

# Preface

In 1998, a special issue of the journal *Mathematical Methods of Operations Research* was published, edited by Guang-Ya Chen and Johannes Jahn. It was devoted solely to optimization problems with set-valued objective functions.

Since then, major breakthroughs have been made including new “set relations,” new solution concepts for set optimization problems and a new framework for a set-valued convex analysis.

The area has been pushed further by the discovery of its relevance for financial mathematics: risk evaluation in markets with “frictions” such as transaction costs or illiquidity effects is best done using set-valued functions. It turns out that results such as the superhedging theorem of Y. Kabanov from 1999 are essentially set-valued duality results, and the dual variables in this superhedging theorem are precisely what the recent theory expects them to be.

Finally, development of algorithms was initiated that can deal with the sometimes scaring complexity of a set-valued objective function and can deliver results which are useful in applications. As a side effect, the theory of vector optimization is not what it used to be: set-valued approaches produced new insides, extensions and in many cases provide methods for repairing unsatisfactory “vector results.” Examples of the latter include duality for linear vector optimization problems and Benson’s now famous algorithm. The latter method was designed for (linear) vector optimization problems, but appropriate extensions allow the computation of infima and even solutions of set optimization problems.

All of this gave rise to a need to summarize the development. This is what motivated the compilation of this volume. The reader may find both surveys with extended bibliographies and original research articles, which provide evidence for the claims above, as well as open questions. The area of “set optimization” is under rapid development, and it is the opinion of the editors that it is becoming a field in its own right: new tools for example from lattice theory (residuation) and new algebraic structures (conlinear spaces of sets) enter the picture. These even shed new light on scalar optimization theory (the objective function is extended real-valued).

Looking at a bigger picture, there are two common denominators in many of the relevant developments in optimization theory. The first is the departure from linear structures in particular on the “image” side. Conlinear spaces of sets are not linear since there is no inverse addition, a feature that is already shared by the extended reals. Modules over  $L^0$  turn out to be fundamental for capturing features of conditional risk measurement in a dynamic framework. We are, therefore, happy to include a contribution from this new field. The second is the utilization of order-complete lattices which leads to a comeback of the notions “infimum” and “supremum”—in particular in vector optimization where the infimum with respect to a vector order is not very useful or does not even exist. This “complete lattice approach” to set optimization complements the “set relation approach” initiated by D. Kuroiwa in the 1990s.

The editors joined this development at an early stage: two of us (Hamel, Löhne) started working on “set relations” in 2001, and were soon followed by the others. A workshop at Humboldt University Berlin, organized by A. Hamel and R. Henrion in 2003, witnessed the first talk about set-valued risk measures from a set optimization perspective, and two theses were completed in 2005 (Hamel’s habilitation, Löhne’s Ph.D.) at Martin Luther University Halle-Wittenberg which paved the way for the “lattice approach” to set optimization.

A regular conference is now devoted to set optimization and finance, see [www.set-optimization.org](http://www.set-optimization.org). The first edition took place in Lutherstadt-Wittenberg, Germany, 2012 the second one in Brunico-Bruneck, Italy, 2014. The third one is planned for 2016.

We thank all contributors of this volume for their effort and their patience. We thank Springer for publishing this volume. We thank all referees who decisively contributed to the scientific quality of the articles. Last but not least, we thank Prof. J. Jahn because he not only contributed to the editorial work of this volume, but already in 2003<sup>1</sup> shared our vision of a new area in optimization emerging, and also gave us the opportunity to publish and to present our results whenever possible and appropriate.

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<sup>1</sup>J. Jahn, “Grundlagen der Mengenoptimierung” (in German). Multi-Criteria-und Fuzzy-Systeme in Theorie und Praxis. Deutscher Universitätsverlag, 2003, 37–71.

Set Optimization and Applications - The State of the Art  
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