Preface

Quantum integrable models play important roles in a variety of fields such as quantum field theory, condensed matter physics, and statistical physics. For decades, a number of theoretical methods have been developed for solving the eigenvalue problem of integrable models. Among them, the three typical and most popular methods are the coordinate Bethe Ansatz method proposed by H. Bethe in 1931, the $T-Q$ method proposed by R.J. Baxter in the early 1970s, and the algebraic Bethe Ansatz method proposed by the Leningrad Group in the late 1970s. These methods have been demonstrated as powerful in solving most of the known quantum integrable models. After Baxter's work on the eight-vertex model, people realized that a special class of quantum integrable models exists in which the $U(1)$ symmetry is broken and, in some cases, obvious reference states are absent. Some well-known examples are the *XYZ* spin chain (or equivalently the eight-vertex model), the quantum Toda chain, the anisotropic spin torus, and the quantum spin chains with nondiagonal boundary fields. Several methods have since been developed to approach this remarkable problem. Among them, two promising ones are Baxter's $T-Q$ method and Sklyanin's separation of variables (SoV) method, which provide efficient tools to treat quantum integrable models with functional analysis.

This book serves as an introduction to the off-diagonal Bethe Ansatz (ODBA) method, a newly developed analytic theory to approach exact solutions of quantum integrable models, especially those with nontrivial boundaries. In any sense, ODBA is not an isolated theory but one based on extensive existing knowledge. Therefore, this book also covers some main ingredients of $T-Q$ relation, algebraic Bethe Ansatz, thermodynamic Bethe Ansatz, fusion techniques and Sklyanin's SoV basis, etc. It is organized in a parallel structure to explain how ODBA works for different types of integrable models. Chapter 1 is devoted to the basic knowledge of quantum integrable models, and Chap. 2 to a comprehensive introduction of the algebraic Bethe Ansatz, the fusion techniques, and the SoV scheme. In addition, the thermodynamic Bethe Ansatz method is introduced as a tool for deriving the physical quantities. Chapter 3 focuses on the application of ODBA in the periodic *XXZ* model and the *XYZ* model, and Chap. 4 on the topological boundary problem using the anisotropic spin torus as example. Chapter 5 studies the exact solution of the spin-$\frac{1}{2}$ chain Hamiltonian with generic open boundaries, which had been a long-standing problem for over two decades. Chapter 6 is devoted to the one-dimensional Hubbard model and the super-symmetric $t-J$ model with generic integrable boundaries. Chapters 7 and 8 focus on the generalizations of ODBA to high-spin integrable models. Chapter 9 is devoted to the Izergin-Korepin model with generic boundaries, a typical integrable model beyond the A-type models. Calculations of some important physical quantities based on the Bethe Ansatz equations, especially the nontrivial boundary contributions, are given in Chaps. 2–5 and the method for retrieving the eigenstates based on the inhomogeneous $T-Q$ relations and the SoV basis is introduced with concrete examples in Chaps. 4 and 5.

In general, the authors aim to introduce topics that are under ongoing research and are developing at a stimulating pace in this fascinating field. These contents are selected for the book according to the authors' own understanding of the topics under discussion. Thus, they devote much attention to methods that work well for the nontrivial boundaries (Research on nontrivial surface effects, including edge states of the quantum Hall effect, surface states of topological insulators, open strings, and stochastic processes in nonequilibrium statistical physics, has become a trend in modern physics. The authors study this problem from the mathematical physics side.). The two-dimensional lattice models and most of the well-established knowledge on the models with periodic and diagonal boundary conditions are not included, since several excellent books have already covered these topics. This book was originally planned for around 100 pages but then was expanded to the present size, thanks to suggestions of numerous colleagues that detailed calculations should be included as much as possible to make it easy to follow the method. Although most of the results contained in this book have been rigorously proven, we still use the word "exactly" in the title as Baxter did for his book, for the reason that some results in this book are not that rigorously proven. For example, the thermodynamic limit is constructed based on reasonable physical arguments. For most models considered in this book, numerical results are provided to support the analytical ones, which is a conventional way for physicists and scientists in other fields to support their proposals, though it may not meet mathematical rigorousness. For physicists, to propose something correct is always more ambitious than to prove it.

Also, the methodology still needs to be developed and leaves some open questions, among which are: how to apply it in graded integrable models and in cyclic integrable models with nontrivial boundaries; how to retrieve the Bethe states and to derive the scalar products of high-rank quantum integrable models, etc. We expect that those issues may undergo significant progress in the near future.

The authors would like to share with you their happiness in undertaking the collaboration, which started in Fall 2012. At that time little was known about ODBA. Without the ensuing teamwork, it would have been impossible to achieve the main original results contained in this book!

We apologize if any important references are omitted. Any such error is definitely due to the limits of the authors' knowledge of the literature.

Acknowledgments Y. Wang would like to acknowledge a number of people who directly or indirectly helped to make the book possible. The first is F.C. Pu, who introduced the author into this interesting field. Under Pu's supervision, the author came to notice the problem of solving those integrable models without $U(1)$ symmetry 30 years ago when introduced to Baxter's papers and Takhtajan and Faddeev's paper about the XYZ model. As a junior graduate student, the author was unable to understand why the model could only be solved for an even number of sites. In 1997, when he became aware of the paper about the open *XYZ* model by H. Fan et al., he gradually realized that Baxter's local gauge transformation (vertex-face transformation or corner matrix) method could also be applied to the *XXZ* spin chain with nondiagonal boundary fields. The idea became clearer and clearer during his collaboration with J. Cao, H.-Q. Lin, and K. Shi. The author's thanks also go to D. Jin and L. Yu who have continually helped, supported, and encouraged him in his career. Fruitful discussions with R.J. Baxter and R.I. Nepomechie about the manuscript are especially acknowledged. Most importantly, the author would like to extend his deep gratitude to his wife Yan, who has been dedicating herself to taking care of the family so that her husband could devote more of his time to work even before the baby was coming!

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All the authors would like to acknowledge some of the referees for their constructive comments on several of the authors' original papers. Remarkably, a question of "how to get the root distribution of the Bethe Ansatz equations" stimulated the authors to write the paper on the thermodynamic limit, while the comments of "how to prove the completeness of the solutions" and "what is the corresponding eigenstate in the homogeneous lattice case" stimulated the authors to propose the two theorems in Chap. 1 and related corollaries and to retrieve Bethe states based on the inhomogeneous $T-Q$ relations. These papers become important parts of the ODBA scheme.

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