane. Its origin is the centroid of the aircraft; axis x_b points to right wing along the horizontal axis of the carrier, axis y_b points to the head of the plane along the longitudinal direction of the carrier, and axis z_b points upward to the carrier along the vertical axis, as shown in Fig. 2.1.

② *Body Coordinate System for Missile*

The missile body coordinate system is fixed with the missile body. Its origin is the centroid of the missile; axis x_b points to the head direction of the missile along the longitudinal axis, axis y_b is vertical to axis x_b and points upward, and the direction of axis z_b is determined by the right-hand rule, as shown in Fig. 2.2.

Fig. 2.2 Body coordinate system for missile

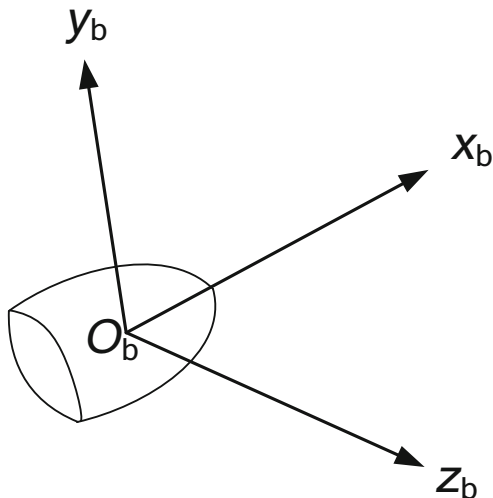
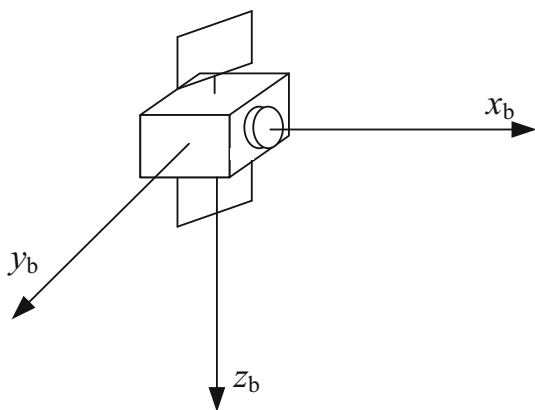


Fig. 2.3 Body coordinate system for satellite



③ Satellite Coordinate System

The satellite coordinate system is fixed with the satellite body. Its origin is the satellite centroid; axes x_b , y_b , and z_b are normally defined on the inertial principle axis of the satellites, which is also known as principle axis coordinate system, as shown in Fig. 2.3.

Platform Coordinate System (Referred to as the p)

For platform inertial navigation system, the platform coordinate system is fixed with the platform which describes the pointing direction of the platform with its origin at the centroid of it. For strapdown inertial navigation system (SINS), the platform coordinate system is realized by direction cosine matrix storing in the computer, thus it is also called “math platform.”

Navigation Coordinate System (Referred to as the n)

The navigation coordinate system is chosen while solving the navigation parameters according to working needs. For SINS, the navigation parameters are not solved within the body coordinate system, while the measurement values of inertial device is generally in the body coordinate system. Therefore, the output of inertial devices must be decomposed into different coordinate system for the convenience of solving navigation parameters, then navigation calculation is carried out, and this coordinate system is the navigation coordinate system. For platform inertial navigation system, the platform coordinate system is the navigation coordinate system.

2.2.2 The Conversion of Coordinate Systems

Currently, the methods commonly used to describe the relation between two coordinate systems are the direction cosine matrix method and quaternion.

The following paragraphs will introduce coordinate conversion system from system n to b by taking aircraft as an example, using the usual rotation sequence $Z \rightarrow X \rightarrow Y$, i.e., first turn the heading angle φ , then the pitch angle θ , and roll angle γ at last.

2.2.2.1 Direction Cosine Matrix Method

Direction cosine matrix is one of the most basic and direct conversion matrix methods between two coordinate systems. According to the above order, the direction cosine matrixes after three rotations are:

$$T_\varphi = \begin{bmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad T_\theta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix}$$

$$T_\gamma = \begin{bmatrix} \cos \gamma & 0 & -\sin \gamma \\ 0 & 1 & 0 \\ \sin \gamma & 0 & \cos \gamma \end{bmatrix}$$

$$C_n^b = T_\gamma \cdot T_\theta \cdot T_\varphi$$

$$= \begin{bmatrix} \cos \gamma \cos \varphi - \sin \gamma \sin \theta \sin \varphi & \cos \gamma \sin \varphi + \sin \gamma \sin \theta \cos \varphi & -\sin \gamma \cos \theta \\ -\cos \theta \sin \varphi & \cos \theta \cos \varphi & \sin \theta \\ \sin \gamma \cos \varphi + \cos \gamma \sin \theta \sin \varphi & \sin \gamma \sin \varphi - \cos \gamma \sin \theta \cos \varphi & \cos \gamma \cos \theta \end{bmatrix}. \quad (2.1)$$

Among them, C_n^b is the coordinate conversion matrix from system n to b .

2.2.2.2 Quaternion

Since the beginning of the 1960s, with the development of SINS, the observation instrument itself translates and rotates with the vehicles in dynamic measurement, which makes the description and solution of the problem very difficult. The quaternion theory classified such problem as the one of a rigid body rotating around a fixed point, which effectively solves this problem, and thus has been widely used.

The Definition of the Quaternion

Definition of quaternion \mathbf{q} :

$$\mathbf{q} = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}$$

where q_0 is the quaternion scalar part, the latter is the quaternion vector part, denoted as q , and the equation can be written as:

$$\mathbf{q} = q_0 + q$$

Define the quaternion mode as 1, and it is written as:

$$N(\mathbf{q}) = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2} = 1.$$

Describing the Vector Rotation with Quaternion

Quaternion can describe a rigid body rotating around a fixed point, which is to describe the rotation of a coordinate or a vector relative to a coordinate system. For quaternion $\mathbf{q} = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}$, it can be written as follows:

$$\mathbf{q} = \cos \frac{\theta}{2} + \sin \frac{\theta}{2} \cos \alpha \mathbf{i} + \sin \frac{\theta}{2} \cos \beta \mathbf{j} + \sin \frac{\theta}{2} \cos \gamma \mathbf{k} \quad (2.2)$$

where θ is the rotation angle, $\cos \alpha$, $\cos \beta$, $\cos \gamma$ is the direction cosine between instantaneous rotation axis and the reference coordinate axis.

Compare the two equations:

$$q_0 = \cos \frac{\theta}{2}, q_1 = \sin \frac{\theta}{2} \cos \alpha, q_2 = \sin \frac{\theta}{2} \cos \beta, q_3 = \sin \frac{\theta}{2} \cos \gamma \quad (2.3)$$

Usually, this type of formula (2.2) is called feature quaternion, referred to as quaternion. The scalar part of the quaternion $\cos \theta/2$ represents the cosine of half the

rotation angle, and its vector part represents the direction of instantaneous rotation axis. Therefore, a quaternion represents both the direction of the axis and the size of the rotation angle; this relation can be realized by the following operation:

$$R^b = q \circ R^n \circ q^* \quad (2.4)$$

Formula (2.4) indicates that the vector R^n is rotated at θ relative to the reference coordinate, instantaneous angle is determined by the quaternion \mathbf{q} and becomes vector R^b after rotation. Vector q^* is the conjugate quaternion of quaternion \mathbf{q} . This is the rotating vector expression of the reference coordinate.

From the Formula (2.4), it can be obtained:

$$R^b = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_2q_3 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} R^n \quad (2.5)$$

So,

$$C_n^b = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_2q_3 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}. \quad (2.6)$$

From formulas (2.1) and (2.6), it can be drawn that the direction cosine matrix and the quaternion methods can both achieve conversion between coordinate systems, in essence, they are equivalent.

2.2.3 Earth Reference Model

2.2.3.1 Main Radius of Curvature of the Earth

The Earth is not a homogeneous sphere, but a nonhomogeneous ellipsoid resulted from the Earth's gravity and centrifugal force. There exist mountains, deep valleys, continents, and oceans on the Earth's surface, thus it has an irregular curve surface and the curvature radius of any point is different. As shown below in Fig. 2.4a, 2.4b, the Plane G is the tangent plane of a point M on the Earth's surface with the normal \vec{n} ; AB is the tangent of point M along the south- north direction, CD is the tangent of point M along the east- west direction. Plane AB and \vec{n} determine the main radius of

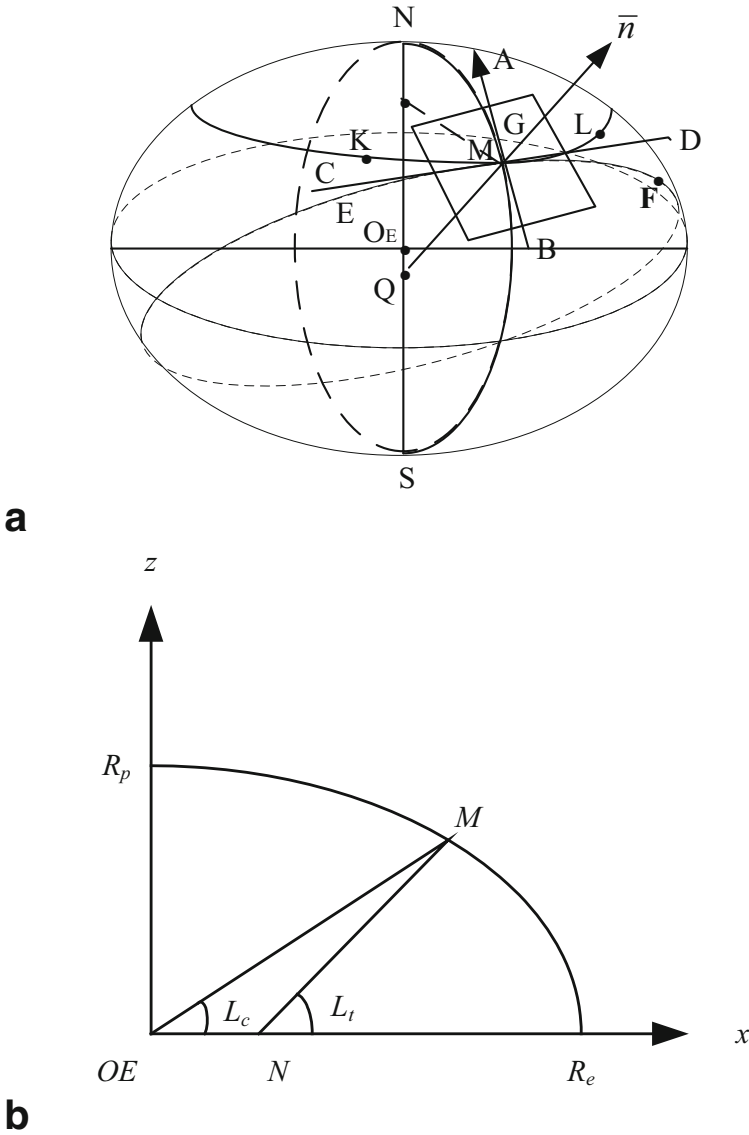


Fig. 2.4 Reference ellipsoid. **a** Primary curvature radius of the Earth reference ellipsoid. **b** Partial cutaway view of the reference ellipsoid

curvature of the meridian plane and the Plane CD and \vec{n} determine the main radius of curvature of the prime plane [1].

From the elliptic equation:

$$\frac{z^2}{R_p^2} + \frac{x^2}{R_e^2} = 1 \quad (2.7)$$

L_t is the angle between the normal of point M and scale plane of reference ellipsoid, we have:

$$\frac{dz}{dx} = -\frac{R_p^2}{R_e^2} \frac{x}{z} = -\cot L_t \quad (2.8)$$

By the

$$\frac{R_p^2}{R_e^2} = 1 - k^2 \quad (2.9)$$

Substitute into Formula (2.8), we have:

$$z = (1 - k^2)x \tan L_t \quad (2.10)$$

Then substitute the above formula into elliptic equations, we have:

$$\frac{(1 - k^2)^2 x^2 \tan^2 L_t}{R_p^2} + \frac{x^2}{R_e^2} = 1$$

Then:

$$x = \frac{R_e \cos L_t}{(1 - k^2 \sin^2 L_t)^{1/2}} \quad (2.11)$$

That is, the expression of the radius curvature can be written as follows:

$$\rho = \left[1 + \left(\frac{dz}{dx} \right)^2 \right]^{3/2} \bigg/ \frac{d^2z}{dx^2} \quad (2.12)$$

From the Formula (2.8), we have:

$$\frac{d^2z}{dx^2} = \frac{1}{\sin^2 L_t} \frac{dL_t}{dx} \quad (2.13)$$

From Formula (2.11), we have:

$$\frac{dx}{dL_t} = -R_e(1 - k^2) \sin L_t / (1 - k^2 \sin^2 L_t)^{3/2} \quad (2.14)$$

Eventually, after finishing, the radius curvature of the Earth in meridian plane can be obtained as:

$$R_M = \frac{R_e(1 - k^2)}{(1 - k^2 \sin^2 L_t)^{3/2}} \quad (2.15)$$

As $e = \frac{R_e - R_p}{R_e}$, obviously, $e < 0$, $k^2 = 2e - e^2$, omitting e^2 :

$$R_M = \frac{R_e(1 - 2e)}{(1 - 2e \sin^2 L_t)^{3/2}} \quad (2.16)$$

Take $(1 - 2e\sin^2 L_t)^{-3/2} \approx 1 + 3e\sin^2 L_t$, then:

$$(1 - 2e\sin^2 L_t)^{-3/2} \approx 1 + 3e\sin^2 L_t. \quad (2.17)$$

Now we find the radius curvature R_N perpendicular to meridian plane with the same normal. According to the law of radius of curvature in any plane surfaces, the relation between radius curvature R_N of the curve at the point of M and radius of latitude circle of the same point is as follows:

$$x = R_N \cos L_t \quad (2.18)$$

Then:

$$R_N = \frac{R_e}{(1 - 2k^2 \sin^2 L_t)^{1/2}} \quad (2.19)$$

Take $k^2 = 2e - e^2$ and omit e^2 . Then take $(1 - 2e\sin^2 L_t)^{-1/2} \approx 1 + e\sin^2 L_t$

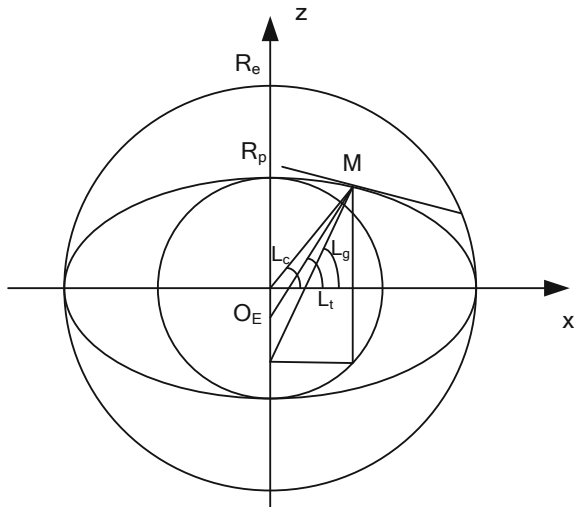
Finally we have:

$$(1 - 2e\sin^2 L_t)^{-1/2} \approx 1 + e\sin^2 L_t. \quad (2.20)$$

2.2.3.2 Several Vertical Lines

The vertical lines mentioned in this book are mainly astronomy, geographic, and geocentric vertical line, shown in Fig. 2.5. Astronomy vertical line is the direction of normal line at the horizontal plane of the Earth of any point on the Earth's surface. The angle between the equatorial plane, the astronomy vertical line is defined as

Fig. 2.5 Several vertical lines and latitude



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