

Chapter 2

Preliminaries: Modeling, Communication Scheme, and Lemmas for NCSs

In this chapter, delay distribution-dependent modeling and discrete event-triggered communication (ETC) scheme are introduced for NCSs studied in this book. The main idea of delay distribution-dependent modeling is to use the nonuniform distribution character of IP-based networks in the modeling of NCSs, therefore, it is helpful to reduce the conservativeness in the analysis and synthesis of NCSs. Compared with the time-triggered communication scheme, the proposed ETC scheme has the advantage in preserving the desired control performance, while saving the limited network bandwidth.

2.1 Delay Distribution-Dependent Modeling

2.1.1 Nonuniform Distribution of IP-based Network Delays

IP-based networks, such as, Internet networks, Ethernet networks, and IP-based wireless sensor networks are generally used in NCSs. Actual IP-based network delays are time-varying and display irregular behavior. To show the characteristics of actual communication delays in IP-based networks, the Round-Trip Time (RTT) delays from different Ethernet network nodes are measured in [82]. Their results have showed nonuniform distribution of the RTT delays in the IP-based networks. For an NCS over 10Mbps IP networks, NCSs over IP networks are simulated in [83] by making use of the open source package NS2. Part of the delays from the sensor-to-controller over 240s are shown in Fig. 2.1. The corresponding histograms of the delays are shown in Fig. 2.2. From Figs. 2.1 and 2.2, one can see that the IP-based communication delay is an approximate of the probability density.

Moreover, Fig. 2.2 shows that the histograms of the delays skew to the left, which indicates that the probability of the large delays are shorter than the probability of the median and mean delays. Actual delays can be much longer than the median and mean, but with much lower probability. Therefore, communication delay is

Fig. 2.1 IP-based network delays from the sensor-to-controller

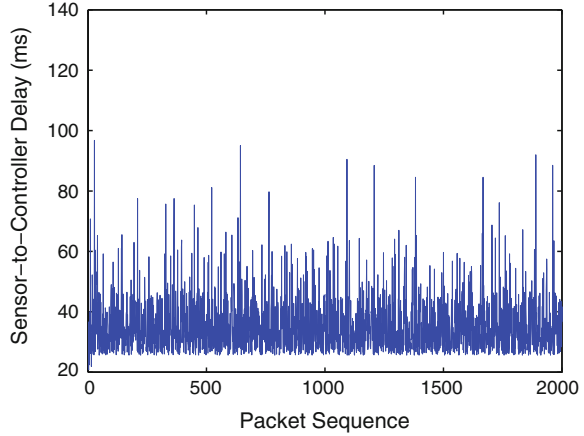
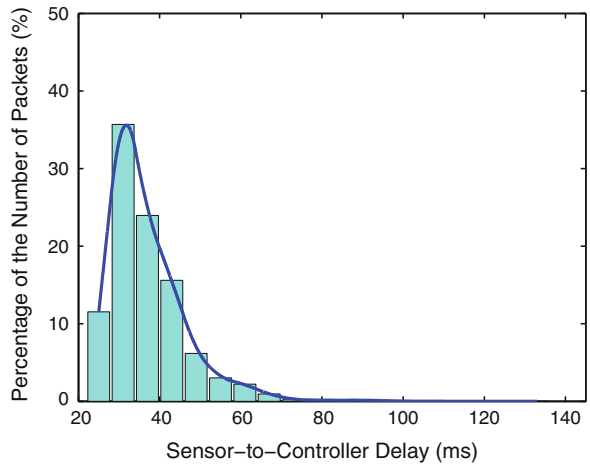


Fig. 2.2 Percentage of the IP-based network delays shown in Fig. 2.1



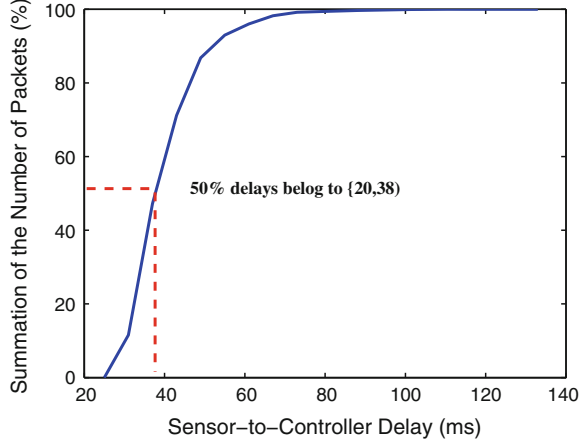
time-varying, nonuniform, and has multifractal nature [82, 84, 85]. Furthermore, accumulative percentage of IP-based network delays is shown in Fig. 2.3.

Compared with the upper bound of delay τ_3 , the middle delay τ_2 reflects more information on the communication network, and $\tau_3 - \tau_2$ is far larger than $\tau_2 - \tau_1$, where τ_1 is lower bound of delay in Fig. 2.3, which also show the nonuniform distribution of the network-induced delay [85]. In the analysis and synthesis of NCSs, if the information of statistical distribution of IP-based network delays is fully utilized, less conservative results are expected to be achieved [86].

For well use of aforementioned nonuniform time-varying communication delays in the analysis and synthesis in NCSs, in the following, a delay distribution-dependent modeling approach is proposed to handle this scenario.

Assume that the network-induced delay $\tau(t)$ is approximately continuous with lower and upper bounds and can be formulated as

Fig. 2.3 Accumulative percentage of the network-induced delays shown in Fig. 2.2



$$\tau_1 \leq \tau(t) \leq \tau_3 < \infty, \quad \forall t \geq 0 \quad (2.1)$$

From the above discussion, it is known that small delays are dominant while large delays are exiguous. Therefore, the probability of small communication delay $\tau(t)$ is bigger than that of large delays. From the control application point of view, this phenomenon can be described using the following conditions:

$$\begin{cases} (1) \tau_1 \leq \tau(t) \leq \tau_3 < \infty, \quad \forall t \geq 0 \\ (2) \text{Prob}[\tau(t) \in [\tau_1, \tau_2]] = \bar{\delta}, \text{Prob}[\tau(t) \in [\tau_2, \tau_3]] = 1 - \bar{\delta} \end{cases} \quad (2.2)$$

where $\bar{\delta} \in [0, 1]$, τ_2 is the bound which means the probability of $\tau(t) \in [\tau_1, \tau_2]$ equals to $\bar{\delta}$.

Define a stochastic variable $\delta(t)$ as

$$\delta(t) = \begin{cases} 1, & t \in \Omega_1 \\ 0, & t \in \Omega_2 \end{cases} \quad (2.3)$$

where $\Omega_1 = \{t : \tau(t) \in [\tau_1, \tau_2]\}$, $\Omega_2 = \{t : \tau(t) \in [\tau_2, \tau_3]\}$. It is clear that $\Omega_1 \cup \Omega_2 = \mathbb{R}^+$ and $\Omega_1 \cap \Omega_2 = \emptyset$. Based on the definitions of Ω_1 and Ω_2 , $t \in \Omega_1$ means the event $\tau(t) \in [\tau_1, \tau_2]$ occurs and $t \in \Omega_2$ means the event $\tau(t) \in [\tau_2, \tau_3]$ occurs.

In this book, the Bernoulli distributed white sequence is used to describe the network quality of service (QoS) related stochastic variable $\delta(t)$.

$$\begin{cases} \text{Prob}\{\delta(t) = 1\} = \mathbb{E}\{\delta(t)\} := \bar{\delta} \\ \text{Prob}\{\delta(t) = 0\} = 1 - \mathbb{E}\{\delta(t)\} := 1 - \bar{\delta} \end{cases} \quad (2.4)$$

The above-described statistical distribution of IP-based network delay is one of the key characteristics of NCS network QoS. The fundamental difficulty from this new description of network QoS is how to improve the QoC under this new framework. In the following section, a class of system controlled over an IP-based network is used to show how to use this nonuniform delay distribution character in NCSs.

2.1.2 A Delay Distribution-Dependent Control Model for NCSs

Consider a class of control systems governed by

$$\dot{x}(t) = g(x(t), u(t), \omega(t)) \quad (2.5)$$

$$x(t) = \rho(t), \quad t \in [-\tau_3, 0] \quad (2.6)$$

where $g(\cdot)$ is a function related with $x(t)$, $u(t)$ and $\omega(t)$; $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are state vector and control input vector, respectively; $\omega(t) \in \mathcal{L}_2[0, \infty)$ denotes the exogenous disturbance signal; $\rho(t)$ is the given function which means the initial condition of the systems on the segment of $t \in [-\tau_3, 0]$.

To make use of the delay distribution information, a delay-dependent feedback control law is designed as:

$$u(t) = \delta(t)Kx(t - \tau_1(t)) + (1 - \delta(t))Kx(t - \tau_2(t)) \quad (2.7)$$

where K is the feedback gain of the network controller and

$$\tau_1(t) = \delta(t)\tau(t), \quad \tau_2(t) = (1 - \delta(t))\tau(t) \quad (2.8)$$

and $\delta(t)$ as described in (2.3).

Substituting (2.7) into (2.5), the following model for the closed-loop control systems can be obtained:

$$\begin{cases} \dot{x}(t) = g(x(t), \delta(t)Kx(t - \tau_1(t)) + (1 - \delta(t))Kx(t - \tau_2(t)), \omega(t)) \\ x(t) = \rho(t), \quad t \in [-\tau_3, 0] \end{cases} \quad (2.9)$$

Notice that two delay regions and white sequence $\delta(t)$ with Bernoulli distribution are considered in (2.9). If more information is available on the network QoS, the results developed in this section can be easily extended to a more general form

$$\dot{x}(t) = g(x(t), \sum_{i=1}^N \delta_i(t)Kx(t - \tau_i(t)), \omega(t)) \quad (2.10)$$

where N is the number of delay classification, $\delta_i(t)$ are stochastic variables which satisfying $\text{Prob}\{\delta_i(t) = 1\} = \mathbb{E}\{\delta_i(t)\} := \bar{\delta}_i$, $\sum_{i=1}^N \bar{\delta}_i = 1$. $\delta_i(t)$ and $\tau_i(t)$ can be obtained through a formula similar to (2.8).

Moreover, when $\bar{\delta} \triangleq 0$ or 1 and lower delay bound τ_1 is not considered, model (2.9) is reduced to the following special case:

$$\dot{x}(t) = g(x(t), Kx(t - \tau(t)), \omega(t)) \quad (2.11)$$

$$x(t) = \rho(t), \quad t \in [-\tau_3, 0] \quad (2.12)$$

This implies that the statistical distribution characteristics of network communication delay $\tau(t)$ is not considered in system analysis and synthesis. This special case has been investigated in [12, 15, 26].

The delay-dependent modeling method mentioned above will be used in Chaps. 3–5 to deal with linear systems, nonlinear systems and large-scale systems, respectively.

2.2 Event-Triggered Transmission Scheme

This section introduces series of ETC schemes applied in this book to reduce the number of transmitted data. Firstly, two discrete ETC schemes are presented with/without considering the data dropouts, respectively. Secondly, an adaptive event-triggered communication scheme is provided to dynamically adjust the event-triggered threshold parameter to reduce more numbers of transmitted packets, while ensuring the desired control performance. Finally, a completed NCS model is provided to link the ETC scheme with the other part of the system to be controlled.

To further development, the following assumptions are needed:

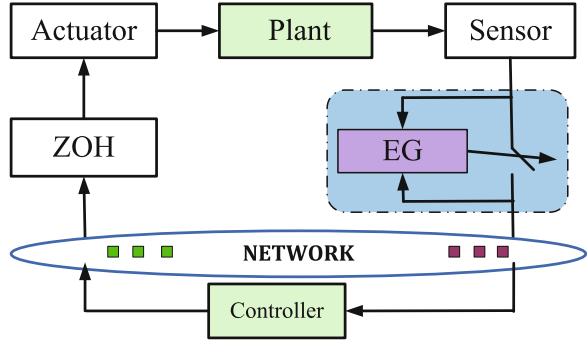
Assumption 1 The sensor is time-triggered with a constant sampling period h . The sampling sequence is described by the set $\mathbb{S}_1 = \{0, h, 2h, \dots, kh\}$, where $k \in \mathbb{N}$.

Assumption 2 Whether the sampled data should be transmitted or not over a communication network is determined by a predetermined communication scheme. The transmission sequence at the sensors is described by the set $\mathbb{S}_2 = \{0, b_1h, b_2h, \dots, b_kh\} \subseteq \mathbb{S}_1$, where $b_k \in \mathbb{N}$. Moreover, part of the data in $\mathbb{S}_2$ may be lost in the communication.

Assumption 3 The controllers and the actuators are event-triggered. The successfully transmitted sampled sequence at the sensors is described by the set $\mathbb{S}_3 = \{0, t_1h, t_2h, \dots, t_kh\} \subseteq \mathbb{S}_2$, where $t_k \in \mathbb{N}$.

Assumption 4 The control input at the actuator is generated by a zero-order holder (ZOH) with the holding time $t \in \Omega \triangleq [t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$, where τ_{t_k} is the communication delay, h is the sampling period, and $t_kh + \tau_{t_k}$ are the instants when the control signal reaches the ZOH.

Fig. 2.4 A conceptual diagram of an event-triggered communication scheme



2.2.1 A Discrete Event-Triggered Communication Scheme

The conceptual diagram of an ETC scheme adopted in this book is shown in Fig. 2.4, where an event generator (EG) is located between the sensor and controller. The EG detects the sampled-data error between the sampled data at the current sampling instant and the sampled data at the latest transmission instant. Whether the sampled data should be transmitted or not is decided by a preselected threshold. The sampled-data transmission should be executed only when the specified threshold is violated [87]. Assume that there is communication delay in the communications but no packet loss is considered, the next transmission instant in the event-triggered transmission scheme is designed as

$$t_{k+1}h = t_kh + \min_{\ell} \{ \ell h \mid e^T(i_kh) \Phi e(i_kh) \geq \delta_1 x^T(t_kh) \Phi x(t_kh) \} \quad (2.13)$$

where $\delta_1 \geq 0$ is a given scalar parameter, Φ is a positive definite weighting matrix to be designed, and $e(i_kh)$ is the error between the two states at the current sampling instant and the latest transmitted sampling instant, i.e.,

$$e(i_kh) = x(i_kh) - x(t_kh) \quad (2.14)$$

where $i_kh = t_kh + \ell h$, $\ell \in \mathbb{N}$; h is the sampling period of the sensor; t_k ($k=1, 2, 3, \dots$) are some integers such that $\{t_1, t_2, t_3, \dots\} \subset \{0, 1, 2, 3, \dots\}$; t_kh is the time at that instant a packet is successfully transmitted from the sensor. The weighting matrix $\Phi > 0$ in (2.13) can also be adjusted to meet the requirement of the event-triggered transmission scheme.

Notice that $e^T(i_kh) \Phi e(i_kh) < \delta_1 x^T(t_kh) \Phi x(t_kh)$ implies that no sampled data is transmitted; $e^T(i_kh) \Phi e(i_kh) \geq \delta_1 x^T(t_kh) \Phi x(t_kh)$ means that a transmission event is generated, and the ZOH uses the latest sampled data as the input of the actuator and holds it until the next transmitted sampled data.

From (2.13), for $t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1})$, the following inequality is satisfied. This will be used in the derivation of the main results in Chaps. 6–8.

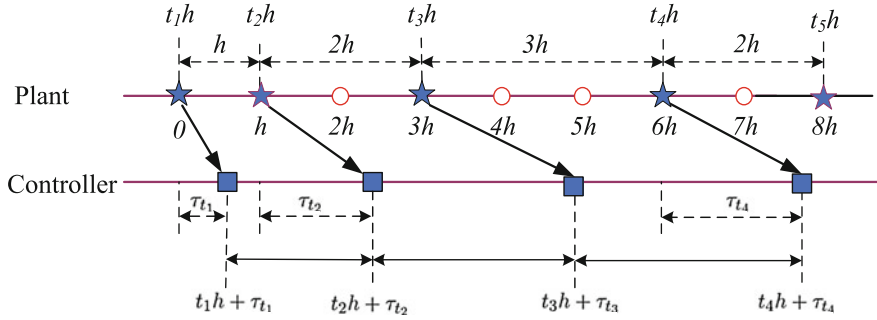


Fig. 2.5 An example of time evolution of the sampling and transmission series

$$e^T(i_k h) \Phi e(i_k h) < \delta_1 x^T(t_k h) \Phi x(t_k h) \quad (2.15)$$

Notice that the communication scheme (2.13) is characterized by the parameters δ_1 , Φ and h , which determine the communication load. How to design these parameters to reduce the amount of data transfer and at the same time to meet the performance requirement is studied in [69]. As a special case, if $\delta_1 \rightarrow 0+$ in (2.13), this leads to $t_{k+1}h = t_k h + h$, which means that all the sampled data should be transmitted, the event-triggered transmission scheme (2.13) is simplified as a time-triggered transmission scheme in [88, 89].

In Fig. 2.5, an example is used to show a typical time evolution with a periodic sampling and an event-triggered transmission scheme. One can see that not all the sampled data is transmitted, only the sampled data satisfying the condition in (2.13) is transmitted. For example, the sampled data at the instants 0, h , $3h$ and $6h$ is transmitted, and the sampled data at other instants is not transmitted.

With the communication scheme (2.13), if there is no packet loss, the transmitter can directly utilize the violation of (2.13) to determine to send packets, while preserving the desired performance of the system under consideration. With packet loss, (2.13) cannot be directly employed to determine if the sampled data should be transmitted. In [69], (2.13) is changed to:

$$b_{k+1}h = b_k h + \min_v \{vh \mid \tilde{e}^T(l_k h) \Phi \tilde{e}(l_k h) > \delta_2 x^T(b_k h) \Phi x(b_k h)\} \quad (2.16)$$

where $\tilde{e}(l_k h) = x(l_k h) - x(b_k h)$, $l_k h = b_k h + vh$, $v \in \mathbb{N}$, $\delta_2 > 0$ is a given scalar parameter, $\Phi > 0$ as defined in (2.13), $b_k h$ is the transmitted sampling instant as defined in Assumption 2.

Notice that the state error and $l_k h$ in (2.16) are different from those in (2.13), and δ_2 in (2.16) should be less than or equal to δ_1 in (2.13) to take into account the extra communication delay caused by the data dropout.

The communication scheme (2.16) is characterized by the parameters δ_2 , Φ and h , which determine the communication load. Also, if $\delta_2 \rightarrow 0+$ in (2.16), this leads

to $t_{k+1}h = t_kh + h$, and this becomes a time-triggered sampling scheme in [88, 89]. How to ensure the allowable number of data dropouts in the communications at the same time to meet the performance requirement is introduced in Chap. 8.

2.2.2 An Adaptive Event-Triggered Communication Scheme

An adaptive event-triggered communication scheme (AETC) is provided in this book to reduce the number of the transmitted data, while preserving the desired control performance. Different from some existing communication schemes with a constant threshold [16, 68, 90], the key feature of AETC is that the threshold condition of AETC can be adaptively adjusted based on the latest available state-dependent information.

Suppose that the occurrence of a transmission depends on a predesigned condition rather than the passing of time, which determines when the next transmission should be taken place. Figure 2.6 depicts a general diagram of the proposed AETC for an NCS, where the adaptive EG has the logic function to determine whether the sampled data should be transmitted to the controller or not; when the sampled data is transmitted over the communication networks, the threshold of ETC scheme is calculated and stored at Storer 1, and the sampled data is also stored at Storer 2 for the next calculation of the state-dependent error. Compared with some existing ETC schemes [16, 67, 68], the main advantage of the proposed AETC is that the threshold condition can be dynamically adjusted to save more limited communication bandwidth, while ensuring the desired control performance. Based the above description, the next transmission instant determined by the above adaptive EG can be expressed as

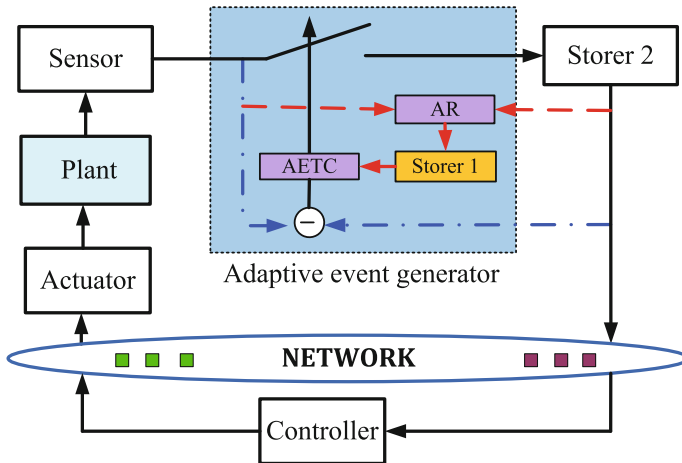


Fig. 2.6 A diagram of an adaptive event-triggered scheme for an NCS. AR adaptive rule; AETC adaptive event-triggered communication scheme

$$t_{k+1}h = t_kh + \min_{\ell \in \mathbb{N}} \{\ell h | e^T(i_kh) \Phi e(i_kh) > \sigma(t_kh) x^T(t_kh) \Phi x(t_kh)\} \quad (2.17)$$

where $\Phi > 0$ is a weighting matrix, and $i_kh = t_kh + \ell h$, $\ell \in \mathbb{N}$, h is the sampling period; t_k ($k = 0, 1, 2, \dots$) are some integers such that $\{t_0, t_1, t_2, \dots\} \subset \{0, 1, 2, \dots\}$; $e(i_kh)$ is the error between the current sampled data $x(i_kh)$ and the latest transmission data $x(t_kh)$, that is

$$e(i_kh) \triangleq x(i_kh) - x(t_kh), i_kh \in (t_k, t_{k+1}] \quad (2.18)$$

Moreover, $\sigma(t_kh)$ in (2.17) is determined by the following adaptive rule

$$\sigma(t_{k+1}h) = \max\{\underbrace{\sigma(t_kh) \left(1 - \frac{2\alpha}{\pi} \text{atan}[\beta(\|x(t_{k+1}h)\| - \|x(t_kh)\|)]\right)}_{\lambda}, \sigma_m\} \quad (2.19)$$

where $\text{atan}(\cdot)$ is the invert tangent function, $0 < \alpha$ and $0 < \beta$ are given constants to adjust the output of $\text{atan}(\cdot)$, σ_m is the given lower bound of $\sigma(t_kh)$, $\sigma(0) = \sigma_m$.

From (2.17) and (2.19), one can see that the transmission events are dependent both on the error $e(i_kh)$ and the latest transmitted states $x(t_kh)$, and on the adjustable threshold $\sigma(t_kh)$.

Different from the ETC scheme (2.13), where the ETC thresholds σ_1 is a preselected constant. One can see that the parameter $\sigma(t_kh)$ in (2.17) can be adaptively adjusted based on (2.19), which is dependent on the current $x(t_{k+1}h)$, the latest $x(t_kh)$, α , β and σ_m simultaneously. Therefore, the proposed AETC (2.17) has the advantage in potentially reducing the number of transmitted packets than those based on (2.19) in [16, 67, 68] while ensuring the desired control performance.

2.2.3 Delay Distribution-Dependent Model of NCSs

Based on the communication schemes mentioned above, this section introduces a communication-dependent delay model of NCSs, which has the advantage in unifying the ETC scheme with the other part of the system to be controlled.

Consider a class of systems governed by

$$\dot{x}(t) = f(x(t), u(t), \omega(t)) \quad (2.20)$$

where $f(\cdot)$ is a function related with $x(t)$, $u(t)$ and $\omega(t)$; $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are state vector and control input vector respectively; $\omega(t) \in \mathcal{L}_2[0, \infty)$ denotes the exogenous disturbance signal; the initial condition of the system (2.20) is given by $x(t_0) = x_0$. Throughout this book, it is assumed that the system (2.20) is controlled over a communication network. For showing the main idea of modeling method, a

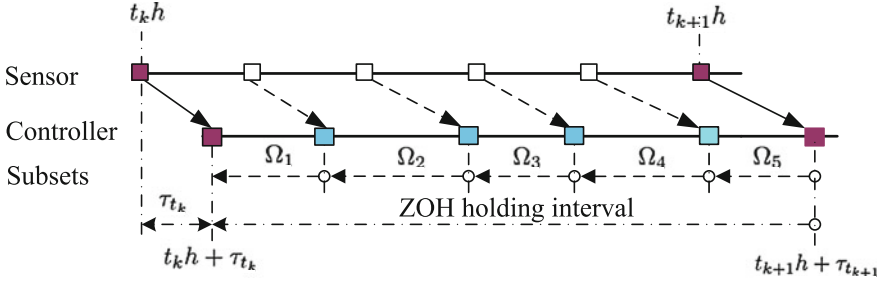


Fig. 2.7 Illustration of subsets of the ZOH. Reprinted from Ref. [69], Copyright 2013, with permission from Elsevier

networked state feedback controller is directly connected to the actuator through a ZOH. This leads to:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t), \omega(t)) \\ u(t) = Kx(t_k h), t \in \Omega \end{cases} \quad (2.21)$$

where K is the state feedback controller gain; τ_{t_k} is the packet transmission delay; h is the sampling period, $\Omega =: [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}})$.

Divide the holding interval of the ZOH $t \in \Omega$ into sampling-interval-like subsets $\Omega_\ell = [i_k h + \tau_{i_k}, i_k h + h + \tau_{i_{k+1}})$ [16], i.e., $\Omega = \cup \Omega_\ell$, where $i_k h = t_k h + \ell h$, $\ell = 0, \dots, t_{k+1} - t_k - 1$ means the sampling instants from the current transmitted sampling instant $t_k h$ to the future transmitted sampling instant $t_{k+1} h$; if ℓ takes the value of $t_{k+1} - t_k - 1$, then $\tau_{i_{k+1}} = \tau_{t_{k+1}}$, otherwise, $\tau_{i_k} = \tau_{t_k}$. See Fig. 2.7 for an illustration. Define $\eta(t) \triangleq t - i_k h$, $t \in \Omega_\ell$. It is clear that $\eta(t)$ is a piecewise linear function satisfying

$$0 < \eta_1 \leq \eta(t) \leq h + \bar{\tau} = \eta_3, t \in \Omega_\ell \quad (2.22)$$

where $\eta_1 = \inf_\ell \{\tau_{i_k}\}$, $\eta_3 = h + \sup_\ell \{\tau_{i_{k+1}}\} = h + \bar{\tau}$; h and $\bar{\tau}$ are the sampling period and the maximum allowable upper communication delay bound, respectively. Then, the control of (2.21) is:

$$u(t) = K(x(t - \eta(t)) - e(i_k h)), t \in \Omega_\ell \quad (2.23)$$

Combining (2.21) and (2.23) leads to a sampled-state-error dependent closed-loop NCS model

$$\dot{x}(t) = f(x(t), K(x(t - \eta(t)) - e(i_k h)), \omega(t)), t \in \Omega_\ell \quad (2.24)$$

where the initial condition of the state $x(t)$ is: $x(t) = \phi(t)$, $t \in [t_0 - \eta_3, t_0]$, $\phi(t_0) = x_0$, and $\phi(t)$ is a continuous function on $[t_0 - \eta_3, t_0]$.

Combining ETC schemes, i.e., (2.13), (2.16), (2.17) and the system model (2.24), communication-dependent analysis and synthesis methods will be used in Chaps. 6–8 to consider the communication and control in a unified framework.

2.3 Main Mathematical Lemmas

In this section, some main lemmas are provided to derive the main results of this book. The proof of these lemmas can be seen in the literature, therefore, they are omitted in this book.

Lemma 2.1 (Schur complement). *For a symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, where S_{11} and $S_{22} \in \mathbb{R}^{n \times n}$. Then the following conditions are equivalent:*

1. $S < 0$;
2. $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;
3. $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 2.2 (Jensen's inequality) [91]. *For any constant matrix $W \in \mathbb{R}^{n \times n}$, $W = W^T \geq 0$, scalar $0 \leq \tau_1 \leq \tau_2$, and vector-valued function $\dot{x}: [-\tau_2, -\tau_1] \rightarrow \mathbb{R}^n$ such that the following integration is well defined, it holds that*

$$-(\tau_2 - \tau_1) \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) W \dot{x}(s) ds \leq - \begin{bmatrix} x(t - \tau_1) \\ x(t - \tau_2) \end{bmatrix}^T \begin{bmatrix} W & * \\ -W & W \end{bmatrix} \begin{bmatrix} x(t - \tau_1) \\ x(t - \tau_2) \end{bmatrix} \quad (2.25)$$

Lemma 2.3 ([92, 93]). *For any constant matrix $R \in \mathbb{R}^{n \times n}$, $R = R^T$, matrix $S \in \mathbb{R}^{n \times n}$, $\begin{bmatrix} R & * \\ S & R \end{bmatrix} \geq 0$, scalars $0 \leq \tau_1 \leq \tau(t) \leq \tau_2$ and vector function $\dot{x}: [-\tau_2, -\tau_1] \rightarrow \mathbb{R}^n$ such that the following integration is well defined, it holds that*

$$-(\tau_2 - \tau_1) \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) R \dot{x}(s) ds \leq - \sum_{i=1}^4 \mathcal{E}_i \quad (2.26)$$

where

$$\begin{aligned} \mathcal{E}_1 &= [x^T(t - \tau_1) - x^T(t - \tau(t))] R [x(t - \tau_1) - x(t - \tau(t))] \\ \mathcal{E}_2 &= [x^T(t - \tau(t)) - x^T(t - \tau_2)] R [x(t - \tau(t)) - x(t - \tau_2)] \\ \mathcal{E}_3 &= [x^T(t - \tau_1) - x^T(t - \tau(t))] S^T [x(t - \tau(t)) - x(t - \tau_2)] \\ \mathcal{E}_4 &= [x^T(t - \tau(t)) - x^T(t - \tau_2)] S [x(t - \tau_1) - x(t - \tau(t))] \end{aligned}$$

Lemma 2.4 ([94]). *Suppose $\tau_m \leq \tau(t) \leq \tau_M$. For any constant matrices $\mathcal{E}_1, \mathcal{E}_2$ and Ω , then*

$$(\tau(t) - \tau_m)\mathcal{E}_1 + (\tau_M - \tau(t))\mathcal{E}_2 + \Omega < 0 \quad (2.27)$$

holds, if and only if

$$(\tau_M - \tau_m)\mathcal{E}_1 + \Omega < 0 \quad (2.28)$$

$$(\tau_M - \tau_m)\mathcal{E}_2 + \Omega < 0 \quad (2.29)$$

hold.

Lemma 2.5 ([85]). For any constant matrix $Q \in \mathbb{R}^{n \times n}$, $Q > 0$, scalars $\tau_1 \leq \tau(t) \leq \tau_3$, and vector function $\dot{x} : [-\tau_3, -\tau_1] \rightarrow \mathbb{R}^n$ such that the following integration is well defined, then it holds that

$$-(\tau_3 - \tau_1) \int_{t-\tau_3}^{t-\tau_1} \dot{x}^T(v) Q \dot{x}(v) dv \leq \zeta_1^T(t) \begin{bmatrix} -Q & Q & 0 \\ * & -2Q & Q \\ * & * & -Q \end{bmatrix} \zeta_1(t) \quad (2.30)$$

where $\zeta_1^T(t) = [x^T(t - \tau_1) \ x^T(t - \tau(t)) \ x^T(t - \tau_3)]$.

Lemma 2.6 ([95]). Let A , L , E , and F be real matrices of appropriate dimensions, with F satisfying $\|F\| \leq 1$. Then we have the following inequalities:

1. For any scalar $\varepsilon > 0$,

$$LFE + E^T F^T L^T \leq \varepsilon^{-1} LL^T + \varepsilon E^T E$$

2. For any matrix $P > 0$ and any scalar $\varepsilon > 0$ such that $P - \varepsilon LL^T > 0$,

$$(A + LFE)^T P^{-1} (A + LFE) \leq A^T (P - \varepsilon LL^T)^{-1} A + \varepsilon^{-1} E^T E$$

Lemma 2.7 ([93]). For any constant matrices $R \in \mathbb{R}^{n \times n}$ and $U \in \mathbb{R}^{n \times n}$, $\mathbb{U} = \begin{bmatrix} R & U^T \\ U & R \end{bmatrix} \geq 0$, scalars $0 \leq \eta_1 \leq \eta(t) \leq \eta_2$, and vector function $\eta : [-\eta_2, -\eta_1] \rightarrow \mathbb{R}^n$, such that the following sum is well defined, then

$$\sum_{j=k-\eta_2}^{k-\eta_1-1} \eta^T(j) R \eta(j) \geq (\eta_2 - \eta_1) \begin{bmatrix} x_2 - x_3 \\ x_3 - x_4 \end{bmatrix}^T \mathbb{U} \begin{bmatrix} x_2 - x_3 \\ x_3 - x_4 \end{bmatrix} \quad (2.31)$$

where $x_2 = \tilde{x}(k - \eta_1)$, $x_3 = \tilde{x}(k - \eta(k))$, $x_4 = \tilde{x}(k - \eta_2)$, $\eta(k) = \tilde{x}(k + 1) - \tilde{x}(k)$.

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