Preface

Numerical integration of differential equations, as an essential tool for investigating the qualitative behaviour of the physical universe, is a very active research area since large-scale science and engineering problems are often modelled by systems of ordinary and partial differential equations, whose analytical solutions are usually unknown even when they exist. Structure preservation in numerical differential equations, known also as geometric numerical integration, has emerged in the last three decades as a central topic in numerical mathematics. It has been realized that an integrator should be designed to preserve as much as possible the (physical/geometric) intrinsic properties of the underlying problem. The design and analysis of numerical methods for oscillatory systems is an important problem that has received a great deal of attention in the last few years. We seek to explore new efficient classes of methods for such problems, that is high accuracy at low cost. The recent growth in the need of geometric numerical integrators has resulted in the development of numerical methods that can systematically incorporate the structure of the original problem into the numerical scheme. The objective of this sequel to our previous monograph, which was entitled "Structure-Preserving Algorithms for Oscillatory Differential Equations", is to study further structure-preserving integrators for multi-frequency oscillatory systems that arise in a wide range of fields such as astronomy, molecular dynamics, classical and quantum mechanics, electrical engineering, electromagnetism and acoustics. In practical applications, such problems can often be modelled by initial value problems of second-order differential equations with a linear term characterizing the oscillatory structure. As a matter of fact, this extended volume is a continuation of the previous volume of our monograph and presents the latest research advances in structure-preserving algorithms for multi-frequency oscillatory second-order differential equations. Most of the materials of this new volume are drawn from very recent published research work in professional journals by the research group of the authors.

Chapter 1 analyses in detail the matrix-variation-of-constants formula which gives significant insight into the structure of the solution to the multi-frequency and multidimensional oscillatory problem. It is known that the Störmer–Verlet formula is a very popular numerical method for solving differential equations. Chapter 2 presents novel improved multi-frequency and multidimensional Störmer–Verlet formulae. These methods are applied to solve four significant problems. For structure-preserving integrators in differential equations, another related area of increasing importance is the computation of highly oscillatory problems. Therefore, Chap. 3 explores improved Filon-type asymptotic methods for highly oscillatory differential equations. In recent years, various energy-preserving methods have been developed, such as the discrete gradient method and the average vector field (AVF) method. In Chap. 4, we consider efficient energy-preserving integrators based on the AVF method for multi-frequency oscillatory Hamiltonian systems. An extended discrete gradient formula for multi-frequency oscillatory Hamiltonian systems is introduced in Chap. 5. It is known that collocation methods for ordinary differential equations have a long history. Thus, in Chap. 6, we pay attention to trigonometric Fourier collocation methods with arbitrary degrees of accuracy in preserving some invariants for multi-frequency oscillatory second-order ordinary differential equations. Chapter 7 analyses the error bounds for explicit ERKN integrators for systems of multi-frequency oscillatory second-order differential equations. Chapter 8 contains an analysis of the error bounds for two-step extended Runge–Kutta–Nyström-type (TSERKN) methods. Symplecticity is an important characteristic property of Hamiltonian systems and it is worthwhile to investigate higher order symplectic methods. Therefore, in Chap. 9, we discuss high-accuracy explicit symplectic ERKN integrators. Chapter 10 is concerned with multi-frequency adapted Runge–Kutta–Nyström (ARKN) integrators for general multi-frequency and multidimensional oscillatory second-order initial value problems. Butcher's theory of trees is widely used in the study of Runge–Kutta and Runge–Kutta–Nyström methods. Chapter 11 develops a simplified tricoloured tree theory for the order conditions for ERKN integrators and the results presented in this chapter are an important step towards an efficient theory of this class of schemes. Structure-preserving algorithms for multi-symplectic Hamiltonian PDEs are of great importance in numerical simulations. Chapter 12 focuses on general approach to deriving local energy-preserving integrators for multi-symplectic Hamiltonian PDEs.

The presentation of this volume is characterized by mathematical analysis, providing insight into questions of practical calculation, and illuminating numerical simulations. All the integrators presented in this monograph have been tested and verified on multi-frequency oscillatory problems from a variety of applications to observe the applicability of numerical simulations. They seem to be more efficient than the existing high-quality codes in the scientific literature.

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