

Chapter 2

Creation of Quantum States of Light

In the previous chapter we described various aspects of quantum states of light. Following a historical order we will now explain how to create quantum states of light, including coherent states, squeezed states, a Schrödinger's cat state, a single-photon state, and a superposition of photon-number states. In the end of this chapter we will explain how to create entanglement.

2.1 Creation of Coherent States of Light

Coherent states of light can in principle be created by a laser. Laser oscillation occurs through amplification of spontaneous emission from the laser material, which corresponds to a vacuum $|0\rangle$. We will not explain in detail, but the process corresponds to the displacement operation $\hat{D}(\alpha)$ in phase space, as shown in Eqs. (1.127) and (1.129). In Eq. (1.129) we used

$$\hat{D}(\alpha) = e^{-ix_0 p_0} e^{i2p_0 \hat{x}} e^{-i2x_0 \hat{p}}, \quad (2.1)$$

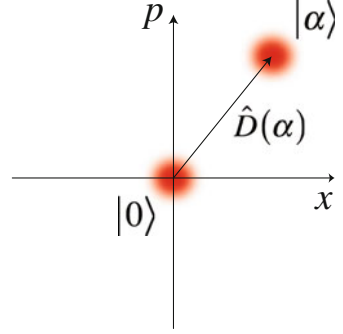
which is equivalent with

$$\hat{D}(\alpha) = e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}}. \quad (2.2)$$

Let's make the displacement operation for a vacuum $|0\rangle$:

$$\begin{aligned} \hat{D}(\alpha)|0\rangle &= e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}}|0\rangle \\ &= e^{\alpha \hat{a}^\dagger} e^{-\alpha^* \hat{a}} e^{-\frac{1}{2}[\alpha \hat{a}^\dagger, -\alpha^* \hat{a}]}|0\rangle \\ &= e^{-\frac{|\alpha|^2}{2}} e^{\alpha \hat{a}^\dagger} e^{-\alpha^* \hat{a}}|0\rangle \\ &= e^{-\frac{|\alpha|^2}{2}} e^{\alpha \hat{a}^\dagger}|0\rangle \end{aligned}$$

Fig. 2.1 A coherent state $|\alpha\rangle$ is created from a vacuum $|0\rangle$ through displacement operation $\hat{D}(\alpha)$



$$\begin{aligned}
 &= e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \hat{a}^{\dagger n} |0\rangle \\
 &= e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \sqrt{n!} |n\rangle \\
 &= e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \\
 &= |\alpha\rangle.
 \end{aligned} \tag{2.3}$$

Here we used the formula

$$e^{\hat{A}+\hat{B}} = e^{\hat{A}} e^{\hat{B}} e^{-[\hat{A}, \hat{B}]/2}. \tag{2.4}$$

Note that this formula holds only when $[\hat{A}, [\hat{A}, \hat{B}]] = [\hat{B}, [\hat{A}, \hat{B}]] = 0$. Figure 2.1 illustrates the displacement operation.

If we take a Hamiltonian for the displacement operation as

$$\hat{H}_\alpha \propto i(\alpha \hat{a}^\dagger - \alpha^* \hat{a}), \tag{2.5}$$

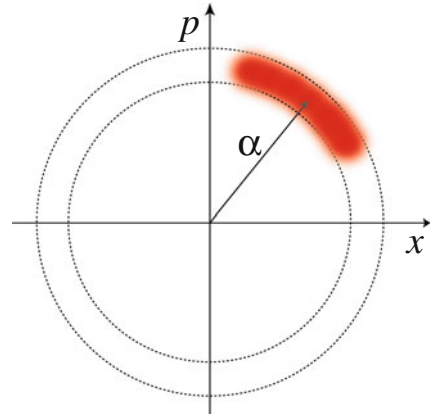
then the “time evolution” of the quantum complex amplitude or the annihilation operator $\hat{a}$ can be described as

$$e^{i \frac{\hat{H}_\alpha}{\hbar} t} \hat{a} e^{-i \frac{\hat{H}_\alpha}{\hbar} t}. \tag{2.6}$$

By using this and Eq.(1.160) we can calculate the “time evolution” of $\hat{a}$ as

$$\begin{aligned}
 \hat{D}^\dagger(\alpha) \hat{a} \hat{D}(\alpha) &= e^{-(\alpha \hat{a}^\dagger - \alpha^* \hat{a})} \hat{a} e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}} \\
 &= \hat{a} - [\alpha \hat{a}^\dagger - \alpha^* \hat{a}, \hat{a}] + \frac{1}{2!} [\alpha \hat{a}^\dagger - \alpha^* \hat{a}, [\alpha \hat{a}^\dagger - \alpha^* \hat{a}, \hat{a}]] + \cdots \\
 &= \hat{a} + \alpha.
 \end{aligned} \tag{2.7}$$

Fig. 2.2 Phase diffusion of an actual laser



Here we can see that the quantum complex amplitude or the annihilation operator $\hat{a}$ changes to $\hat{a} + \alpha$ through the displacement operation $\hat{D}(\alpha)$. This situation can be understood from Fig. 2.1. In short, the displacement operation $\hat{D}(\alpha)$ corresponds to laser oscillation itself when it acts on a vacuum $|0\rangle$. This situation can be seen in the Hamiltonian of the displacement operation ($\hat{H}_\alpha$) given in Eq. (2.5). In Eq. (2.5) $\alpha\hat{a}^\dagger$ and $\alpha^*\hat{a}$ can be regarded as stimulated emission and absorption, respectively.

In the case of an actual laser there is phase diffusion on top of stimulated emission and absorption. So the actual picture of a laser should be the one shown in Fig. 2.2. Although there are even some technical (classical) noises on top of the phase diffusion, the quantum noise is dominant in the high-frequency region. Also, when we want to cancel out the phase diffusion, we use a coherent beam as the reference. More precisely, we usually use a coherent beam from the same laser for the local oscillator light for homodyne measurement to cancel out the phase diffusion.

There is another way to create coherent states of light. We can create coherent states of light at the sideband of the laser frequency by using amplitude or phase modulators. The advantage of this method is that technical (classical) noises are intrinsically very small at higher frequencies and it is easy to realize a situation in which the quantum noise is dominant. So we can create an ideal coherent state with this method. As an example we show an experimental result of quadrature amplitude measurement of a coherent state which is created with this method. Laser light was divided into two beams, and one of the beams was used for a local oscillator beam and the other was modulated to create a coherent state at the modulation sideband. Figure 2.3 shows the experimental phase dependence of amplitude for the coherent state. We can see that it is qualitatively a coherent state. As a reference we show the result of the same experiment for a vacuum $|0\rangle$ in Fig. 2.4. We can see that the quantum fluctuations in Figs. 2.3 and 2.4 look similar. More quantitatively we can calculate the photon-number distribution for the coherent state with the data presented in Fig. 2.3. The result is shown in Fig. 2.5. We can see a Poisson distribution there, which is supposed to follow Eq. (1.40). Finally we show a Wigner function calculated from Fig. 2.3 as Fig. 2.6.

Fig. 2.3 Experimental result of phase dependence of amplitude for a coherent state

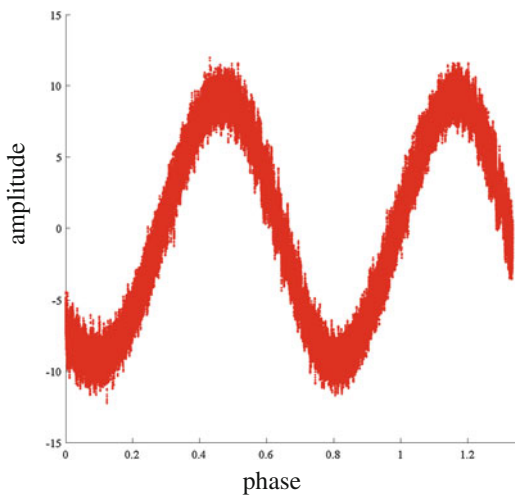


Fig. 2.4 Experimental result of phase dependence of amplitude for a vacuum

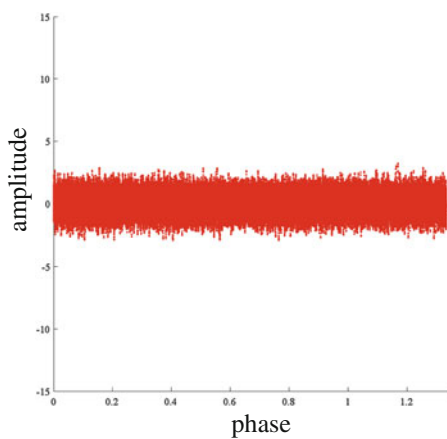


Fig. 2.5 Photon-number distribution calculated with the data presented in Fig. 2.3

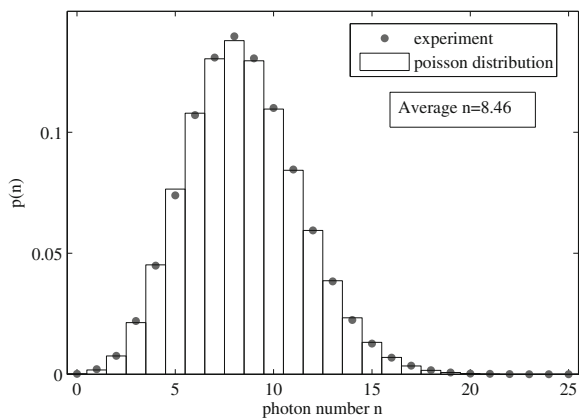


Fig. 2.6 Wigner function calculated with the data presented in Fig. 2.3

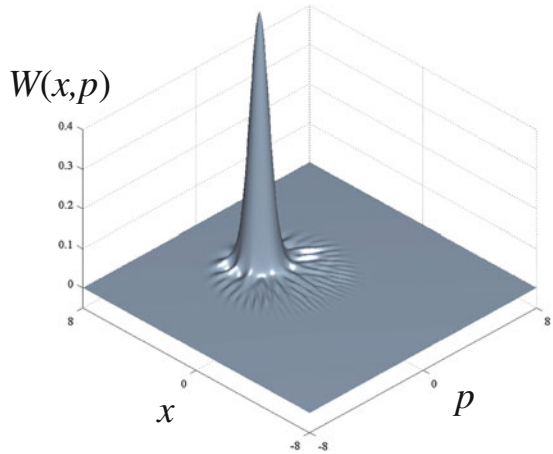
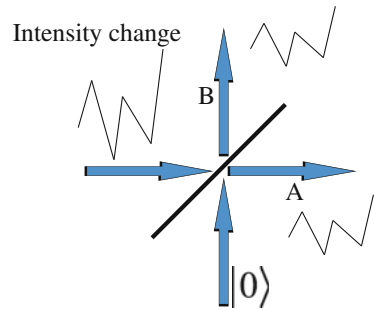


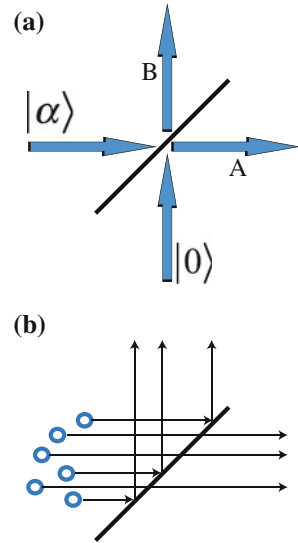
Fig. 2.7 An intensity-fluctuating beam enters a 50/50 beam splitter



We consider quantum fluctuation here. We showed an experimental result of phase dependence of amplitude for a vacuum $|0\rangle$ in Fig. 2.4, which corresponds to quantum fluctuation or quantum noise. The experiment was done with balanced homodyne measurement, where measured light mode $\hat{a}_1$ was in a vacuum state $|0\rangle$, i.e., no input and the other input $\hat{a}_2$ was used for a local oscillator in a coherent state $|\alpha\rangle$. It might seem funny at a first glance. The reason is the following. Some light beam enters a 50/50 beam splitter. It can be in a coherent state but it can be in other states as well. In any case, the two output beams have the same intensity fluctuations. So naively speaking the output of the balanced homodyne measurement should be zero, because the same signals should be totally cancelled out when we took the difference signal of two homodyne currents. However, we got Fig. 2.4. What happened here? Of course, it can be easily solved when we use the particle picture of photons (Fig. 2.7).

We can explain the situation with Fig. 2.8b. It is the particle picture. In this picture individual photons have no correlation and randomly enter the 50/50 beam splitter. Each photon has a 50 %-chance of transmission and a 50 %-chance of reflection at the beam splitter. Of course, a reflected photon cannot be transmitted and a transmitted photon cannot be reflected. So the output two beams have opposite-phase intensity noises. Since we take the difference of them to get a balanced homodyne signal, we

Fig. 2.8 A light beam in a coherent state enters a 50/50 beam splitter. **a** State picture, $|\alpha\rangle$ and $|0\rangle$. **b** Particle picture



consequently take the sum of the opposite-phase intensity noises. As a result, we can get the noise originated from the particle picture of photons, which corresponds to the quantum noise or vacuum noise shown in Fig. 2.4. In other words, the existence of this noise is a proof of the existence of photons. Moreover, since the photon's (particle's) temporal mode is a time-domain delta function $\delta(t)$, the noise spectrum becomes uniform or white in frequency domain. Thus, it is often called “shot noise”.

2.2 Creation of a Squeezed Vacuum

A coherent state can be regarded as a classical electro-magnetic field plus quantum noise, which corresponds to a classical state of light in some sense. So the “true” quantum state of light created for the very first time would be a squeezed vacuum $\hat{S}(r)|0\rangle$. As mentioned in the preface, although it was created by Slusher et al. in 1985 [1] for the very first time with a third-order nonlinear process, most people nowadays use a degenerate parametric process (second-order nonlinearity), which was realized by Wu et al. in 1986 [2]. It is because the parametric process is far more efficient than the third-order nonlinear process. We will thus explain the degenerate parametric process in this section.

Figure 2.9 shows the schematic of the degenerate parametric process. In the process, strong pump light of frequency 2ω (complex amplitude E_3) and input light of frequency ω (complex amplitude E_{in}) are coupled in a second-order nonlinear crystal ($\chi^{(2)}$) and create the difference frequency component of frequency $\omega = 2\omega - \omega$ (complex amplitude E_2). This component couples coherently with the input light and creates the E_{out} output. The input-output relation is

Fig. 2.9 Degenerate parametric process

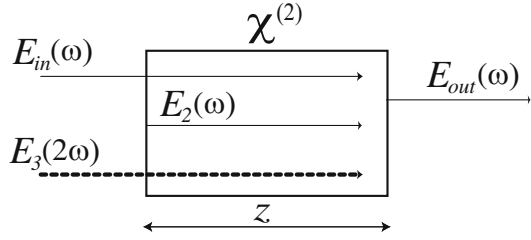
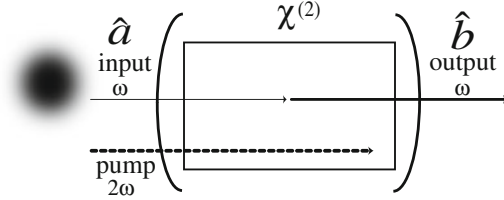


Fig. 2.10 Degenerate optical parametric process with a cavity, which corresponds to the Bogoliubov transformation, i.e., squeezing operation



$$E_{\text{out}} = E_{\text{in}} \cosh r + E_{\text{in}}^* \sinh r, \quad (2.8)$$

where

$$r = \frac{z}{2c} \cdot \frac{\omega}{n} |\chi^{(2)} E_3|, \quad (2.9)$$

r is the squeezing parameter, z is the length of the nonlinear crystal, c is the speed of light, n is the refractive index of the nonlinear crystal, and $\chi^{(2)}$ is the nonlinear coefficient.

From $|\cosh r|^2 - |\sinh r|^2 = 1$, we can see that Eq.(2.8) corresponds to the Bogoliubov transformation in Eqs.(1.154) or (1.162). When we take $\hat{a}$ as a complex amplitude of a light field, Eq.(2.8) is the Bogoliubov transformation itself. We can make a Bogoliubov transformation by using a degenerate parametric process. Note that we cannot get very big r when we use ordinary nonlinear crystals. So we usually use a cavity to enhance the nonlinearity to get bigger r , which corresponds to lengthening the effective crystal length z . A schematic of the process is shown in Fig. 2.10.

We usually create a squeezed vacuum $\hat{S}(r)|0\rangle$ in actual experiments. It is because we can have a vacuum input $|0\rangle$ even without preparing some input state. Although we inevitably will suffer from losses, e.g., coupling losses in the case of some actual input state, we do not have to be worried about losses in the case of a vacuum input. Losses always introduce a vacuum, that always makes the coupling efficiency 100 % for a vacuum input! So it is easy to make a high-level squeezing operation when the input is a vacuum. Moreover, squeezing parameter r should go to infinity depending on the pump power as can be seen in Eq.(2.9). However, the observable r is not so big in the real experiments. We will explain the reason below.

We have to make a balanced homodyne measurement to detect squeezed light, which was explained in Sect. 1.3. It is because a balanced homodyne measurement is a phase-sensitive measurement and we can independently measure squeezed and anti-squeezed components. We use the setup shown in Fig. 1.4 for homodyne measurements and obtain $\langle \hat{I}_2 - \hat{I}_1 \rangle$ of Eq. (1.69) to get the marginal distributions. To get the squeezing parameter r we usually obtain $\langle (\hat{I}_2 - \hat{I}_1)^2 \rangle$ by using a spectrum analyzer. In that case, we use Eq. (1.69) and get

$$\langle (\hat{I}_2 - \hat{I}_1)^2 \rangle = 4|\alpha|^2 \cdot {}_1\langle \psi | (\hat{x}_1 \cos \theta + \hat{p}_1 \sin \theta)^2 | \psi \rangle_1. \quad (2.10)$$

Since the input state is a squeezed vacuum $\hat{S}(r)|0\rangle$, then $|\psi\rangle_1 = \hat{S}(r)|0\rangle_1$. So the output of the spectrum analyzer should be

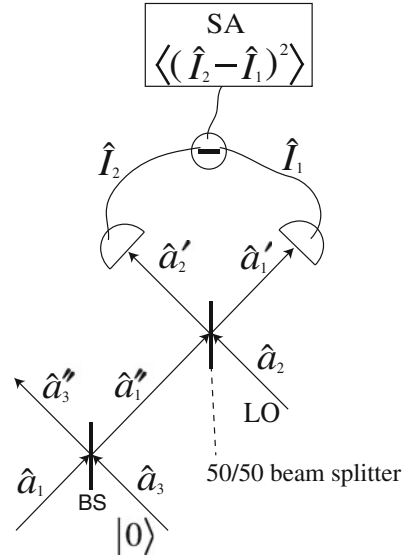
$$\begin{aligned} \langle (\hat{I}_2 - \hat{I}_1)^2 \rangle &= 4|\alpha|^2 \cdot {}_1\langle 0 | \hat{S}_1^\dagger(r) (\hat{x}_1 \cos \theta + \hat{p}_1 \sin \theta)^2 \hat{S}_1(r) | 0 \rangle_1 \\ &= 4|\alpha|^2 \left[{}_1\langle 0 | \hat{S}_1^\dagger(r) \hat{x}_1^2 \hat{S}_1(r) | 0 \rangle_1 \cos^2 \theta \right. \\ &\quad \left. + {}_1\langle 0 | \hat{S}_1^\dagger(r) \hat{p}_1^2 \hat{S}_1(r) | 0 \rangle_1 \sin^2 \theta \right. \\ &\quad \left. + {}_1\langle 0 | \hat{S}_1^\dagger(r) (\hat{x}_1 \hat{p}_1 + \hat{p}_1 \hat{x}_1) \hat{S}_1(r) | 0 \rangle_1 \sin \theta \cos \theta \right] \\ &= 4|\alpha|^2 \left({}_1\langle 0 | \hat{x}_1^2 e^{-2r} | 0 \rangle_1 \cos^2 \theta + {}_1\langle 0 | \hat{p}_1^2 e^{2r} | 0 \rangle_1 \sin^2 \theta \right) \\ &= |\alpha|^2 (e^{-2r} \cos^2 \theta + e^{2r} \sin^2 \theta). \end{aligned} \quad (2.11)$$

Here we used Eqs. (1.165)–(1.168). From the result we should see a periodic structure corresponding to squeezed and anti-squeezed quadratures (e^{-2r} and e^{2r}) depending on the phase of the local-oscillator beam for the balanced homodyne measurement. Here e^{-2r} and e^{2r} correspond to variances $\langle (\Delta x_1)^2 \rangle$ and $\langle (\Delta p_1)^2 \rangle$. It is because ${}_1\langle 0 | \hat{S}_1^\dagger(r) \hat{x}_1 \hat{S}_1(r) | 0 \rangle_1 = {}_1\langle 0 | \hat{S}_1^\dagger(r) \hat{p}_1 \hat{S}_1(r) | 0 \rangle_1 = 0$ which corresponds to that the averaged amplitude is zero.

As shown above, we can measure the squeezing level and determine the squeezing parameter r . Now we have to think about the influence losses have on squeezing. A squeezed vacuum is a superposition of even-photon states as shown in previous sections. When a squeezed vacuum is exposed to loss, the even-photon nature or nonclassicality is degraded. Especially the e^{-2r} component is degraded. We will consider the consequences of this below.

We make a model shown in Fig. 2.11 to think about the influence of loss. In this figure we have a fictitious beam splitter (BS) on top of the setup of Fig. 1.4, which represents losses. Here the input for the homodyne measurement is $\hat{a}_1$ and loss is represented by a vacuum $|0\rangle_3$ from the BS. The input after having losses is $\hat{a}'_1$.

Fig. 2.11 Power measurement with a spectrum analyzer (SA) for balanced homodyne current. The input beam $\hat{a}_1$ experiences losses, which is represented by mode $\hat{a}_3$ (a vacuum $|0\rangle$) and a fictitious beam splitter (BS)



When we set the local-oscillator phase $\theta = 0$, the output from a spectrum analyzer $\langle (\hat{I}_2 - \hat{I}_1)^2 \rangle$ can be calculated with Eq. (2.10) as follows:

$$\begin{aligned}
 \langle (\hat{I}_2 - \hat{I}_1)^2 \rangle &= 4|\alpha|^2 {}_3\langle 0| \otimes {}_1\langle \psi| e^{-i\Theta \hat{L}_2} \hat{x}_1^2 e^{i\Theta \hat{L}_2} |\psi\rangle_1 \otimes |0\rangle_3 \\
 &= 4|\alpha|^2 {}_3\langle 0| \otimes {}_1\langle \psi| [\hat{x}_1^2 \cos^2(\Theta/2) + \hat{x}_3^2 \sin^2(\Theta/2)] |\psi\rangle_1 \otimes |0\rangle_3 \\
 &= 4|\alpha|^2 \left[{}_1\langle \psi| \hat{x}_1^2 |\psi\rangle_1 \cos^2(\Theta/2) + {}_3\langle 0| \hat{x}_3^2 |0\rangle_3 \sin^2(\Theta/2) \right] \\
 &= 4|\alpha|^2 \left[{}_1\langle \psi| \hat{x}_1^2 |\psi\rangle_1 \cos^2(\Theta/2) + \frac{1}{4} \sin^2(\Theta/2) \right]. \quad (2.12)
 \end{aligned}$$

Here we used Eq. (1.59)¹ and ${}_3\langle 0| \hat{x}_3 |0\rangle_3 = 0$. From the result we can see that some portion of the output of the spectrum analyzer is replaced by a vacuum variance of $1/4$ by the ratio of the reflectivity of the fictitious beam splitter $\sin^2(\Theta/2)$. Of course, we can calculate the case of non-zero θ , where the result is similar.

In summary, we can obtain the following result from the measurement of a squeezed vacuum with loss;

$$|\alpha|^2 \left[(e^{-2r} \cos^2 \theta + e^{2r} \sin^2 \theta) \cos^2(\Theta/2) + \sin^2(\Theta/2) \right]. \quad (2.13)$$

When the vacuum fluctuation is set to be one, the squeezing level becomes

$$e^{-2r} \cos^2(\Theta/2) + \sin^2(\Theta/2). \quad (2.14)$$

¹We set $\hat{L}_2 = \frac{1}{2i} (\hat{a}_1^\dagger \hat{a}_3 - \hat{a}_1 \hat{a}_3^\dagger)$.

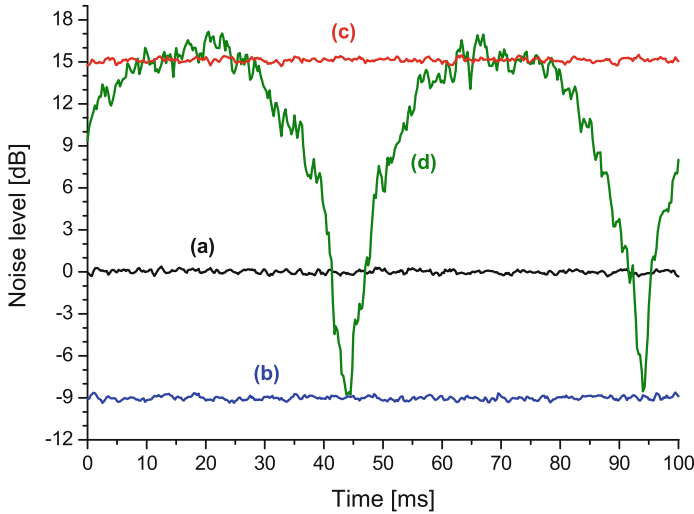


Fig. 2.12 An experimental result of measurements of a squeezed vacuum [3]. It was a world record in squeezing in 2007. **a** Vacuum fluctuation, **b** measurement result when the local oscillator phase was locked at the squeezed quadrature, **c** measurement result when the local oscillator phase was locked at the anti-squeezed quadrature, **d** the local oscillator phase was scanned

The anti-squeezing level becomes

$$e^{2r} \cos^2 (\Theta/2) + \sin^2 (\Theta/2). \quad (2.15)$$

We will check them with an experimental result below.

Figure 2.12 shows the experimental result from a measurements on a squeezed vacuum. The squeezing level is -9 dB (0.126) and the anti-squeezing level is 15 dB (31.6). It was a world record in squeezing in 2007. We can understand these values by using Eqs. (2.14) and (2.15). In Eq. (2.15) we can neglect the influence of loss. It is because the vacuum fluctuation is minute compared to the anti-squeezing. So we take

$$31.6 \approx e^{2r}, \quad (2.16)$$

and get

$$e^{-2r} = \frac{1}{31.6} \approx 0.0316. \quad (2.17)$$

From Eq. (2.14) we get

$$0.126 \approx 0.0316 \times \cos^2 (\Theta/2) + \sin^2 (\Theta/2). \quad (2.18)$$

So we get $\sin^2(\Theta/2) = 0.1$ with $\cos^2(\Theta/2) + \sin^2(\Theta/2) = 1$. From these considerations we can conclude that the squeezing level before having a loss was -15 dB (0.0316) and the total loss was 10%. Note that we usually take an observed squeezing level as the experimental squeezing level. So in the above case we usually take the experimental squeezing level as $e^{-2r'} = 0.126$ and the experimental squeezing parameter as $r' = 1.04$.

We mention the history of experimental squeezing level here. The world record in squeezing, reported by Kimble's group at Caltech was -6 dB ($e^{-2r'} = 0.25$) from 1992 to 2006 [4]. Since this record was not broken for 14 years, it became common sense that -6 dB of squeezing would be a physical limit. However, the author's group bravely tried to improve the squeezing level and finally realized -7 dB of squeezing in 2006 [5], and then the world-wide race of chasing a high level of squeezing begun. It is a typical example of "common sense is nonsense".

In 2007 the author's group succeeded in -9 dB of squeezing as shown in Fig. 2.12. In 2008 Schnabel's group at Hannover reported -10 dB [6]. By 2013 a couples of groups in the world reported -13 dB [7]. In any case, the important point was how to reduce pump-induced losses. As previously mentioned, losses degrade the squeezing level. It is trivial now but it was not trivial until 2006.

Before closing this section we show examples of experimental squeezed vacua in various ways. Figure 2.13 shows the phase dependence of amplitude for a squeezed vacuum, Fig. 2.14 shows the photon number distribution calculated with the data presented in Figs. 2.13, and 2.15 shows the Wigner function calculated with the data presented in Fig. 2.13. Figure 2.13 agrees well with the one of theoretical -10 dB of squeezing shown in Fig. 1.51, and we can see the even-photon nature in Fig. 2.14. Figure 2.15 also agrees well with the one for a theoretical Wigner function shown in Fig. 1.50.

Fig. 2.13 An example of phase dependence of amplitude for an experimental squeezed vacuum

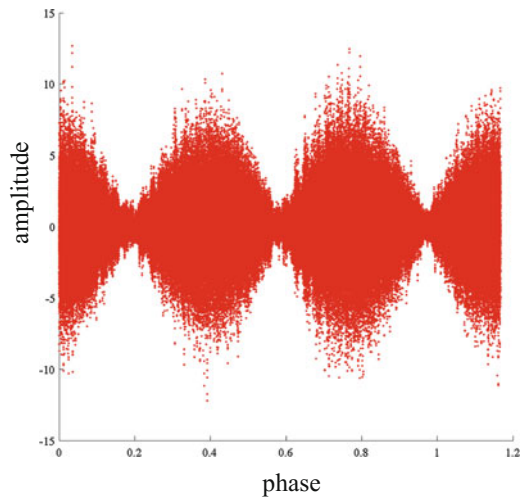


Fig. 2.14 Photon number distribution calculated with the data presented in Fig. 2.13

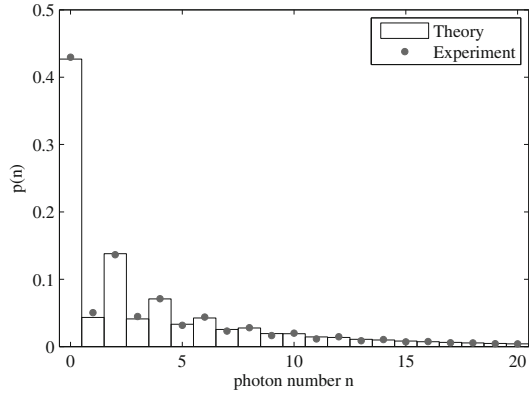
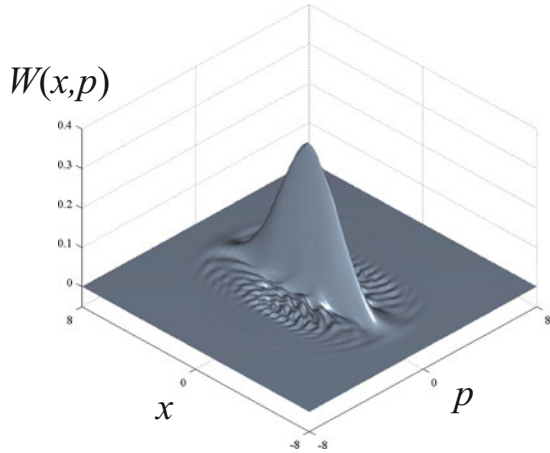


Fig. 2.15 Wigner function calculated with the data presented in Fig. 2.13



2.3 Creation of a Single-Photon State

There are several ways to generate a single-photon state. However, the only way to get the negativity of a single-photon Wigner function is the heralded method, which will be explained below. Of course, in that situation we should see the phase dependence of amplitude for a single-photon state $|1\rangle$ as shown in Fig. 2.16.

In the case of a heralded way to generate a single-photon state, we use a -3 dB-squeezed vacuum $\hat{S}(\ln 2/2)|0\rangle$ in Eq. (1.152). Note that we use a non-degenerate squeezed vacuum in this case, not an ordinary degenerate one. For the non-degenerate case, two photons of state $|2\rangle$ are created in different modes A and B, for example two orthogonally polarized modes. These photon pairs can be created with some particular phase-matching condition. So Eq. (1.152) is modified and we get the $|\text{PDC}\rangle$ state:

$$|\text{PDC}\rangle = 0.971|0\rangle_A|0\rangle_B + 0.229|1\rangle_A|1\rangle_B + \dots \quad (2.19)$$

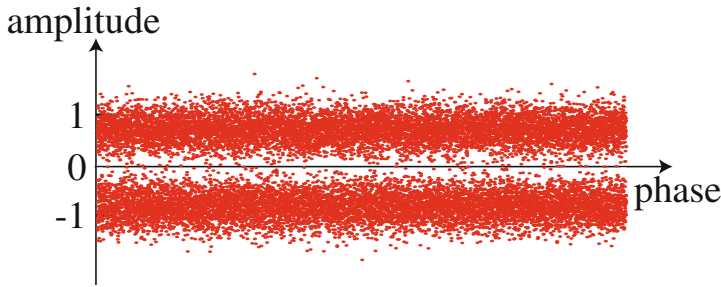


Fig. 2.16 Phase dependence of amplitude for a single-photon state $|1\rangle$ (Fig. 1.12)

This state is a vacuum $|0\rangle$ with a probability of 0.971^2 and is a photon pair A,B with a probability of 0.229^2 . That means it is a vacuum $|0\rangle$ most of the time and a photon pair A,B with a very small probability. We sometimes call it “parametric downconversion” (PDC). Of course, it is just a squeezed vacuum with a different phase-matching condition.

A photon pair created by parametric downconversion can be divided into two photons in different paths. For example, we can use a polarization beam splitter when the two photons have orthogonal polarizations. If we detect a photon in beam A for heralding creation of a single-photon state, then we can get a single-photon state $|1\rangle$ in beam B. Figure 2.17 shows the scheme of heralded single-photon creation.

Figure 2.18 shows an example of phase dependence of amplitude for an experimental single-photon state $|1\rangle$. The Wigner function calculated with the data presented in Fig. 2.18 is shown in Fig. 2.19, and the photon-number distribution calculated with the data presented in Fig. 2.18 is shown in Fig. 2.20. We can see the similarity between Figs. 2.16 and 2.18 and reasonable agreement between the theoretical prediction and the experimental result. We can clearly see the negativity of the experimental single-photon Wigner function and it tells us that the state is a highly pure experimental single-photon state. Actually, the portion of calculated single-photon component is more than 80 % as shown in Fig. 2.20.

Fig. 2.17 Scheme of heralded single-photon creation. If we detect a photon in beam A, then we can get a single-photon state $|1\rangle$ in beam B

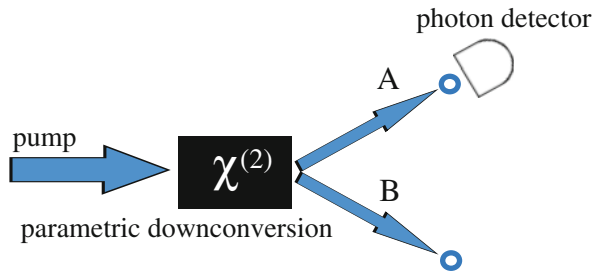


Fig. 2.18 An example of phase dependence of amplitude for an experimental single-photon state $|1\rangle$. Note that we usually use $\hbar = 1$ for this type of plot for some reason

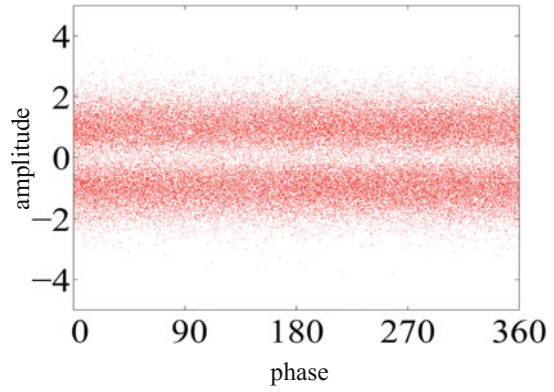


Fig. 2.19 Wigner function calculated with the data presented in Fig. 2.18

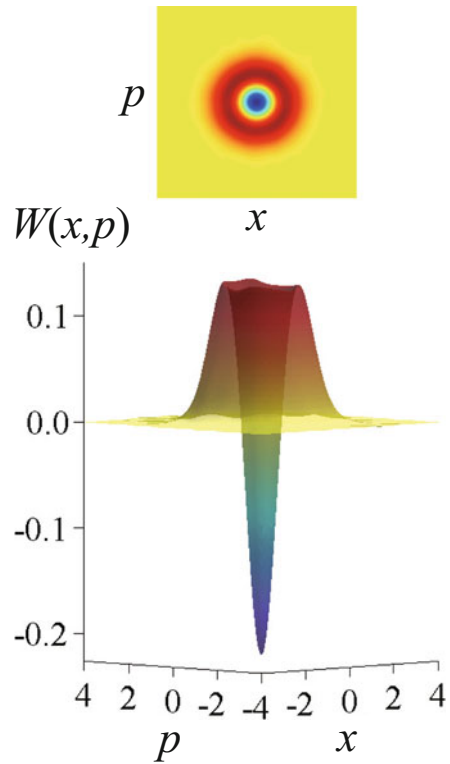
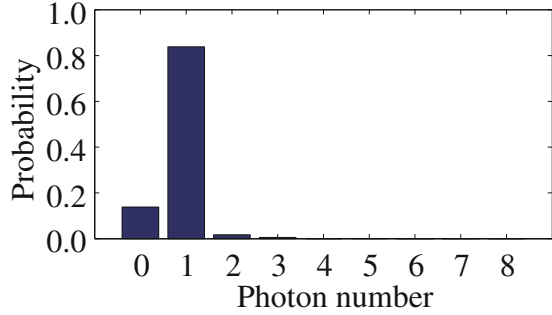


Fig. 2.20 Photon-number distribution calculated with the data presented in Fig. 2.18



2.4 Creation of a Minus Cat State

We will explain how to create one of the Schrödinger’s cat states, a minus cat state, in this section. We consider a heralded creation similar to the previous section. Note that the word “herald” is replaced by “photon subtraction” in this section. As seen in Eq. (1.150), a squeezed vacuum is a superposition of even photons. On the other hand, a minus cat state is a superposition of odd photons. So we can create a minus cat state by subtracting a single photon from a squeezed vacuum.

Let’s think about this example. From Eq. (1.151) we can describe a -4 dB-squeezed vacuum $|\text{sqz} - 4 \text{ dB}\rangle$ (-4 dB-squeezing: $e^{-2r} = 0.398$) as

$$|\text{sqz} - 4 \text{ dB}\rangle = 0.950|0\rangle + 0.289|2\rangle + 0.108|4\rangle + 0.0424|6\rangle + \dots \quad (2.20)$$

Figure 2.21 shows the phase dependence of amplitude of -4 dB-squeezed vacuum $|\text{sqz} - 4 \text{ dB}\rangle$ calculated with above equation. The envelope of the plot agrees well with the one of a minus cat state in Fig. 1.30.

We subtract a single photon from the -4 dB-squeezed vacuum $|\text{sqz} - 4 \text{ dB}\rangle$. From Eq. (2.20) we get without normalization

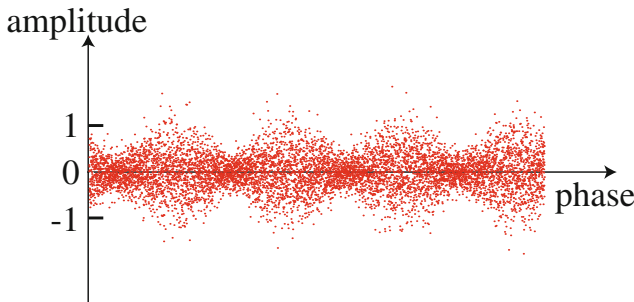


Fig. 2.21 Phase dependence of amplitude of the -4 dB-squeezed vacuum $|\text{sqz} - 4 \text{ dB}\rangle$ calculated with Eq. (2.20)

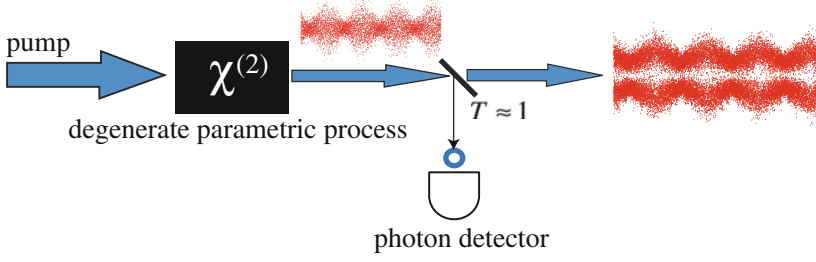


Fig. 2.22 We can create the minus cat state $|\alpha = 1\rangle - |\alpha = -1\rangle$ with single-photon subtraction from the -4 dB-squeezed vacuum $|\text{sqz} - 4 \text{ dB}\rangle$

$$|\text{sqz} - 4 \text{ dB}(1\text{subtract})\rangle = 0.289|1\rangle + 0.108|3\rangle + 0.0424|5\rangle + \dots \quad (2.21)$$

For the normalization we have to divide it by $\sqrt{0.289^2 + 0.108^2 + 0.0424^2 + \dots} \approx 0.311$. Then we get

$$|\text{sqz} - 4 \text{ dB}(1\text{subtract})\rangle \approx 0.929|1\rangle + 0.347|3\rangle + 0.136|5\rangle + \dots \quad (2.22)$$

This state agrees well with the state $|\alpha = 1\rangle - |\alpha = -1\rangle$, whose equation is the following (Eq. (1.119)):

$$\begin{aligned} N_{\alpha-}(|\alpha = 1\rangle - |\alpha = -1\rangle) &= \sqrt{\frac{2e}{e^2 - 1}} \left(|1\rangle + \frac{1}{\sqrt{3!}}|3\rangle + \frac{1}{\sqrt{5!}}|5\rangle + \dots \right) \\ &= 0.922|1\rangle + 0.377|3\rangle + 0.00769|5\rangle + \dots \end{aligned} \quad (2.23)$$

Actually, when we calculate the phase dependence of amplitude of the state $|\text{sqz} - 4 \text{ dB}(1\text{subtract})\rangle$, we cannot see any difference from Fig. 1.30. So we can create the minus cat state $|\alpha = 1\rangle - |\alpha = -1\rangle$ with single-photon subtraction from the -4 dB-squeezed vacuum $|\text{sqz} - 4 \text{ dB}\rangle$ as shown in Fig. 2.22. The intuitive explanation of the single-photon subtraction is the following. When we detect a photon at the photon detector, we eliminate the probability of no photon or a vacuum $|0\rangle$ for the other output from the $T \sim 1$ beam splitter. So we can eliminate the probability of small amplitude of the -4 dB-squeezed vacuum $|\text{sqz} - 4 \text{ dB}\rangle$ in Fig. 2.21.

We explain the schematic of Fig. 2.22 a bit in detail. First we create a squeezed vacuum by using a degenerate parametric process. Here we set the squeezing level so as to create a minus cat state as seen above (-4 dB or so). Then the squeezed vacuum enters a beam splitter with transmissivity $T \sim 1$. The reflected beam contains one photon at most because the reflectivity is very small. Therefore, when we detect a photon at the photon detector, it follows that we subtract a single photon from the squeezed vacuum. So we can get a minus cat state. The process is similar to heralding creation of a single photon in some sense.

Fig. 2.23 An example of phase dependence of amplitude of an experimental minus cat state $|\alpha = 1\rangle - |\alpha = -1\rangle$, which was created by single-photon subtraction

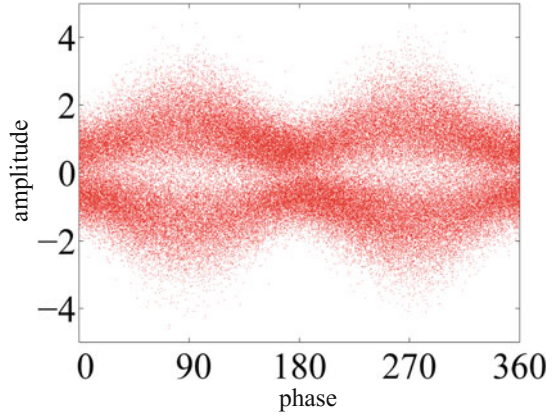


Fig. 2.24 Wigner function calculated with the data presented in Fig. 2.23

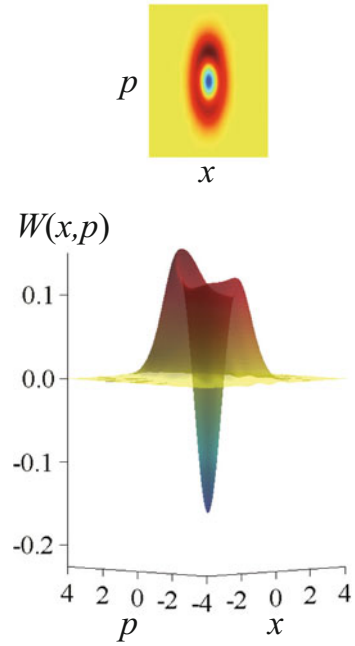


Figure 2.23 shows an example of phase dependence of amplitude of an experimental minus cat state $|\alpha = 1\rangle - |\alpha = -1\rangle$, which was created by single-photon subtraction. The calculated Wigner function with the data presented in Fig. 2.23 is shown in Fig. 2.24 and the calculated photon-number distribution with the data presented in Fig. 2.23 is shown in Fig. 2.25

Figures 2.23 and 1.30 agree well with each other, which means that the experimental result and the theoretical calculation agree well with each other. We can clearly see two opposite-phase coherent states and elimination around zero-amplitude in

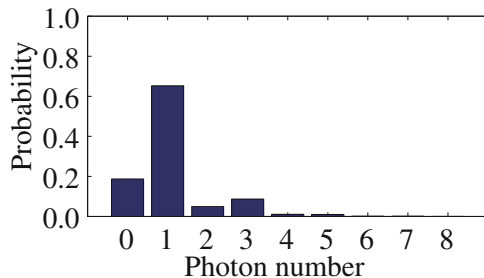


Fig. 2.25 Photon-number distribution calculated with the data presented in Fig. 2.23

the experimental result. Moreover, we can see the negativity and two peaks of the Wigner function in Fig. 2.24, which agree well with Fig. 1.34 (90-degrees phase space rotated: switch x and p). In Fig. 2.25 we can see the odd-photon nature of a minus cat state.

2.5 Creation of a Superposition of Photon-Number States

We can create a superposition of photon-number states with heralding. We will describe that in this section. The simplest example is a superposition of a vacuum $|0\rangle$ and a single-photon state $|1\rangle$.

Figure 2.26 shows the schematic. A photon pair A, B is created by parametric downconversion. As mentioned in Sect. 2.3, it is just a vacuum most of the time and very rarely a photon pair. Photon A enters a beam splitter with transmissivity $T \approx 1$ and the output is combined with a very weak coherent beam, in which the

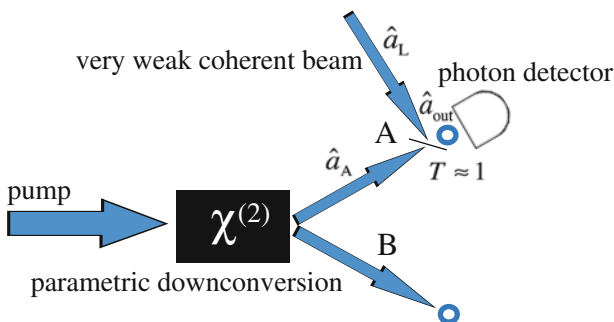


Fig. 2.26 Creation of a superposition of a vacuum $|0\rangle$ and a single-photon state $|1\rangle$. A photon pair A, B is created by parametric downconversion. Photon A enters a beam splitter with transmissivity $T \approx 1$ and the output is combined with a very weak coherent beam, in which the average photon number is much less than one. When we detect a photon at the photon detector, we can get a superposition of a vacuum $|0\rangle$ and a single-photon state $|1\rangle$ in beam B

average photon number is much less than one. When we detect a photon at the photon detector, we can get superposition of a vacuum $|0\rangle$ and a single-photon state $|1\rangle$ in beam B. Note that we assume that we can neglect the probability of simultaneous existence of photons in both beam A and the very weak coherent beam.

We try to make a naive explanation of the mechanism for creation of a superposition of a vacuum $|0\rangle$ and a single-photon state $|1\rangle$ here. When we detect a photon, there are two possibilities. One is that the photon was photon A created by parametric downconversion. Another possibility is that the photon came from the very weak coherent beam. Fortunately or unfortunately the photon detector cannot distinguish the two possibilities, and that is why we can create the superposition. This explanation is really naive and we have to consider entanglement explained in Sect. 1.12 to make the precise explanation. From now on we will deal with entanglement or a two-mode squeezed vacuum (Eq. (1.210)) and try to explain the mechanism for creation of a superposition of a vacuum $|0\rangle$ and a single-photon state $|1\rangle$.

The word “parametric downconversion” is used when the pump in Fig. 2.26 is very weak. In the general case for the pump it becomes a two-mode squeezed vacuum (Eq. (1.210)) explained in Sect. 1.12. We reintroduce Eq. (1.210) here:

$$|EPR^*\rangle = \sqrt{1-q^2} \sum_{n=0}^{\infty} q^n |n\rangle_A \otimes |n\rangle_B. \quad (2.24)$$

Moreover, since a beam splitter matrix with transmissivity T is

$$\begin{pmatrix} \sqrt{T} & \sqrt{1-T} \\ -\sqrt{1-T} & \sqrt{T} \end{pmatrix}, \quad (2.25)$$

the input-output relation of the beam splitter should satisfy

$$\begin{pmatrix} \hat{a}_{\text{out}} \\ \hat{a}_E \end{pmatrix} = \begin{pmatrix} \sqrt{T} & \sqrt{1-T} \\ -\sqrt{1-T} & \sqrt{T} \end{pmatrix} \begin{pmatrix} \hat{a}_A \\ \hat{a}_L \end{pmatrix}, \quad (2.26)$$

where $\hat{a}_A$ represents beam A, $\hat{a}_L$ represents the very weak coherent beam, $\hat{a}_{\text{out}}$ represents one of the output beams from the beam splitter, which is going to the photon detector, and $\hat{a}_E$ represents the other output beam. By using above equations we get

$$\hat{a}_{\text{out}} = \sqrt{T}\hat{a}_A + \sqrt{1-T}\hat{a}_L. \quad (2.27)$$

Here we assume that the very weak coherent beam is in a coherent state $|\beta\rangle$. In the limit of $T \rightarrow 1$ and when we assume $|\beta| \rightarrow \infty$, $\sqrt{1-T}\beta \rightarrow \alpha$, and $|\alpha| \gg 1$, we can treat $\sqrt{1-T}\hat{a}_L$ as α . It is because we can neglect the quantum fluctuations when $|\beta| \gg 1$. So in the limit of $T \rightarrow 1$ and $|\beta| \rightarrow \infty$, we get

$$\hat{a}_{\text{out}} = \hat{a}_A + \alpha. \quad (2.28)$$

It is obvious that we can make a displacement operation $\hat{D}(\alpha)$ on mode $\hat{a}_A$. We can check it by Eq. (2.7), where we calculate displacement operation in the Heisenberg picture, $\hat{D}^\dagger(\alpha)\hat{a}\hat{D}(\alpha)$.

From above considerations we can see that we are making a displacement operation on one of the two-mode squeezed vacuum in the setup of Fig. 2.26. So the process can be written as

$$\begin{aligned}
 & \hat{D}_A(\alpha)\sqrt{1-q^2}\sum_{n=0}^{\infty}q^n|n\rangle_A\otimes|n\rangle_B \\
 &= \sqrt{1-q^2}\sum_{n=0}^{\infty}q^n\hat{D}_A(\alpha)|n\rangle_A\otimes|n\rangle_B \\
 &= \sqrt{1-q^2}\sum_{n=0}^{\infty}q^n\sum_{m=0}^{\infty}\frac{(\alpha\hat{a}^\dagger-\alpha^*\hat{a})^m}{m!}|n\rangle_A\otimes|n\rangle_B. \tag{2.29}
 \end{aligned}$$

When the pump is very weak ($q \ll 1$) and the displacement is very small ($|\alpha| \ll 1$), we get

$$\begin{aligned}
 & \sqrt{1-q^2}\sum_{n=0}^{\infty}q^n\sum_{m=0}^{\infty}\frac{(\alpha\hat{a}^\dagger-\alpha^*\hat{a})^m}{m!}|n\rangle_A\otimes|n\rangle_B \\
 & \approx (1+\alpha\hat{a}^\dagger-\alpha^*\hat{a})|0\rangle_A\otimes|0\rangle_B + (1+\alpha\hat{a}^\dagger-\alpha^*\hat{a})q|1\rangle_A\otimes|1\rangle_B \\
 & = (|0\rangle_A + \alpha|1\rangle_A)\otimes|0\rangle_B + q(|1\rangle_A + \alpha\sqrt{2}|2\rangle_A - \alpha^*|0\rangle_A)\otimes|1\rangle_B \\
 & = |0\rangle_A|0\rangle_B + \alpha|1\rangle_A|0\rangle_B + q|1\rangle_A|1\rangle_B + \sqrt{2}q\alpha|2\rangle_A|1\rangle_B - q\alpha^*|0\rangle_A|1\rangle_B \\
 & \approx |0\rangle_A|0\rangle_B + |1\rangle_A\otimes(\alpha|0\rangle_B + q|1\rangle_B), \tag{2.30}
 \end{aligned}$$

where we neglected the terms of $q\alpha$ ($q\alpha^*$) for the last nearly-equality. Therefore when a photon is detected in beam A, beam B becomes²

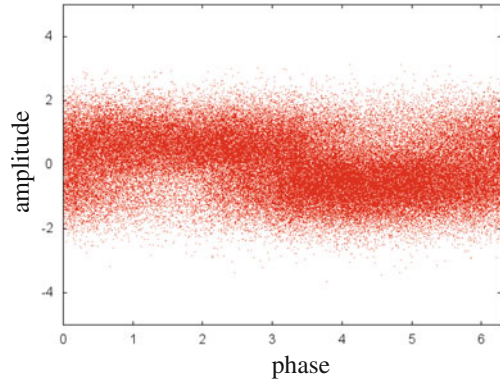
$$\alpha|0\rangle_B + q|1\rangle_B. \tag{2.31}$$

We can thus create a superposition of a vacuum $|0\rangle$ and a single-photon state $|1\rangle$. Here we can say that the displacement operation on beam A was transferred to beam B because of entanglement between beams A and B. Moreover, we can get an arbitrary superposition of a vacuum $|0\rangle$ and a single-photon state $|1\rangle$ by tuning the pump power q and the displacement α . Note that when we set $\alpha = 0$ the output (Eq. (2.31)) becomes $q|1\rangle_B$. We can see that it is heralding creation of a single photon with parametric downconversion in this case.

Figure 2.27 shows an example of phase dependence of amplitude of an experimental superposition of a vacuum $|0\rangle$ and a single-photon state $|1\rangle$ (one cycle). This figure agrees well with Figs. 1.22 and 1.23, which are theoretically calculated superposi-

²It is not normalized. We should only use the ratio between α and q .

Fig. 2.27 An example of phase dependence of amplitude of an experimental superposition of a vacuum $|0\rangle$ and a single-photon state $|1\rangle$ (one cycle). Note that the method of getting this figure is similar to Fig. 2.26 but not exactly the same



tion states of a vacuum $|0\rangle$ and a single-photon state $|1\rangle$ ($(|0\rangle + |1\rangle)/\sqrt{2}$). Moreover, various types of the superposition is examined as shown in Fig. 2.28, where the states are represented by the Wigner functions. These are cases of $|\alpha| : |q| = 1 : 1$ and they are called “qubits” (quantum bits). The results agree well with Fig. 1.44, which is a theoretical calculation. Of course, we can create other states than qubits with just tuning the pump power q and the displacement α .

By using a similar methodology we can create a superposition of many photon-number states. Figure 2.29 shows how to create an arbitrary superposition up to a

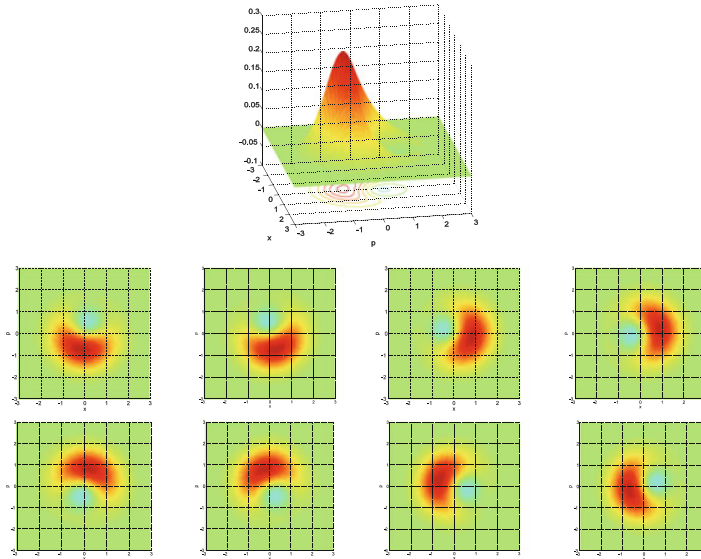


Fig. 2.28 Examples of the Wigner functions of an experimental superposition of a vacuum and a single-photon state $\alpha|0\rangle + q|1\rangle$ ($|\alpha| : |q| = 1 : 1$). It is called (single-rail) qubit

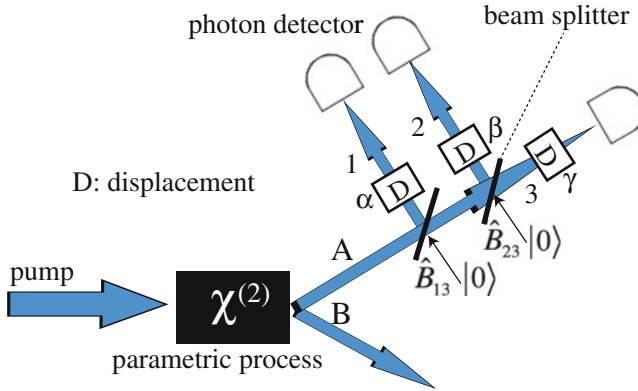


Fig. 2.29 Heralded creation of an arbitrary superposition up to a three-photon state $|3\rangle$. Two-mode squeezed vacuum (beams A, B) is created by a parametric process. Beam A is divided into three beams. Each beam has displacement operation realized by a beam splitter with $T \approx 1$ and a very weak coherent beam as shown in Fig. 2.26. When we have triple coincidence of three photon detector clicks, we can get superposition up to a three-photon state $|3\rangle$

three-photon state $|3\rangle$. In this scheme we use a methodology similar to Fig. 2.26, where we use entanglement and the displacement operation. The “D” in Fig. 2.29 corresponds to the displacement operation and can be realized by a beam splitter with $T \approx 1$ and a very weak coherent beam as shown in Fig. 2.26.

We will explain the mechanism of the heralded creation of an arbitrary superposition up to a three-photon state $|3\rangle$ with Fig. 2.29. We label the three output beams from the two-beam-splitter network as 1, 2, and 3 and describe the two beam-splitter operators as $\hat{B}_{13}$ and $\hat{B}_{23}$ as shown in Fig. 2.29. We set transmissivity of the beam splitters to $T_{13} = 2/3$ and $T_{23} = 1/2$. So the beam splitter transformation becomes

$$\hat{B}_{13}^\dagger \begin{pmatrix} \hat{a}_1 \\ \hat{a}_3 \end{pmatrix} \hat{B}_{13} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_3 \end{pmatrix} \quad (2.32)$$

$$\hat{B}_{23}^\dagger \begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} \hat{B}_{23} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix}. \quad (2.33)$$

When beam A is in a photon-number state $|n\rangle$, the input state to the two-beam-splitter network can be described as $|0\rangle_1 \otimes |0\rangle_2 \otimes |n\rangle_3 \equiv |0, 0, n\rangle$, because there is no input beam from the other ports of the two-beam-splitter network than beam A as shown in Fig. 2.29.

By using the equations above we can describe the state just before the three photon detectors in Fig. 2.29 as

$$\hat{D}_1(\alpha)\hat{D}_2(\beta)\hat{D}_3(\gamma)\hat{B}_{23}\hat{B}_{13}\sqrt{1-q^2}\sum_{n=0}^{\infty}q^n|0,0,n\rangle\otimes|n\rangle_B. \quad (2.34)$$

To calculate this, we first calculate the following equations by using Eq. (1.193).

$$\begin{aligned}\hat{B}_{13}|0,1\rangle &= \sum_{k_2=0}^1 \binom{1}{k_2} \left(\sqrt{\frac{1}{3}}\right)^{k_2} \left(\sqrt{\frac{2}{3}}\right)^{1-k_2} \sqrt{k_2!(1-k_2)!}|k_2,1-k_2\rangle \\ &= \sqrt{\frac{2}{3}}|0,1\rangle + \sqrt{\frac{1}{3}}|1,0\rangle.\end{aligned} \quad (2.35)$$

$$\begin{aligned}\hat{B}_{13}|0,2\rangle &= \frac{1}{\sqrt{2}!} \sum_{k_2=0}^2 \binom{2}{k_2} \left(\sqrt{\frac{1}{3}}\right)^{k_2} \left(\sqrt{\frac{2}{3}}\right)^{2-k_2} \sqrt{k_2!(2-k_2)!}|k_2,2-k_2\rangle \\ &= \frac{2}{3}|0,2\rangle + \frac{2}{3}|1,1\rangle + \frac{1}{3}|2,0\rangle.\end{aligned} \quad (2.36)$$

$$\begin{aligned}\hat{B}_{13}|0,3\rangle &= \frac{1}{\sqrt{3}!} \sum_{k_2=0}^3 \binom{3}{k_2} \left(\sqrt{\frac{1}{3}}\right)^{k_2} \left(\sqrt{\frac{2}{3}}\right)^{3-k_2} \sqrt{k_2!(3-k_2)!}|k_2,3-k_2\rangle \\ &= \frac{2\sqrt{2}}{3\sqrt{3}}|0,3\rangle + \frac{2}{3}|1,2\rangle + \frac{\sqrt{2}}{3}|2,1\rangle + \frac{1}{3\sqrt{3}}|3,0\rangle.\end{aligned} \quad (2.37)$$

$$\begin{aligned}\hat{B}_{23}|0,1\rangle &= \sum_{k_2=0}^1 \binom{1}{k_2} \left(\sqrt{\frac{1}{2}}\right)^{k_2} \left(\sqrt{\frac{1}{2}}\right)^{1-k_2} \sqrt{k_2!(1-k_2)!}|k_2,1-k_2\rangle \\ &= \sqrt{\frac{1}{2}}|0,1\rangle + \sqrt{\frac{1}{2}}|1,0\rangle.\end{aligned} \quad (2.38)$$

$$\begin{aligned}\hat{B}_{23}|0,2\rangle &= \frac{1}{\sqrt{2}!} \sum_{k_2=0}^2 \binom{2}{k_2} \left(\sqrt{\frac{1}{2}}\right)^{k_2} \left(\sqrt{\frac{1}{2}}\right)^{2-k_2} \sqrt{k_2!(2-k_2)!}|k_2,2-k_2\rangle \\ &= \frac{1}{2}|0,2\rangle + \frac{1}{\sqrt{2}}|1,1\rangle + \frac{1}{2}|2,0\rangle.\end{aligned} \quad (2.39)$$

$$\begin{aligned}\hat{B}_{23}|0,3\rangle &= \frac{1}{\sqrt{3}!} \sum_{k_2=0}^3 \binom{3}{k_2} \left(\sqrt{\frac{1}{2}}\right)^{k_2} \left(\sqrt{\frac{1}{2}}\right)^{3-k_2} \sqrt{k_2!(3-k_2)!}|k_2,3-k_2\rangle \\ &= \frac{1}{2\sqrt{2}}|0,3\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|1,2\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|2,1\rangle + \frac{1}{2\sqrt{2}}|3,0\rangle.\end{aligned} \quad (2.40)$$

By using above equations we get

$$\begin{aligned}
& \hat{B}_{23} \hat{B}_{13} \sum_{n=0}^{\infty} q^n |0, 0, n\rangle \otimes |n\rangle_B \\
&= |0, 0, 0\rangle \otimes |0\rangle_B \\
&+ q \left[\sqrt{\frac{1}{3}} |0, 0, 1\rangle + \sqrt{\frac{1}{3}} |0, 1, 0\rangle + \sqrt{\frac{1}{3}} |1, 0, 0\rangle \right] \otimes |1\rangle_B \\
&+ q^2 \left[\frac{1}{3} |0, 0, 2\rangle + \frac{\sqrt{2}}{3} |0, 1, 1\rangle + \frac{1}{3} |0, 0, 2\rangle \right. \\
&\quad \left. + \frac{\sqrt{2}}{3} |1, 0, 1\rangle + \frac{\sqrt{2}}{3} |1, 1, 0\rangle + \frac{1}{3} |2, 0, 0\rangle \right] \otimes |2\rangle_B \\
&+ q^3 \left[\frac{1}{3\sqrt{3}} |0, 0, 3\rangle + \frac{1}{3} |0, 1, 2\rangle + \frac{1}{3} |0, 2, 1\rangle \right. \\
&\quad + \frac{1}{\sqrt{3}} |1, 0, 2\rangle + \frac{\sqrt{2}}{3} |1, 1, 1\rangle + \frac{1}{3} |1, 2, 0\rangle \\
&\quad \left. + \frac{1}{3} |2, 0, 1\rangle + \frac{1}{3} |2, 1, 0\rangle + \frac{1}{3\sqrt{3}} |3, 0, 0\rangle \right] \otimes |3\rangle_B + \dots \quad (2.41)
\end{aligned}$$

Now we make three displacement operations on this state. We assume $|\alpha| \ll 1$, $|\beta| \ll 1$, $|\gamma| \ll 1$, and $q \ll 1$ as before, and then we can make the following approximation:

$$\hat{D}(\alpha) \approx 1 + \alpha \hat{a}^\dagger - \alpha^* \hat{a}. \quad (2.42)$$

By discarding terms of fourth order and higher of α , β , γ , and q and also discarding terms that does not have at least one photon in each beam (1, 2, 3) we get

$$\begin{aligned}
& \hat{D}_1(\alpha) \hat{D}_2(\beta) \hat{D}_3(\gamma) \hat{B}_{23} \hat{B}_{13} \sqrt{1-q^2} \sum_{n=0}^{\infty} q^n |0, 0, n\rangle \otimes |n\rangle_B \\
&\rightarrow \alpha\beta\gamma |1, 1, 1\rangle \otimes |0\rangle_B \\
&+ q \left[\sqrt{\frac{1}{3}} \alpha\beta |1, 1, 1\rangle + \sqrt{\frac{1}{3}} \alpha\gamma |1, 1, 1\rangle + \sqrt{\frac{1}{3}} \beta\gamma |1, 1, 1\rangle \right] \otimes |1\rangle_B \\
&+ q^2 \left[\frac{\sqrt{2}}{3} \alpha |1, 1, 1\rangle + \frac{\sqrt{2}}{3} \beta |1, 1, 1\rangle + \frac{\sqrt{2}}{3} \gamma |1, 1, 1\rangle \right] \otimes |2\rangle_B \\
&+ q^3 \left[\frac{\sqrt{2}}{3} |1, 1, 1\rangle \right] \otimes |3\rangle_B \\
&= |1, 1, 1\rangle \otimes \left[\alpha\beta\gamma |0\rangle_B + q\sqrt{\frac{1}{3}} (\alpha\beta + \alpha\gamma + \beta\gamma) |1\rangle_B \right. \\
&\quad \left. + q^2 \frac{\sqrt{2}}{3} (\alpha + \beta + \gamma) |2\rangle_B + q^3 \frac{\sqrt{2}}{3} |3\rangle_B \right]. \quad (2.43)
\end{aligned}$$

Therefore, when we have a triple coincidence count of the three photon detectors, the state of beam B becomes

$$\alpha\beta\gamma|0\rangle_B + q\sqrt{\frac{1}{3}}(\alpha\beta + \alpha\gamma + \beta\gamma)|1\rangle_B + q^2\frac{\sqrt{2}}{3}(\alpha + \beta + \gamma)|2\rangle_B + q^3\frac{\sqrt{2}}{3}|3\rangle_B. \quad (2.44)$$

Here the coefficients originally appearing in beam A appear in beam B after the triple coincidence count. Of course, it is caused by entanglement between beams A and B. Note that the state of Eq. (2.44) is not normalized. So only the ratio between the coefficients has meaning.

We describe the state $|\psi\rangle$ that we want to create as

$$|\psi\rangle = \sum_{n=0}^3 c_n |n\rangle. \quad (2.45)$$

To create arbitrary states we have to control c_0 , c_1 , c_2 , and c_3 with the experimental parameters α , β , γ , and q . We can see that it is possible because of the following equations, which can be obtained from the comparison between Eqs. (2.44) and (2.45):

$$\alpha\beta\gamma = c_0, \quad (2.46)$$

$$q\sqrt{\frac{1}{3}}(\alpha\beta + \beta\gamma + \gamma\alpha) = c_1, \quad (2.47)$$

$$q^2\frac{\sqrt{2}}{3}(\alpha + \beta + \gamma) = c_2, \quad (2.48)$$

$$q^3\frac{\sqrt{2}}{3} = c_3. \quad (2.49)$$

By using the above equation we get

$$q = \left(\frac{3}{\sqrt{2}} c_3 \right)^{\frac{1}{3}}. \quad (2.50)$$

We can get the other experimental parameters by solving the following third order equation:

$$(x - \alpha)(x - \beta)(x - \gamma) = 0. \quad (2.51)$$

It can be rearranged as follows:

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0. \quad (2.52)$$

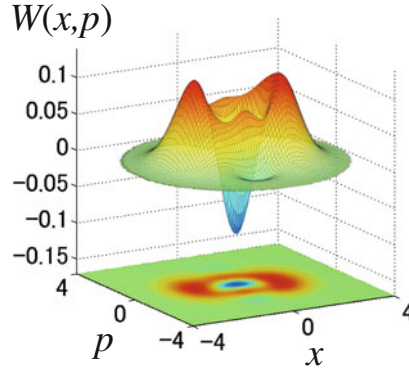


Fig. 2.30 Wigner function of an experimental superposition of a single-photon state $|1\rangle$ and a three-photon state $|3\rangle$ created by the scheme in Fig. 2.29 [8]. It corresponds to a minus cat state with $\alpha = \sqrt{2}$, $N_{\alpha-}(|\alpha\rangle - |-\alpha\rangle)$. It should have at least three negative parts in the Wigner function and we can see them in this figure. Note that we can see only one negative part in Fig. 1.34, where $\alpha = 1$

So we can get α , β , and γ from q , which we already got in Eq. (2.50), by using the following third order equation:

$$x^3 - \frac{3c_2}{\sqrt{2}q^2}x^2 + \frac{\sqrt{3}c_1}{q}x - c_0 = 0. \quad (2.53)$$

Therefore, we can create an arbitrary superposition up to a three-photon state $|3\rangle$ by tuning the pump power and the three displacement parameters. In principle we can extend this methodology up to an arbitrary photon-number state. However, the coincidence rate becomes very small when the photon number increases. So the practical limit is a four-photon state.

Figures 2.30 and 2.31 show examples of Wigner functions of experimental superposition states up to a three-photon state created by the scheme of Fig. 2.29. Figure 2.30 shows Wigner function of an experimental superposition of a single-photon state $|1\rangle$ and a three-photon state $|3\rangle$. As mentioned in Sects. 1.7 and 2.4, a minus cat state $N_{\alpha-}(|\alpha\rangle - |-\alpha\rangle)$ is a superposition of odd photon-number states, and can be created by using single-photon subtraction from a squeezed vacuum. However, since the portions of a two-photon state $|2\rangle$ and a four-photon state $|4\rangle$ in a squeezed vacuum cannot be changed independently, it is impossible to get a minus cat state with $|\alpha| > 1$ and high fidelity by using single-photon subtraction. So we use the scheme in Fig. 2.29 instead of single-photon subtraction to create a “bigger” minus cat state with $|\alpha| > 1$. For example, we can create a “bigger” minus cat state with $\alpha = \sqrt{2}$, which is

$$|1\rangle + \frac{1}{\sqrt{3}}|3\rangle \quad (2.54)$$

without the normalizing factor.

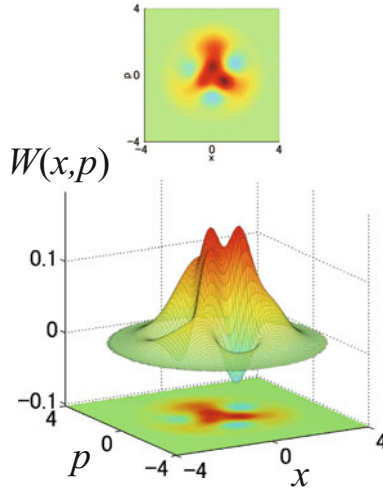


Fig. 2.31 Wigner function of an experimental superposition of a vacuum $|0\rangle$ and a three-photon state $|3\rangle$ created by the scheme of Fig. 2.29 [8]. It might be called “three-headed cat state”, because it corresponds to a superposition of three coherent states

Figure 2.30 shows the Wigner function of an experimental superposition of a single-photon state $|1\rangle$ and a three-photon state $|3\rangle$ created by the scheme in Fig. 2.29. It corresponds to a minus cat state with $\alpha = \sqrt{2}$, $N_{\alpha-}(|\alpha\rangle - |-\alpha\rangle)$ as mentioned above. Its Wigner function should at least have three negative parts as we can see in Fig. 2.30. Note that we can see only one negative part in Fig. 1.34, where $\alpha = 1$. Moreover, the fidelity of the Wigner function in Fig. 2.30 is rather high for a minus cat state with $\alpha = \sqrt{2}$.

Another example is shown in Fig. 2.31, which corresponds to a superposition of three coherent states, $|\alpha\rangle + |\alpha e^{i\frac{2\pi}{3}}\rangle + |\alpha e^{-i\frac{2\pi}{3}}\rangle$. We create this state by the scheme in Fig. 2.29. Here we use the equation

$$|\alpha\rangle + |\alpha e^{i\frac{2\pi}{3}}\rangle + |\alpha e^{-i\frac{2\pi}{3}}\rangle \propto |0\rangle + \frac{\alpha^3}{\sqrt{6}}|3\rangle + \dots, \quad (2.55)$$

where $\alpha = -1.2i$. We can clearly see three peaks and dips rotated by 120° , which correspond to three coherent states and quantum interference fringes between them, respectively. So it is obvious that three coherent states are superposed in this result.

Before closing this section, we show an experimental three photon state $|3\rangle$, which is also created by the scheme in Fig. 2.29. The example is shown in Fig. 2.32.

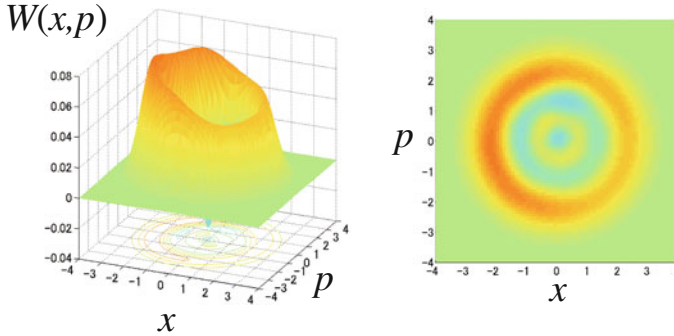


Fig. 2.32 Wigner function of an experimental three-photon state $|3\rangle$ created by the scheme in Fig. 2.29 [8]. The experimental Wigner function agrees with the theoretical one in Fig. 1.39

2.6 Creation of Quantum Entanglement

As mentioned in the previous section, we need to have quantum entanglement between beams A and B for heralding creation of quantum states, and the most important entanglement would be a two-mode squeezed vacuum. It is the nonclassical correlation that causes the photon number of beam B to become n given that the one of beam A is n . Of course, we have other types of correlations, e.g., amplitudes, as shown in Eq. (1.205). In this section we consider a two-mode squeezed vacuum from this point of view.

As shown in Sect. 1.12 the EPR state $|EPR\rangle$ is a maximally entangled state. So if we make an x -measurement on beam A and get the value of x then we know the value of beam B is also x without measuring it ($\hat{x}_A - \hat{x}_B \rightarrow 0$). Similarly, if we make a p -measurement on beam A and get the value of p then we know the value of beam B is $-p$ without measuring it ($\hat{p}_A + \hat{p}_B \rightarrow 0$). Figure 2.33 shows this concept.

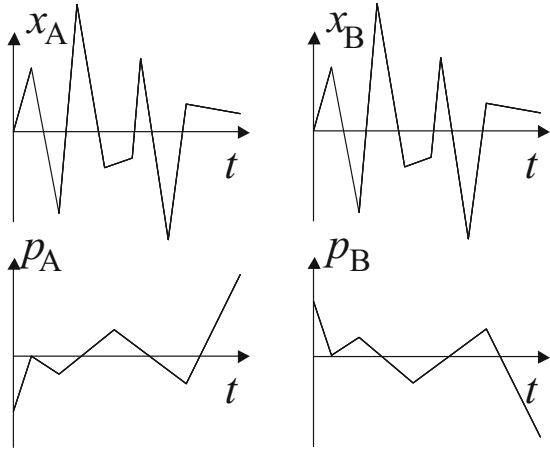
So far we have mainly dealt with pure states, where we can use a wavefunction. We extend the story to a general state here. In Sect. 1.12 we explained that the EPR state is the simultaneous eigenstate of $\hat{x}_A - \hat{x}_B$ and $\hat{p}_A + \hat{p}_B$ with zero eigenvalues. It can be described with quantum mechanical expectation values $\langle \rangle$ as

$$\langle [\Delta(\hat{x}_A - \hat{x}_B)]^2 \rangle + \langle [\Delta(\hat{p}_A + \hat{p}_B)]^2 \rangle = 0. \quad (2.56)$$

The point here is that we can use the above condition for all types of states. It is really powerful for experiments.

As mentioned in Sect. 1.12, it is impossible to create a perfect EPR state because we need an infinite amount of energy to do so. However, as also mentioned in Sect. 1.12, we can create an entangled state or a two-mode squeezed vacuum without infinite amount of energy. So we think about loosening the condition of Eq. (2.56).

Fig. 2.33 Concept of EPR-type correlation between x_A and x_B and between p_A and p_B . Each point corresponds to a measurement. x is correlated and p is anti-correlated



It is well known that beams A and B are entangled if the following condition holds [9, 10]:

$$\langle [\Delta(\hat{x}_A - \hat{x}_B)]^2 \rangle + \langle [\Delta(\hat{p}_A + \hat{p}_B)]^2 \rangle < 1. \quad (2.57)$$

Now let's think about the case where beams A and B are not entangled (separable) and both are in a vacuum state. In this case, since the overall state is $|0\rangle_A \otimes |0\rangle_B$ and $\langle \Delta \hat{A}^2 \rangle = \langle \psi | \hat{A}^2 | \psi \rangle - \langle \psi | \hat{A} | \psi \rangle^2$, we get

$$\begin{aligned} & \langle [\Delta(\hat{x}_A - \hat{x}_B)]^2 \rangle + \langle [\Delta(\hat{p}_A + \hat{p}_B)]^2 \rangle \\ &= {}_B\langle 0 | \otimes {}_A\langle 0 | (\hat{x}_A - \hat{x}_B)^2 | 0 \rangle_A \otimes | 0 \rangle_B \\ & \quad - [{}_B\langle 0 | \otimes {}_A\langle 0 | (\hat{x}_A - \hat{x}_B) | 0 \rangle_A \otimes | 0 \rangle_B]^2 \\ & \quad + {}_B\langle 0 | \otimes {}_A\langle 0 | (\hat{p}_A + \hat{p}_B)^2 | 0 \rangle_A \otimes | 0 \rangle_B \\ & \quad - [{}_B\langle 0 | \otimes {}_A\langle 0 | (\hat{p}_A + \hat{p}_B) | 0 \rangle_A \otimes | 0 \rangle_B]^2 \\ &= 1. \end{aligned} \quad (2.58)$$

Here we used Eqs. (1.22), (1.23), (1.24), and (1.25) with $\alpha = 0$. Obviously the inequality of Eq. (2.57) does not hold.

As an example in which the inequality of Eq. (2.57) holds, we consider a two-mode squeezed vacuum $|EPR^*\rangle_{AB}$. By using the “two-mode squeezing operator” $\hat{S}'_{AB}(r)$ shown in Sect. 1.12 we will show that a two-mode squeezed vacuum satisfies the inequality of Eq. (2.57). The “two-mode squeezing operator” $\hat{S}'_{AB}(r)$ is

$$\hat{S}'_{AB}(r) = \hat{B}_{HBS*} \exp \left[\frac{r}{2} (\hat{a}_A^{\dagger 2} - \hat{a}_A^2) \right] \exp \left[\frac{r}{2} (\hat{a}_B^2 - \hat{a}_B^{\dagger 2}) \right]. \quad (2.59)$$

By using this operator a two-mode squeezed vacuum can be described as $|\text{EPR}^*\rangle_{AB} = \hat{S}'_{AB}(r)|0\rangle_A \otimes |0\rangle_B$, which is shown in Eq.(1.210). So we can calculate $\langle [\Delta(\hat{x}_A - \hat{x}_B)]^2 \rangle + \langle [\Delta(\hat{p}_A + \hat{p}_B)]^2 \rangle$ for a two-mode squeezed vacuum as follows:

$$\begin{aligned}
& \langle [\Delta(\hat{x}_A - \hat{x}_B)]^2 \rangle + \langle [\Delta(\hat{p}_A + \hat{p}_B)]^2 \rangle \\
&= {}_{AB}\langle \text{EPR}^* | (\hat{x}_A - \hat{x}_B)^2 | \text{EPR}^* \rangle_{AB} \\
&\quad - [{}_{AB}\langle \text{EPR}^* | (\hat{x}_A - \hat{x}_B) | \text{EPR}^* \rangle_{AB}]^2 \\
&\quad + {}_{AB}\langle \text{EPR}^* | (\hat{p}_A + \hat{p}_B)^2 | \text{EPR}^* \rangle_{AB} \\
&\quad - [{}_{AB}\langle \text{EPR}^* | (\hat{p}_A + \hat{p}_B) | \text{EPR}^* \rangle_{AB}]^2 \\
&= {}_B\langle 0 | \otimes {}_A\langle 0 | \hat{S}_{AB}^{\prime\dagger}(r) (\hat{x}_A - \hat{x}_B)^2 \hat{S}'_{AB}(r) | 0 \rangle_A \otimes | 0 \rangle_B \\
&\quad - [{}_B\langle 0 | \otimes {}_A\langle 0 | \hat{S}_{AB}^{\prime\dagger}(r) (\hat{x}_A - \hat{x}_B) \hat{S}'_{AB}(r) | 0 \rangle_A \otimes | 0 \rangle_B]^2 \\
&\quad + {}_B\langle 0 | \otimes {}_A\langle 0 | \hat{S}_{AB}^{\prime\dagger}(r) (\hat{p}_A + \hat{p}_B)^2 \hat{S}'_{AB}(r) | 0 \rangle_A \otimes | 0 \rangle_B \\
&\quad - [{}_B\langle 0 | \otimes {}_A\langle 0 | \hat{S}_{AB}^{\prime\dagger}(r) (\hat{p}_A + \hat{p}_B) \hat{S}'_{AB}(r) | 0 \rangle_A \otimes | 0 \rangle_B]^2. \tag{2.60}
\end{aligned}$$

Here the following equations holds:

$$\begin{aligned}
\hat{S}_{AB}^{\prime\dagger}(r) (\hat{x}_A - \hat{x}_B)^2 \hat{S}'_{AB}(r) &= \hat{S}_{AB}^{\prime\dagger}(r) (\hat{x}_A - \hat{x}_B) \hat{S}'_{AB}(r) \hat{S}_{AB}^{\prime\dagger}(r) (\hat{x}_A - \hat{x}_B) \hat{S}'_{AB}(r) \\
&= \left(\frac{e^r \hat{x}_A + e^{-r} \hat{x}_B}{\sqrt{2}} - \frac{e^{-r} \hat{x}_A - e^r \hat{x}_B}{\sqrt{2}} \right)^2 \\
&= 2e^{-2r} \hat{x}_B^2, \tag{2.61}
\end{aligned}$$

$$\begin{aligned}
\hat{S}_{AB}^{\prime\dagger}(r) (\hat{x}_A - \hat{x}_B) \hat{S}'_{AB}(r) &= \frac{e^r \hat{x}_A + e^{-r} \hat{x}_B}{\sqrt{2}} - \frac{e^{-r} \hat{x}_A - e^r \hat{x}_B}{\sqrt{2}} \\
&= \sqrt{2} e^{-r} \hat{x}_B, \tag{2.62}
\end{aligned}$$

$$\begin{aligned}
\hat{S}_{AB}^{\prime\dagger}(r) (\hat{p}_A + \hat{p}_B)^2 \hat{S}'_{AB}(r) &= \hat{S}_{AB}^{\prime\dagger}(r) (\hat{p}_A + \hat{p}_B) \hat{S}'_{AB}(r) \hat{S}_{AB}^{\prime\dagger}(r) (\hat{p}_A + \hat{p}_B) \hat{S}'_{AB}(r) \\
&= \left(\frac{e^{-r} \hat{p}_A + e^r \hat{p}_B}{\sqrt{2}} + \frac{e^{-r} \hat{p}_A - e^r \hat{p}_B}{\sqrt{2}} \right)^2 \\
&= 2e^{-2r} \hat{p}_A^2, \tag{2.63}
\end{aligned}$$

and

$$\begin{aligned}
\hat{S}_{AB}^{\prime\dagger}(r) (\hat{p}_A + \hat{p}_B) \hat{S}'_{AB}(r) &= \frac{e^{-r} \hat{p}_A + e^r \hat{p}_B}{\sqrt{2}} + \frac{e^{-r} \hat{p}_A - e^r \hat{p}_B}{\sqrt{2}} \\
&= \sqrt{2} e^{-r} \hat{p}_A. \tag{2.64}
\end{aligned}$$

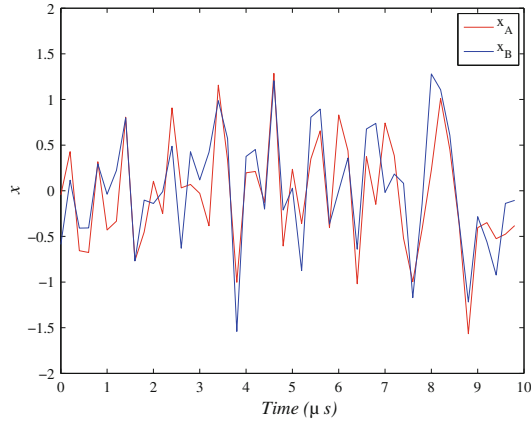


Fig. 2.34 An example of experimental EPR-type correlation or a two-mode squeezed vacuum [11]. Time dependences of x are shown for beams A and B . Each point corresponds to a measurement. We can see strong correlation between x_A and x_B

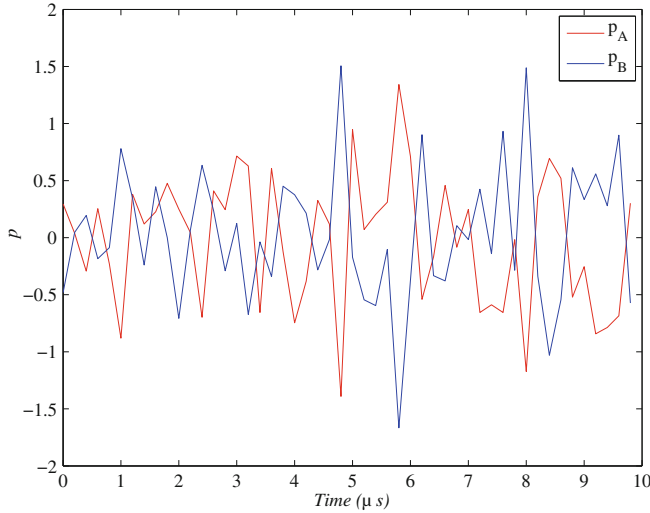


Fig. 2.35 An example of experimental EPR-type correlation or a two-mode squeezed vacuum [11]. Time dependences of p are shown for beams A and B . Each point corresponds to a measurement. We can see strong anti-correlation between p_A and p_B

Note that we use Eqs. (1.157) and (1.209) here.

From these equations we can get

$$\langle [\Delta(\hat{x}_A - \hat{x}_B)]^2 \rangle + \langle [\Delta(\hat{p}_A + \hat{p}_B)]^2 \rangle = e^{-2r} \quad (2.65)$$

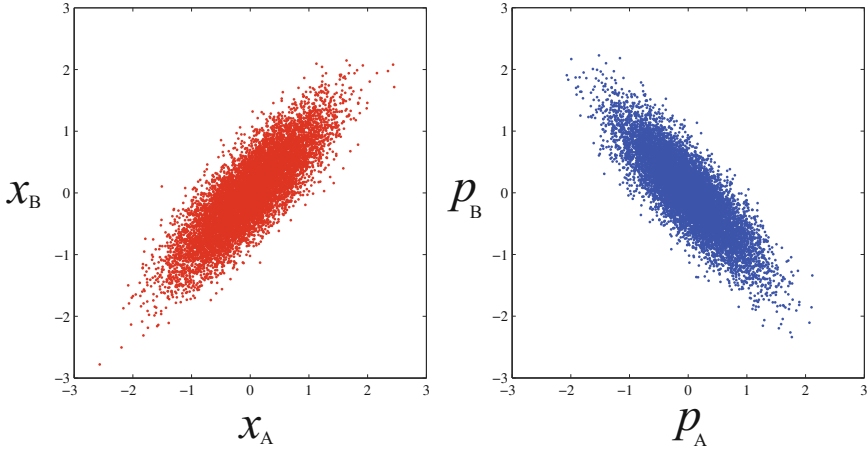


Fig. 2.36 An example of experimental EPR-type correlation or a two-mode squeezed vacuum [11]. We can see strong correlation for x and anti-correlation for p . The degree of (anti-)correlation corresponds to the resource squeezing level e^{-2r}

for a two-mode squeezed vacuum. So if $r > 0$ then beams A and B are entangled. In other words, when the inequality of Eq.(2.57) holds for a two-mode squeezed vacuum $|\text{EPR*}\rangle_{AB}$, we should see the nonclassical correlation of x ($\hat{x}_A - \hat{x}_B \rightarrow 0$) and anti-correlation of p ($\hat{p}_A + \hat{p}_B \rightarrow 0$) between beams A and B. Although, they are of course not perfect correlations. They become perfect in the limit of $r \rightarrow \infty$, where r is the squeezing parameter. Again, r cannot reach infinity, because it needs infinite amount of energy. However, we can increase the r to be high enough. As mentioned in Sect. 2.2, the world record of squeezing level at the moment is -13 dB. That means we can get $r = 1.5$. Although $r = 1.5$ seems to be very small, the inequality is scaled by e^{-2r} and it is already very small. In that sense even when $r = 1.5$, we can say it is big enough.

We show a last example of experimental EPR-type correlation or a two-mode squeezed vacuum here. The time dependences of x are shown for beams A and B in Fig. 2.34. We can see a strong correlation between x_A and x_B . The time dependences of p are shown for beams A and B in Fig. 2.35. We can see a strong anti-correlation between p_A and p_B . Figure 2.36 shows the correlation plots. So we can see a strong correlation for x and a strong anti-correlation for p . The degree of (anti-)correlation corresponds to the resource squeezing level e^{-2r} . In this experiment we use -4 dB of squeezing which corresponds to $r = 0.46$. Even with $r = 0.46$ we can get a reasonably strong EPR-type of correlation. Note that the name “two-mode squeezing” originates from the shapes of the correlation plots of Fig. 2.36.

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