Preface

By a 'scientific law', we mean an equation in which the variables represent physical or geometrical quantities such as mass, length, (absolute) temperature or time duration. Examples are the Ideal Gas Laws, Coulomb's Law, the Lorentz-FitzGerald Contraction, Beer's Law, or van der Waals' Equation (see e.g. Hix and Alley, 1958), the last three playing important roles in this book. Examples in geometry are many, such as the Pythagorean Theorem and the equations of a parabola and of the volume of a sphere.

As suggested by these examples, we restrict consideration to laws linking ratio scale quantities. In such cases, which represent the large majority in science and geometry, fixing the units suffices to specify the variables in an equation[1].

The second word in the title of our book, 'meaningfulness', refers to a fundamental invariance property that is satisfied by practically all scientific or geometrical laws. Stated informally, it is that: *the mathematical form of a law is not altered by a change of units*, for example from meters to centimeters or miles, or from kilograms to grams or pounds.

There are three important reasons for requiring meaningfulness in the formulation of scientific laws. The first two are obvious.

1) Permitting non-meaningful laws would result in a scientific Tower of Babel, in which the mathematical form of a model would depend upon the particular measurement units favored by a community of scientists, promoting confusion.

2) More importantly, as the units of a scientific variable have no representation in nature, any non-meaningful expression would be a poor representation of reality.

3) Taking meaningfulness as an axiom of a scientific theory may lead to weakening, or even replacing, the other axioms, resulting in a refocusing of the theory, and conceivably, leading to a deeper understanding of the basic mechanisms at work.

For instance, under meaningfulness, the Pythagorean Theorem follows from assuming that the hypothenuse of a right triangle is an associative,

[1] Our concepts and results can surely be extended to other cases, such as equations involving interval scale variables. However, this will be left for later work.

symmetric and homogeneous function of the two sides of the triangle (see Theorem 7.1.1 on page 85). For another example, in case of the Lorentz-FitzGerald Contraction, our analysis, based on a meaningfulness condition, led us to assign a prominent place to a particular 'quasi upper convexity' class of transformations (see Theorem 8.6.1 on page 125).

A critical feature of our approach to the formalization of meaningfulness lies in our notation: we systematically index the laws by the units of all the variables. For example, we write $L_{\alpha,\beta}(\ell, v)$ for the function of the Lorentz-FitzGerald Contraction evaluated for a length ℓ and a speed v, with α and β denoting the units of the two variables. This notation[2] is highly unconventional. (Typically, the units of the variables are either fixed and then ignored in the notation or, in dimensional analysis, the dependence on the unit is implicit in the concept of "physical quantity.") This device allows an explicit formalization of an invariance with respect to the unit(s) and is essential for the derivation of our results. Indeed, an important benefit is derived from such a notation: representing the units as added variables in our equations makes them more amenable to functional equation techniques, many examples of which are contained in our book.

A disclaimer is in order here. Obviously, we are not proposing that the admittedly elaborate notation of our book should become the standard for scientific discourse. Rather, we are advocating its use in the context of meaningfulness arguments. This notation could then be discarded as soon as all its useful consequences have been derived[3].

Together with more purely scientific considerations, the methods used here, suitably extended and generalized, may perhaps be instrumental in a search for suitable mathematical expressions for scientific laws. A more ambitious goal would be the systematic construction of a catalogue of possible meaningful scientific laws in terms of their invariance properties.

However, our book is only the beginning of the development of a general theory analyzing scientific functions from the standpoint of intuitive, abstract theories about the phenomena. For example, we mostly deal with what we call *self-transforming* collections of scientific functions of two variables, that is, functions having the scale of their output identical to the scale of their first variable, such as the Lorentz-FitzGerald Contraction, Beer's Law, or the Pythagorean Theorem. While many scientific functions are self-transforming, many are not, and so our theory will need further elaboration.

Because our book is meant to be accessible to anyone with a scientific background equivalent to a master's degree in mathematics, we have included a

---

[2] Which is also used by Louis Narens (see e.g. Falmagne and Narens, 1983; Narens, 2007, 2002).

[3] In fact, our book provides the mechanism for discarding the notation in the guise of what we call the "*initial code*."

chapter on functional equations because its results are essential to our derivations, and the subject is not always part of the standard university mathematics curriculum.

We have benefited from useful discussions on the topic of meaningfulness with many people. We mention in particular János Aczél, Scott Brown, Jean-Paul Doignon, Geoff Iverson, David Krantz, Duncan Luce, Jeff Matayoshi, Louis Narens, Fred Roberts and Pat Suppes. We are especially grateful to János Aczél for his many detailed comments on part of this work over the years. We also thank, for their always helpful efficiency, Mrs. Glaunsinger, Mrs. Fischer, and Dr. Engesser, all from Springer-Verlag.

Lastly, we thank our respective spouses, Dina and Rose, for their kind and unwavering support.

Jean-Claude Falmagne
Irvine, CA

Christopher Doble
Tustin, CA

June 30, 2015

http://www.springer.com/978-3-662-46097-9

On Meaningful Scientific Laws
Falmagne, J.-C.; Doble, C.
2015, XIII, 170 p., Hardcover
ISBN: 978-3-662-46097-9