Preface

The aim of this book is mainly to give an account of recent advancements in the study of multiplication operators on the Bergman space with a new point of view. The main focus will be on commutants and reducing subspaces of multiplication operators on function spaces along with relevant von Neumann algebras. Relevant techniques include complex analysis, complex geometry, operator theory, and group theory, which altogether yield a fascinating interplay and reveal some natural connections between them. The results and methods involved can be applied to most of the other analytic function spaces.

The last four decades have seen dramatic progress in the afore-mentioned topic on commutants and reducing subspaces of multiplication operators, including two main phases of development.

The first phase is mainly concerned with the topic of commutants and reducing subspaces of analytic multiplication operators on the Hardy space of the unit disk in the seventies of the last century. Several remarkable advances in this period were achieved mainly by Abrahamse and Douglas [AD], Cowen [Cow1, Cow2, Cow3], Baker et al. [BDU], Deddens and Wong [DW], Nordgren [Nor] and Thomson [T1, T2, T3, T4], etc.

A natural theme is to consider the case of the Bergman space. As well known, the approaches of the corresponding problems on the Bergman space depend heavily on metric structure of the space and relevant function-theoretic characters, and hence the case of the Bergman space diverges considerably from that of the Hardy space.

In the case of the Bergman space, the relevant topic began with Zhu's conjecture on numbers of minimal reducing subspaces of finite Blaschke product multiplication operators [Zhu1] in 2000. This research is presently experiencing a period of intense development. Most notably, during the past dozen years a lot of remarkable achievements had been made in this direction [DSZ, DPW, GH1, GH2, GH4, GSZZ, SZZ1, SZZ2], etc.

The topic of the Bergman space has been the focus of considerable attention from the authors in the past years. Briefly, the goal of this book is to give an account of the latest developments on commutants and reducing subspaces of multiplication operators on both the Hardy space and the Bergman space, and von Neumann

algebras generated by multiplication operators on the Bergman space. It is shown that types of such von Neumann algebras turn out to be closely related to the geometric property of the symbols of the corresponding multiplication operators.

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