

Chapter 2

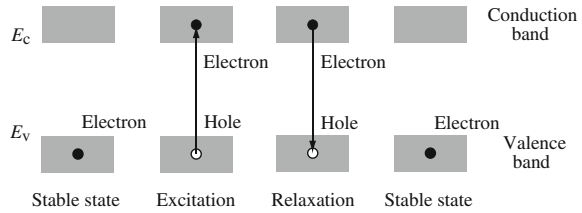
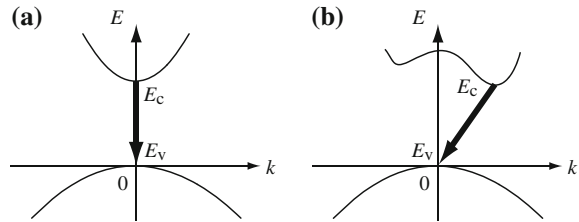
Optical Transitions

2.1 Introduction

Among energy states, the state with the lowest energy is most stable. Therefore, the electrons in semiconductors tend to stay in low energy states. If they are excited by thermal energy, light, or electron beams, the electrons absorb these energies and transit to high energy states. These *transitions* of the electrons from low energy states to high energy states are called *excitations*. High energy states, however, are unstable. As a result, to take stable states, the electrons in high energy states *transit* to low energy states in certain *lifetimes*. These transitions of the excited electrons from high energy states to low energy states are referred to as *relaxations*. The excitation and relaxation processes between the valence band and the conduction band are shown in Fig. 2.1.

In semiconductors, the transitions of electrons from high energy states to low energy states are designated *recombinations* of the electrons and the holes. In the recombinations of the electrons and the holes, there are *radiative recombinations* and *nonradiative recombinations*. The radiative recombinations emit *photons*, and the energies of the photons correspond to a difference in the energies between the initial and final energy states related to the transitions. In contrast, in the nonradiative recombinations, the *phonons* are emitted to crystal lattices or the electrons are trapped in the defects, and the transition energy is transformed into forms other than light. The *Auger processes* are also categorized as nonradiative recombinations. To obtain high efficiency semiconductor light emitting devices, we have to minimize the nonradiative recombinations. However, to enhance modulation characteristics, the nonradiative recombination centers may be intentionally induced in the active layers, because they reduce the carrier lifetimes (see Sect. 5.1).

Let us consider the transitions of the electrons from the bottom of the conduction band to the top of the valence band. A semiconductor, in which the bottom of the conduction band and the top of the valence band are placed at a common wave vector \mathbf{k} , is the *direct transition* semiconductor. A semiconductor, in which the bottom of the conduction band and the top of the valence band have different \mathbf{k} -values,

Fig. 2.1 Excitation and relaxation**Fig. 2.2** **a** Direct and **b** indirect transition semiconductors

is the *indirect transition* semiconductor. These direct and indirect transitions are schematically shown in Fig. 2.2. In transitions of the electrons, the energy and the momentum are conserved, respectively. Therefore, the phonons do not take part in direct transitions. Because the wave vector k of the phonons is much larger than that of the photons, the phonon transitions accompany the indirect transitions to satisfy the momentum conservation law. Hence, in the direct transitions, the transition probabilities are determined by only the electron transition probabilities. In contrast, in the indirect transitions, the transition probabilities are given by a product of the electron transition probabilities and the phonon transition probabilities. As a result, the transition probabilities of the direct transitions are much higher than those of the indirect transitions. Consequently, the direct transition semiconductors are superior to the indirect ones for light emitting devices.

2.2 Light Emitting Processes

Light emission due to the radiative recombinations is called the *luminescence*. According to the lifetime, the excitation methods, and the energy states related to the transitions, light emitting processes are classified as follows.

2.2.1 Lifetime

With regard to the lifetime, there are two light emissions: *fluorescence*, with a short lifetime of 10^{-9} – 10^{-3} s, and *phosphorescence*, with a long lifetime of 10^{-3} s to one day.

2.2.2 Excitation

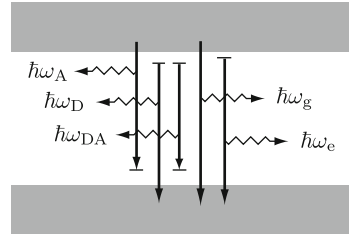
Luminescence due to optical excitation (pumping) is *photoluminescence*, which is widely used to characterize materials. Optical excitation is also used to pump dye lasers (for example, Rhodamine 6G and Coumalin) and solid-state lasers (e.g., YAG and ruby). When the photon energy of the pumping light is $\hbar\omega_1$ and that of the luminescence is $\hbar\omega_2$, the luminescence with $\hbar\omega_2 < \hbar\omega_1$ is called *Stokes luminescence* and that with $\hbar\omega_2 > \hbar\omega_1$ is designated *anti-Stokes luminescence*. Luminescence caused by electrical excitation is *electroluminescence*, which has been used for panel displays. In particular, luminescence by current injection is called *injection-type electroluminescence*; it has been used for light emitting diodes (LEDs) and semiconductor lasers or laser diodes (LDs). In such injection-type optical devices, the carriers are injected into the active layers by forward bias across the pn junctions. Note that the current (carrier) injection is also considered the excitation, because it generates a lot of high energy electrons. The luminescence due to electron beam irradiation is *cathodoluminescence*, which has been adopted to characterize materials. The luminescence induced by mechanical excitation using stress is *triboluminescence*, and that by thermal excitation is *thermoluminescence*. Luminescence during a chemical reaction is referred to chemiluminescence; it has not been reported in semiconductors.

2.2.3 Transition States

Figure 2.3 shows light emission processes between various energy states. They are classified into impurity recombinations, interband recombinations, and exciton recombinations.

In *impurity recombinations*, there is recombination between the electron in the conduction band and the empty acceptor level with the photon energy of $\hbar\omega_A$, recombination between the electron in the donor level and the hole in the valence band with the photon energy of $\hbar\omega_D$, and recombination between the electron in the donor level and the empty acceptor level with the photon energy of $\hbar\omega_{DA}$. By observing light emissions due to these recombinations at extremely low temperatures, information on doping characteristics is obtained.

Fig. 2.3 Light emission processes



In *interband recombinations* between the conduction and valence bands, light emission in the vicinity of the band edges with the photon energy of $\hbar\omega_g$ is dominant. This *band edge emission* is used in most LEDs and LDs composed of III–V group semiconductors.

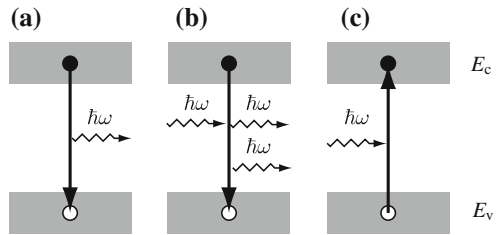
The *exciton recombination*, with the photon energy of $\hbar\omega_e$, is the recombination of the electron generated by decay of an exciton and the hole in the valence band.

2.3 Spontaneous Emission, Stimulated Emission, and Absorption

Figure 2.4 schematically shows *radiations* and *absorption*. In the radiations, there are *spontaneous emission* and *stimulated emission* (or *induced emission*). As shown in Fig. 2.4a, spontaneous emission is a radiative process in which an excited electron decays in a certain lifetime and a photon is emitted. Note that spontaneous emission takes place irrespective of incident lights. In contrast, in the stimulated emission illustrated in Fig. 2.4b, an incident light induces a radiative transition of an excited electron. The emitted light due to the stimulated emission has the same *wavelength*, *phase*, and *direction* as the incident light. Therefore, the light generated by the stimulated emission is highly *monochromatic*, *coherent*, and *directional*. In the stimulated emission, one incident photon generates two photons; one is the incident photon itself, and the other is an emitted photon due to the stimulated emission. As a result, the incident light is amplified by the stimulated emission. The absorption depicted in Fig. 2.4c is a process that the electron transits from a lower energy state to a higher one by absorbing energy from the incident light. Because this transition is induced by the incident light, it is sometimes called *induced absorption*. It should be noted that spontaneous absorption does not exist; this will be explained in Chap. 8.

When a light is incident on a material, the stimulated emission and the absorption simultaneously take place. In *thermal equilibrium*, there are more electrons in a lower energy state than in a higher one, because the lower energy state is more stable than the higher one. Therefore, in thermal equilibrium, only the absorption is observed when a light is incident on a material. In order to obtain a net optical gain, we have to make the number of electrons in a higher energy state larger than in the lower one. This condition is referred to as *inverted population*, or, *population inversion* because the electron population is inverted compared with that in thermal equilibrium. In

Fig. 2.4 Radiation and absorption: **a** spontaneous emission, **b** stimulated emission, and **c** absorption



semiconductors, the population inversion is obtained only in the vicinity of the band edges by excitation of the electrons through optical pumping or electric current injection. The population inversion generates many electrons at the bottom of the conduction band and many holes at the top of the valence bands.

The laser oscillators use fractions of the spontaneous emission as the optical input and amplify the fractions by the stimulated emission under population inversion. Once the optical gains exceed the optical losses in the laser oscillators, *laser oscillations* take place. The term *laser* is an acronym for “light amplification by stimulated emission of radiation” and is used as a noun in a *laser oscillator*. Note that an emitted light from a laser oscillator is not a laser but a *laser light* or a *laser beam*. As a back formation from laser, “lase” is used as a verb meaning “to emit coherent light.”

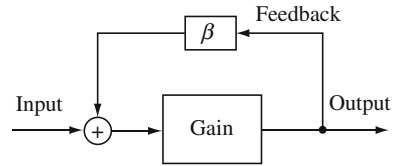
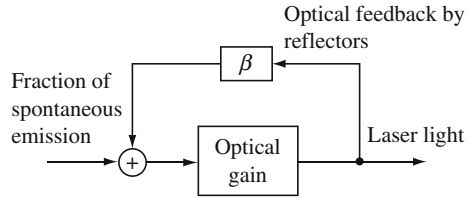
Among semiconductor light emitting devices, LEDs use spontaneous emission and are applied to remote-control transmitters, switch lights, brake lights, displays, and traffic signals. In contrast, LDs are oscillators of lights using stimulated emission and are used as light sources for lightwave communications, compact discs (CDs), magneto-optical discs (MOs), digital video discs (DVDs), laser beam printers, laser pointers, bar-code readers, and so on.

2.4 Optical Gains

2.4.1 Lasers

Let us consider the relationships between *laser oscillators* and *optical gains*. First, we review a general oscillator, as shown in Fig. 2.5. An oscillator has a *gain* and amplifies an input. Also, the oscillator returns a fraction of an output to an input port through a *feedback* loop. The returned one is repetitively amplified, and oscillation starts when a net gain exceeds an internal loss of the oscillator.

Figure 2.6 shows a laser oscillator where a fraction of the spontaneous emission is used as the input and the *optical gain* is produced by the stimulated emission. To feed back light, *optical resonators* or *optical cavities*, which are composed of *reflectors* or *mirrors*, are adopted. Due to this configuration, characteristics of lasers (for brevity,

Fig. 2.5 Oscillator**Fig. 2.6** Laser oscillator

lasers are used for laser oscillators here) are affected by the optical gains and optical resonators. Many lasers use optical resonators, but nitrogen lasers, whose optical gains are very large, can start laser oscillations even without optical resonators. It should be noted that all semiconductor lasers use optical resonators.

As explained earlier, a fraction of the spontaneous emission is used as the input of a laser. Note that all the spontaneously emitted lights cannot be used as the input, because they have different wavelengths, phases, and propagation directions. Among these lights, only the lights, which have wavelengths within the optical gain spectrum and satisfy the resonance conditions of the optical resonators, can be the sources of laser oscillations. Other spontaneous emissions, which do not satisfy the resonance conditions of optical resonators, are readily emitted outward without obtaining sufficient optical gains for laser oscillations. The lights amplified by the stimulated emission have the same wavelength, phase, and propagation direction as those of the input lights. Therefore, the laser lights are highly monochromatic, bright, coherent, and directional.

2.4.2 Optical Gains

2.4.2.1 Carrier Distribution

We derive equations for the optical gains of semiconductor lasers. We suppose that many electrons are excited to the conduction band by optical pumping or electrical current injection. In this condition, the *carrier distribution* is in *nonthermal equilibrium*, and there are many electrons in the conduction band and many holes in the valence band. As a result, one *Fermi level* E_F cannot describe the distribution functions of the electrons and holes. In this case, it is useful to determine the distribution functions by assuming that the electrons in the conduction band and the holes in the

valence band are separately governed by *Fermi–Dirac distribution*. For this purpose, we introduce *quasi-Fermi levels* E_{Fc} and E_{Fv} defined as

$$\begin{aligned} n &= N_c \exp\left(-\frac{E_c - E_{Fc}}{k_B T}\right), \\ p &= N_v \exp\left(-\frac{E_{Fv} - E_v}{k_B T}\right), \\ N_c &= 2 \left(\frac{2 \pi m_e^* k_B T}{h^2}\right)^{3/2}, \\ N_v &= 2 \left(\frac{2 \pi m_h^* k_B T}{h^2}\right)^{3/2}. \end{aligned} \quad (2.1)$$

Here, n and p are the electron concentration and the hole concentration, respectively; E_c and E_v are the energy of the bottom of the conduction band and that of the top of the valence band, respectively; $k_B = 1.3807 \times 10^{-23}$ J/K is *Boltzmann constant*; T is an absolute temperature; N_c and N_v are the *effective density of states* for the electrons and that for the holes, respectively; m_e^* and m_h^* are the effective mass of the electrons and that of the holes, respectively; and h is Planck's constant.

From (2.1), these quasi-Fermi levels E_{Fc} and E_{Fv} are written as

$$\begin{aligned} E_{Fc} &= E_c + k_B T \ln\left(\frac{n}{N_c}\right), \\ E_{Fv} &= E_v - k_B T \ln\left(\frac{p}{N_v}\right). \end{aligned} \quad (2.2)$$

We express a distribution function for the electrons in the valence band with the energy E_1 as f_1 and that for the electrons in the conduction band with the energy E_2 as f_2 . Using E_{Fc} and E_{Fv} , we can express f_1 and f_2 as

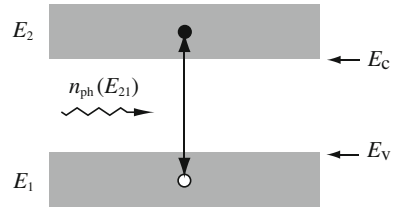
$$\begin{aligned} f_1 &= \frac{1}{\exp[(E_1 - E_{Fv}) / (k_B T)] + 1}, \\ f_2 &= \frac{1}{\exp[(E_2 - E_{Fc}) / (k_B T)] + 1}. \end{aligned} \quad (2.3)$$

It should be noted that the distribution function for the holes in the valence band is given by $[1 - f_1]$.

2.4.2.2 Optical Transition Rates

As shown in Fig. 2.7, we assume that a light, which has a photon energy of $E_{21}=E_2-E_1$ and a photon density of $n_{ph}(E_{21})$, interacts with a direct-transition

Fig. 2.7 Schematic model for radiation and absorption



semiconductor. Under this assumption, we calculate the *transition rates* for the stimulated emission, the absorption, and the spontaneous emission. In Fig. 2.7, E_c is the energy for the bottom of the conduction band, and E_v is the energy for the top of the valence band. Note that this figure shows the energy bands for a certain value of k , and the horizontal line has no physical meaning.

(i) Stimulated Emission Rate

The *stimulated emission rate per unit volume* $r_{21}(\text{stim})$ is given by a product of

- the transition probability per unit time from E_2 to E_1 : B_{21} ,
- the electron concentration in a state with the energy E_2 : n_2 ,
- the hole concentration in a state with the energy E_1 : p_1 ,
- the photon density: $n_{ph}(E_{21})$.

The concentration n_2 of the electron, which occupies a state with the energy E_2 in the conduction band, is expressed as

$$n_2 = \rho_c(E_2 - E_c) f_2, \quad (2.4)$$

where $\rho_c(E_2 - E_c)$ is the density of states, which is a function of $E_2 - E_c$, and f_2 is the distribution function in (2.3).

The concentration p_1 of the hole, which occupies a state with the energy E_1 in the valence band, is written as

$$p_1 = \rho_v(E_v - E_1) [1 - f_1], \quad (2.5)$$

where $\rho_v(E_v - E_1)$ is the density of states, which is a function of $E_v - E_1$, and f_1 is the distribution function in (2.3).

Therefore, the stimulated emission rate $r_{21}(\text{stim})$ is obtained as

$$\begin{aligned} r_{21}(\text{stim}) &= B_{21} n_2 p_1 n_{ph}(E_{21}) \\ &= B_{21} n_{ph}(E_{21}) \rho_c(E_2 - E_c) \rho_v(E_v - E_1) f_2 [1 - f_1]. \end{aligned} \quad (2.6)$$

(ii) Absorption Rate

The *absorption rate per unit volume* $r_{12}(\text{abs})$ is given by a product of

the transition probability per unit time from E_1 to E_2 : B_{12} ,
 the concentration of an empty state with the energy E_2 : p_2 ,
 the electron concentration in a state with the energy E_1 : n_1 ,
 the photon density: $n_{\text{ph}}(E_{21})$.

The concentration p_2 of an empty state, which is not occupied by the electrons with the energy E_2 in the conduction band, is expressed as

$$p_2 = \rho_c(E_2 - E_c)[1 - f_2]. \quad (2.7)$$

The concentration n_1 of the electron, which occupies a state with the energy E_1 in the valence band, is written as

$$n_1 = \rho_v(E_v - E_1)f_1. \quad (2.8)$$

As a result, the absorption rate $r_{12}(\text{abs})$ is obtained as

$$\begin{aligned} r_{12}(\text{abs}) &= B_{12}p_2n_1n_{\text{ph}}(E_{21}) \\ &= B_{12}n_{\text{ph}}(E_{21})\rho_c(E_2 - E_c)\rho_v(E_v - E_1)f_1[1 - f_2]. \end{aligned} \quad (2.9)$$

(iii) Spontaneous Emission Rate

The *spontaneous emission rate per unit volume* $r_{21}(\text{spon})$ is independent of the incident photon density and given by a product of

the transition probability per unit time from E_2 to E_1 : A_{21} ,
 the electron concentration in a state with the energy E_2 : n_2 ,
 the hole concentration in a state with the energy E_1 : p_1 .

Using (2.4) and (2.5), the spontaneous emission rate $r_{21}(\text{spon})$ is obtained as

$$\begin{aligned} r_{21}(\text{spon}) &= A_{21}n_2p_1 \\ &= A_{21}\rho_c(E_2 - E_c)\rho_v(E_v - E_1)f_2[1 - f_1]. \end{aligned} \quad (2.10)$$

2.4.2.3 Optical Transition in Thermal Equilibrium

In thermal equilibrium, the carrier distributions are described by one Fermi level E_F , and the radiations balance the absorption. In other words, a sum of the stimulated

emission rate and the spontaneous emission rate is equal to the absorption rate. Using the optical transition rates for the stimulated emission, the spontaneous emission, and the absorption, we obtain

$$\begin{aligned} r_{21}(\text{stim}) + r_{21}(\text{spon}) &= r_{12}(\text{abs}), \\ E_{\text{Fc}} &= E_{\text{Fv}} = E_{\text{F}}. \end{aligned} \quad (2.11)$$

Substituting (2.6), (2.9), and (2.10) into (2.11), we have

$$n_{\text{ph}}(E_{21}) = \frac{A_{21}}{B_{12} \exp[E_{21}/(k_{\text{B}}T)] - B_{21}}, \quad (2.12)$$

while the *blackbody radiation theory* gives

$$n_{\text{ph}}(E_{21}) = \frac{8 \pi n_{\text{r}}^3 E_{21}^2}{h^3 c^3 \exp[E_{21}/(k_{\text{B}}T)] - h^3 c^3}. \quad (2.13)$$

Here, n_{r} is the effective refractive index of a material, h is Planck's constant, and $c = 2.99792458 \times 10^8$ m/s is the speed of light in a vacuum.

Comparison of (2.12) and (2.13) results in

$$\begin{aligned} A_{21} &= Z(E_{21})B, \quad B_{21} = B_{12} = B, \\ Z(E_{21}) &= \frac{8 \pi n_{\text{r}}^3 E_{21}^2}{h^3 c^3}, \end{aligned} \quad (2.14)$$

which is known as *Einstein's relation*, and A_{21} is called *Einstein's A coefficient*; and B is called *Einstein's B coefficient*. It should be noted that $B_{21} = B_{12} = B$, and A_{21} is proportional to B .

We consider the physical meaning of $Z(E_{21})$ in (2.14) in the following. We suppose that a light field E_{L} is written as

$$E_{\text{L}} = A_0 \exp[i(\omega t - \mathbf{k} \cdot \mathbf{r})], \quad (2.15)$$

where A_0 is a complex amplitude, ω is an angular frequency, t is a time, \mathbf{k} is a wave vector, and \mathbf{r} is a position vector. We also suppose that the periodic boundary condition is satisfied on the surfaces of a cube with a side length L . Because the electric fields at $x = 0$ and $x = L$ are the same, an eigenmode is given by

$$\exp(0) = \exp(i k_x L) = 1, \quad \therefore k_x = \frac{2 \pi}{L} n_x, \quad (2.16)$$

where k_x is an x -component of the wave vector \mathbf{k} and n_x is an integer. Similarly, we obtain

$$k_y = \frac{2 \pi}{L} n_y, \quad k_z = \frac{2 \pi}{L} n_z, \quad (2.17)$$

where n_y and n_z are integers. Hence, $k^2 = k_x^2 + k_y^2 + k_z^2$ is expressed as

$$k^2 = \left(\frac{2\pi}{L} \right)^2 (n_x^2 + n_y^2 + n_z^2). \quad (2.18)$$

Using the effective refractive index n_r of the material, the angular frequency ω is given by

$$\omega = \frac{kc}{n_r}. \quad (2.19)$$

Therefore, the energy $E = \hbar\omega$ is written as

$$E = \hbar\omega = \frac{2\pi\hbar c}{n_r L} (n_x^2 + n_y^2 + n_z^2)^{1/2} = \frac{\hbar c}{n_r L} (n_x^2 + n_y^2 + n_z^2)^{1/2}, \quad (2.20)$$

which takes discrete values, because n_x , n_y , and n_z are integers. However, when L is much larger than the wavelength of the light, $n_x^2 + n_y^2 + n_z^2$ becomes a huge number, and many modes exist near each other. When we consider a sphere with a radius of $R = (n_x^2 + n_y^2 + n_z^2)^{1/2}$, the number of combinations (n_x, n_y, n_z) is equal to a volume of the sphere $4\pi R^3/3$. For each combination (n_x, n_y, n_z) , there exist two modes corresponding to their polarizations, which are perpendicular to each other. As a result, the number of modes whose energy is placed between 0 and E is given by

$$2 \times \frac{4\pi}{3} \left(\frac{n_r E L}{\hbar c} \right)^3 = \frac{8\pi n_r^3 E^3}{3\hbar^3 c^3} L^3. \quad (2.21)$$

The number of modes with their energies between E and $E + dE$ is obtained as a derivative of (2.21) with respect to E , which is expressed as

$$\frac{8\pi n_r^3 E^2}{\hbar^3 c^3} L^3 dE. \quad (2.22)$$

Because a volume of the cube is L^3 , dividing (2.22) by L^3 gives the number of modes per unit volume, that is, the *mode density* $m(E) dE$ as

$$m(E) dE = \frac{8\pi n_r^3 E^2}{\hbar^3 c^3} dE. \quad (2.23)$$

From (2.23) and the third equation in (2.14), it is found that we have $m(E_{21}) = Z(E_{21})$. This relation suggests that the spontaneous emission takes place for all the modes with their energy placed between E and $E + dE$ while the stimulated emission and absorption occur only for the mode corresponding to the incident light.

Note that the derivation of A and B assumes that both $r_{21}(\text{stim})$ and $r_{12}(\text{abs})$ are proportional to the photon density $n_{\text{ph}}(E_{21})$. If we assume that both $r_{21}(\text{stim})$ and $r_{12}(\text{abs})$ are proportional to the energy density of the photons as Einstein did [1], other formulas for A and B will be obtained.

2.4.2.4 Definitions of the Mode Density

In the preceding discussion, we explained the mode density $m(E) dE$ with the energies placed between E and $E + dE$. Here, we will derive mode densities $m(\omega) d\omega$ with the angular frequencies of a light between ω and $\omega + d\omega$, and $m(\nu) d\nu$ with the light frequencies between ν and $\nu + d\nu$. Using the (effective) refractive index of the material n_r and the speed of light in a vacuum c , we have

$$k = \frac{n_r \omega}{c} = \frac{2 \pi n_r \nu}{c}. \quad (2.24)$$

Substituting (2.24) into (2.18) and then calculating a volume $4 \pi R^3/3$ of a sphere with the radius $R = (n_x^2 + n_y^2 + n_z^2)^{1/2}$ results in

$$2 \times \frac{4 \pi}{3} R^3 = \frac{n_r^3 \omega^3}{3 \pi^2 c^3} L^3 = \frac{8 \pi n_r^3 \nu^3}{3 c^3} L^3, \quad (2.25)$$

where a factor 2 corresponds to the directions of the polarizations. As a result, dividing (2.25) by L^3 and then differentiating with respect to ω or ν leads to

$$m(\omega) d\omega = \frac{n_r^3 \omega^2}{\pi^2 c^3} d\omega, \quad m(\nu) d\nu = \frac{8 \pi n_r^3 \nu^2}{c^3} d\nu. \quad (2.26)$$

2.4.2.5 Net Stimulated Emission

Let us consider the excited conditions where many free carriers exist. In these non-thermal equilibrium conditions, radiations do not balance with absorption anymore. When a light is incident on a material, the stimulated emission and the absorption simultaneously take place. Therefore, the net stimulated emission rate $r^0(\text{stim})$ is given by

$$\begin{aligned} r^0(\text{stim}) &= r_{21}(\text{stim}) - r_{12}(\text{abs}) \\ &= B n_{\text{ph}}(E_{21}) \rho_c(E_2 - E_c) \rho_v(E_v - E_1) [f_2 - f_1] \\ &= \frac{A_{21}}{Z(E_{21})} n_{\text{ph}}(E_{21}) \rho_c(E_2 - E_c) \rho_v(E_v - E_1) [f_2 - f_1]. \end{aligned} \quad (2.27)$$

From (2.27), to obtain the *net stimulated emission* $r^0(\text{stim}) > 0$, we need

$$f_2 > f_1, \quad (2.28)$$

which indicates the population inversion in the semiconductors. With the help of (2.3), (2.28) reduces to

$$E_{F_c} - E_{F_v} > E_2 - E_1 = E_{21}, \quad (2.29)$$

which is known as the *Bernard–Duraffourg relation*. In semiconductor lasers, a typical carrier concentration for $r^0(\text{stim}) > 0$ is of the order of 10^{18} cm^{-3} .

2.4.2.6 Net Stimulated Emission Rate and Optical Gains

The optical power gain coefficient per unit length g is defined as

$$\frac{dI}{dz} = gI, \quad (2.30)$$

where I is the light intensity per unit area and z is a coordinate for a propagation direction of the light. Because the light intensity is proportional to the square of the electric field, the amplitude gain coefficient g_E , which is the gain coefficient for the electric field, is given by $g_E = g/2$. From the definition, I is expressed as

$$\begin{aligned} I &= v E_{21} n_{\text{ph}}(E_{21}) = v \hbar \omega n_{\text{ph}}(E_{21}), \\ v &= \frac{d\omega}{dk} = \frac{c}{n_r}, \\ \frac{dI}{dt} &= v E_{21} \frac{dn_{\text{ph}}(E_{21})}{dt} = v E_{21} r^0(\text{stim}), \end{aligned} \quad (2.31)$$

where v is a group velocity of the light in the material, $E_{21} = \hbar \omega$ is the photon energy, and $n_{\text{ph}}(E_{21})$ is the photon density.

Here we consider a relationship between the derivatives of I with respect to a position and a time. Because the position z is a function of the time t , we have

$$\begin{aligned} \frac{dI}{dz} &= \frac{dt}{dz} \frac{dI}{dt} = \left(\frac{dz}{dt} \right)^{-1} \frac{dI}{dt} = \frac{1}{v} \frac{dI}{dt}, \\ \frac{dI}{dz} &= gI = g v E_{21} n_{\text{ph}}(E_{21}). \end{aligned} \quad (2.32)$$

From (2.31) and (2.32), we have the relation

$$r^0(\text{stim}) = v g n_{\text{ph}}(E_{21}). \quad (2.33)$$

As a result, it is found that a large $r^0(\text{stim})$ leads to a large g . Using (2.27), (2.31), and (2.33), we can also express g as

$$g = \frac{r^0(\text{stim})}{n_{\text{ph}}(E_{21})} \frac{n_r}{c} = \frac{n_r}{c} B \rho_c(E_2 - E_c) \rho_v(E_v - E_1) [f_2 - f_1]. \quad (2.34)$$

From the *time-dependent quantum mechanical perturbation theory* (see Appendix C), Einstein's B coefficient in (2.14), which is the transition rate due to the stimulated emission for a photon in a free space, is given by

$$B = \frac{e^2 h}{2m^2 \varepsilon_0 n_r^2 E_{21}} |\langle 1 | \mathbf{p} | 2 \rangle|^2. \quad (2.35)$$

Here, e is the elementary charge; h is Planck's constant; m is the electron mass; ε_0 is permittivity in a vacuum; n_r is the refractive index of the material; E_{21} is the photon energy; \mathbf{p} is the momentum operator; and $|1\rangle$ and $|2\rangle$ are the wave functions of the valence band and the conduction band in a steady state, respectively.

Substituting (2.35) into (2.14), we can write Einstein's A coefficient as

$$A_{21} = \frac{4 \pi e^2 n_r E_{21}}{m^2 \varepsilon_0 h^2 c^3} |\langle 1 | \mathbf{p} | 2 \rangle|^2. \quad (2.36)$$

Using the *dipole moment* μ , (2.35) and (2.36) can be rewritten as follows: The momentum operator \mathbf{p} is expressed as

$$\mathbf{p} = m \frac{d\mathbf{r}}{dt}. \quad (2.37)$$

As a result, we obtain

$$\frac{1}{m} \langle 1 | \mathbf{p} | 2 \rangle = \frac{d}{dt} \langle 1 | \mathbf{r} | 2 \rangle = i \omega \langle 1 | \mathbf{r} | 2 \rangle, \quad (2.38)$$

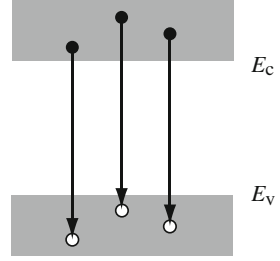
where $\mathbf{r} \propto e^{i\omega t}$ was assumed. With the help of (2.38), Einstein's A and B coefficients are written as

$$\begin{aligned} B &= \frac{\pi e^2 \omega}{\varepsilon_0 n_r^2} |\langle 1 | \mathbf{r} | 2 \rangle|^2 = \frac{\pi \omega}{\varepsilon_0 n_r^2} |\mu|^2, \\ A_{21} &= \frac{2 n_r \omega^3}{\varepsilon_0 h c^3} |\mu|^2, \\ |\mu|^2 &= |\langle 1 | e \mathbf{r} | 2 \rangle|^2. \end{aligned} \quad (2.39)$$

Note that many textbooks on quantum electronics use (2.39) as Einstein's A coefficient. Expressions on Einstein's B coefficient depend on the definitions of the stimulated emission rate whether it is proportional to the photon density or to the energy density. In the preceding explanations, Einstein's B coefficient is defined to be in proportion to the photon density. Therefore, the quantum mechanical transition rate for the stimulated emission is equal to Einstein's B coefficient.

In semiconductors, due to the energy band structures, transitions take place between various energy states, as shown in Fig. 2.8. If we put $E_{21} = E$ and $E_2 - E_c = E''$, the electron energy in the valence band E' for the allowed transition

Fig. 2.8 Transition with a constant photon energy



is given by $E' = E'' - E$. Therefore, by integrating (2.27) with respect to E'' , the net stimulated emission rate $r^0(\text{stim})$ in the semiconductors is obtained as

$$r^0(\text{stim}) = \frac{e^2 h n_{\text{ph}}(E)}{2m^2 \varepsilon_0 n_r^2 E} \int_0^\infty |\langle 1|\mathbf{p}|2\rangle|^2 \rho_c(E'') \rho_v(E') [f_2(E'') - f_1(E')] dE'', \quad (2.40)$$

where (2.35) was used. Similarly, the optical power gain coefficient $g(E)$ is given by

$$g(E) = \frac{e^2 h}{2m^2 \varepsilon_0 n_r c E} \int_0^\infty |\langle 1|\mathbf{p}|2\rangle|^2 \rho_c(E'') \rho_v(E') [f_2(E'') - f_1(E')] dE''. \quad (2.41)$$

From (2.10) and (2.36), the spontaneous emission rate $r_{21}(\text{spon})$ is expressed as

$$r_{21}(\text{spon}) = \frac{4\pi e^2 n_r E}{m^2 \varepsilon_0 h^2 c^3} \int_0^\infty |\langle 1|\mathbf{p}|2\rangle|^2 \rho_c(E'') \rho_v(E') f_2(E'') [1 - f_1(E')] dE''. \quad (2.42)$$

When the excitation is weak, the absorption is observed, and the net absorption rate $r^0(\text{abs})$ is written as

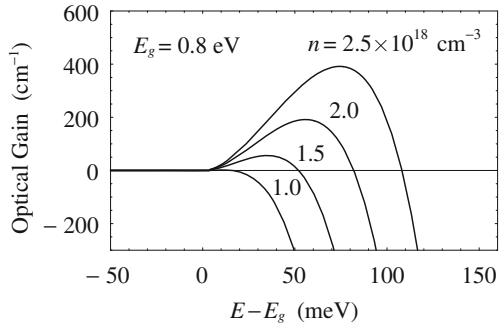
$$r^0(\text{abs}) = r_{12}(\text{abs}) - r_{21}(\text{stim}) = -r^0(\text{stim}). \quad (2.43)$$

Therefore, the optical power absorption coefficient $\alpha(E)$ and the optical power gain coefficient $g(E)$ are related as

$$\alpha(E) = -g(E). \quad (2.44)$$

Figure 2.9 shows the calculated optical power gain coefficient $g(E)$ in (2.41) as a function of $E - E_g$ for the bulk structures with the bandgap energy of $E_g = 0.8 \text{ eV}$. With an increase in the carrier concentration n , the gain peak shifts toward a higher energy (shorter wavelength). This shift of the gain peak is caused by the *band filling effect* [2], in which the electrons in the conduction band and the holes in the valence band occupy each band from the band edges. Because higher energy states are more dense than lower energy states, the optical gain in higher energy states is larger than that in lower energy ones.

Fig. 2.9 Optical power gain coefficient



Here, let us consider the optical power gain coefficient $g(E)$ from other viewpoints. In (2.41), $\rho_c(E'')\rho_v(E')$ is considered to be the density of state pairs related to the optical transitions. The density of state pairs is also expressed using the law of momentum conservation (*k-selection rule*) and the law of energy conservation ($E_{21} = E_2 - E_1$). Under the *k-selection rule*, optical transitions take place only for $\mathbf{k} = \mathbf{k}_1 = \mathbf{k}_2$. Therefore, we can define the density of state pairs or the reduced density of states $\rho_{\text{red}}(E_{21})$ as

$$\rho_{\text{red}}(E_{21}) dE_{21} \equiv \rho_v(E_1) dE_1 = \rho_c(E_2) dE_2, \quad (2.45)$$

where

$$dE_{21} = dE_1 + dE_2. \quad (2.46)$$

From (2.45) and (2.46), we have

$$\rho_{\text{red}}(E_{21}) = \left[\frac{1}{\rho_v(E_1)} + \frac{1}{\rho_c(E_2)} \right]^{-1}. \quad (2.47)$$

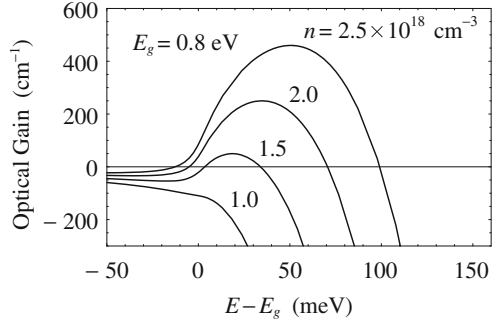
If the conduction and valence bands are parabolic, the energies E_1 and E_2 are written as

$$E_1 = E_v - \frac{\hbar^2 k^2}{2m_h^*}, \quad (2.48)$$

$$E_2 = E_c + \frac{\hbar^2 k^2}{2m_e^*}, \quad (2.49)$$

where m_h^* and m_e^* are the effective masses of the holes and the electrons, respectively. As a result, E_1 and E_2 are rewritten as

Fig. 2.10 Optical power gain coefficient under the \mathbf{k} -selection rule



$$E_1 = E_v - \frac{m_e^*}{m_e^* + m_h^*} (E_{21} - E_g), \quad (2.50)$$

$$E_2 = E_c + \frac{m_h^*}{m_e^* + m_h^*} (E_{21} - E_g). \quad (2.51)$$

Also, once the initial state of the optical transitions is given, the final state is determined by E_{21} and \mathbf{k} . Hence, the number of state pairs per volume associated with the optical transitions for the photon energies between E_{21} and $E_{21} + dE_{21}$ is given by

$$\rho_{\text{red}}(E_{21}) [f_2(E_{21}) - f_1(E_{21})] dE_{21}. \quad (2.52)$$

We assume that the energy states related to optical transitions have an energy width of \hbar/τ_{in} , and the spectral shape of Lorentzian such as

$$L(E_{21}) = \frac{1}{\pi} \frac{\hbar/\tau_{\text{in}}}{(E_{21} - E)^2 + (\hbar/\tau_{\text{in}})^2}, \quad (2.53)$$

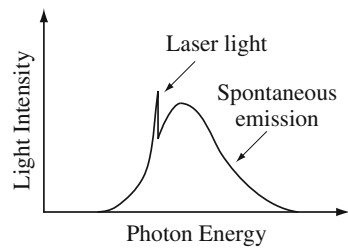
where τ_{in} is the relaxation time due to electron scatterings and transitions. In this case, the optical power gain coefficient $g(E)$ is expressed as

$$g(E) = \frac{e^2 \hbar}{2m^2 \epsilon_0 n_r c E} \int_0^\infty |\langle 1|\mathbf{p}|2 \rangle|^2 \rho_{\text{red}}(E_{21}) [f_2(E_{21}) - f_1(E_{21})] L(E_{21}) dE_{21}, \quad (2.54)$$

which is plotted in Fig. 2.10 for the bulk structures with $\tau_{\text{in}} = 10^{-13}$ s. It is found that relaxation is equivalent to band tailing, which contributes to optical transitions in $E < E_g$.

Because f_2 and $[1 - f_1]$ are always positive, spontaneously emitted lights distribute in a higher energy than the gain peak. Figure 2.11 schematically shows spectra for the stimulated emission (laser light) and the spontaneously emitted lights; its horizontal line indicates photon energy. Note that the photon energies increase due to the band filling effect with enhancement of the excitation, irrespective of spontaneous emission or stimulated emission.

Fig. 2.11 Stimulated emission and spontaneous emission



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