Preface

As M.V. Fedoryuk, a renowned expert in asymptotic analysis in the 1980s, once lamented ([F]), global asymptotic analysis of higher order differential equations was thought to be impossible to construct in his days. At such a time H.L. Berk, W.M. Nevins, and K.V. Roberts published a remarkable paper in J. Math. Phys., **23** (1982), which shows that the traditional Stokes geometry cannot globally describe the Stokes phenomena of WKB solutions of higher order differential equations; a "new" Stokes curve is necessary for the complete description. Later T. Aoki, T. Kawai, and Y. Takei discovered the notion of a virtual turning point by applying microlocal analysis to Borel transformed WKB solutions; a "new" Stokes curve is a Stokes curve emanating from a virtual turning point. An important point is that a virtual turning point is intrinsically defined in the sense that it does not depend on the argument of the large parameter contained in the equation. At the same time, as the qualifier "virtual" indicates, a virtual turning point cannot be detected by a cosmetic study of ordinary WKB solutions; we need the conversion of the study to the one on a different space, the Borel plane on which the Borel transformed WKB solutions are analyzed. This is the reason why a virtual turning point was not found before the advent of the exact WKB analysis, the analysis of Borel transformed WKB solutions.

The aim of the monograph is to explain the core part of this novel and important notion so that it may be appreciated not only by mathematicians but also physicists and engineers and be practically used in concrete problems. To be more concrete, we present in Chap. 2 several concrete figures of Stokes geometry related to some higher order Painlevé equations (the Noumi-Yamada system), and we analyze in Chap. 3 the non-adiabatic transition problems for three-levels (the generalized three-level Landau-Zener model). In both subjects, the reader will be impressed by the importance of the role of virtual turning points in their analysis. We also note that the employment of graph-theoretic notions in Chap. 2 is a natural and reasonable approach in view of the practical way of locating virtual turning points (Sect. 1.6). The results reported in Chaps. 2 and 3 are still in progress; for example, much remains to be done in investigating the role of the total value integral of a tree (Sect. 2.4) in the study of the Noumi-Yamada system, and the more precise study of connection coefficients with the help of the exact steepest descent method (Appendix) has only just been instigated. We hope to come back to these problems in the future, but we feel it important to publish this short monograph now so that virtual turning points may find their proper place in the tool box of a mathematical scientist.

In ending this preface we would like to thank several people for their help without which this could not have been completed in this form. The most important person is Prof. T. Aoki, whose collaboration with two of us (T.K. and Y.T.) is the essential core of this monograph. We also thank sincerely Prof. T. Nishimoto, who kindly called the attention of (some of) Aoki, Kawai, and Takei to the paper [BNR] in a private conversation at the occasion of an RIMS conference; it was really an excellent instruction. Further, we are gratefully indebted to Dr. Shinji Sasaki for having allowed us to include in Sects. 3.2 and 3.3 his unpublished results on the effect of the virtual turning points in calculating the transition probabilities for three-levels. Therewith, we are most obliged to him for having drawn many figures together with several important comments on the draft of this monograph. The heartiest thanks of one of us, T. Kawai, also goes to Ms. K. Kohno for her excellent typing.

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Kyoto

Naofumi Honda
Takahiro Kawai
Yoshitsugu Takei

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Honda, N.; Kawai, T.; Takei, Y.

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