Chapter 2
Transmitter Receiver Techniques

Abstract The transmission over a wireless channel is restricted to a certain range of frequencies around the some central carrier frequency. The wire is a low-pass filter and hence the carrier frequency for the wireline channel is $f_c = 0$. This restriction immediately poses some questions about the design of the wireless communication systems. The foremost question is how is reliable communication related to the carrier frequency? Is the communication strategy and hence the transmitter–receiver design particular to the specific carrier frequency? Do we have to design the system based on f_c? It turns out that we can always work in with the baseband signal (i.e., the signal with $f_c = 0$) even for the wireless communication and then convert the baseband signal into the passband signal (a signal that is centered around some nonzero carrier frequency) with the desired carrier frequency. This makes the design of the transmitter and receiver transparent to the carrier frequency. Thus, only the front end of the system needs to be changed if we change f_c. Also, since the bandwidth of the signal W (typically in KHz) is smaller than the carrier frequency f_c (typically in MHz), the design of DAC and ADC becomes much easier and modular. The focus of this chapter is on the conversion of the baseband signal to the passband signal and vice versa.

Keywords Convolution · Cross-correlation · Autocorrelation · Power spectral density · Baseband signals · Passband signals · Upconversion · Complex envelope · Downconversion · Power and energy spectra · White noise thermal noise

2.1 Notation and Basics of Baseband and Passband Signals

If the continuous-time complex waveform x(t) then we indicate the Fourier transform of x(t) as

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi f t}\mathrm{d}t \tag{2.1}$$

B.B. Purkayastha and K.K. Sarma, *A Digital Phase Locked Loop based Signal and Symbol Recovery System for Wireless Channel*,
Signals and Communication Technology, DOI 10.1007/978-81-322-2041-1_2

and the inverse Fourier transform of X(f) is x(t) itself as indicated by the following expression

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\mathrm{d}f \tag{2.2}$$

2.1.1 Convolution

Convolution has been a standard topic in engineering and computing science for some time, but only since the early 1990s has it been widely available to computer music composers, thanks largely to the theoretical descriptions by Curtis Roads (1996), and the Sound Hack software of Tom Erbe that made this technique accessible.

Convolving two waveforms in the time domain means that you are multiplying their spectra (i.e., frequency content) in the frequency domain. By “multiplying” the spectra we mean that any frequency that is strong in both signals will be very strong in the convolved signal, and conversely any frequency that is weak in either input signal will be weak in the output signal.

In practice, the calculation is usually performed by point-by-point multiplication of the two signals in the Fourier domain. First, the Fourier transform of each signal is obtained. Then the two Fourier transforms are multiplied point-by-point by the rules for complex multiplication and the result is then inverse Fourier transformed. Fourier transforms are usually expressed in terms of complex numbers, with real and imaginary parts; if the Fourier transform of the first signal is $a + ib$, and the Fourier transform of the second signal is $c + id$, then the product of the two Fourier transforms is $(a + ib)(c + id) = (ac - bd) + i(bc + ad)$. Although this seems to be a round-about method, it turns out to be faster then the shift-and-multiply algorithm when the number of points in the signal is large. Convolution can be used as a powerful and general algorithm for smoothing and differentiation.

Convolution function is used to model filters; it is almost the same as correlation, except that one of the signals is mirrored in time before multiplication takes place. Essentially, if the input signal correlates well with the spectrum of the filter impulse response, then the value of the output signal is high; if not, the signal is attenuated.

The length of the impulse response may be thought of as the “echo” or delay effect of the filter. For each point in time, the operation of convolution involves the reflection of the signal (the “echo”) in the filter window, multiplication of the reflected signal with the impulse response, and obtaining the sum of the product sequence for this window. The reason for reflecting the signal can be made intuitively clear.

Assume that the filter has length M_1 and slopes downward, and that the signal has length M_2 and is just a sequence of unit pulses of constant amplitude. Then the first pulse passes through starting at time t_1, producing a copy of the impulse response as the output signal. But point for point, this is added to a copy of the impulse response at for the next pulse, and so on.

This means that in general, the value of the output signal will rise from zero until a stable value is reached when the number of pulses in the signal sequence reaches the number of positions in the pulse response, and will continue at this stable value until the end of the pulse train arrives and the output signal starts to decrease again, and the output signal finally reach zero again at position at time $M_1 + M_2$. The rise in amplitude at the beginning, and the fall at the end are known as the end effects.The number of values which are relevant for defining the convolution of the two signals is thus $M_1 + M_2 - 1$.

Reflection occurs because the input signal meets the filter response 'end to end', like streams of traffic moving in opposite directions, and not, unlike operations such as correlation or Fourier series addition, completely synchronously, like streams of traffic moving in the same direction.

Convolution in the time domain corresponds to multiplication in the frequency domain, i.e., to a cascade of systems performing the Fourier transformation of two signals, multiplication and inverse Fourier transformation of the product.

The Convolution of two signal x(t) and y(t) in time domain is expressed as

$$w(t) = \int_{-\infty}^{\infty} x(k)y(t-k)\mathrm{d}k \tag{2.3}$$

and in frequency domain convolution is:

$$W(f) = X(f)Y(f) \tag{2.4}$$

2.1.2 Cross-Correlation

Correlation determines the degree of similarity between two signals. If the signals are identical, then the correlation coefficient is 1; if they are totally different, the correlation coefficient is 0, and if they are identical except that the phase is shifted by exactly 180° (i.e., mirrored), then the correlation coefficient is -1. Cross-correlation is the method which basically underlies implementations of the Fourier transformation: signals of varying frequency and phase are correlated with the input signal, and the degree of correlation in terms of frequency and phase represents the frequency and phase spectrums of the input signal.

Cross-correlation of two signal $x(t)$ and $y(t)$ in time domain is expressed as

$$w(t) = \int_{-\infty}^{\infty} x^*(k)y(t+k)\mathrm{d}k \tag{2.5}$$

and in frequency domain cross-correlation of two signals $x(t)$ and $y(t)$ is:

$$W(f) = X^*(f)Y(f) \tag{2.6}$$

where the asterisk denotes the complex conjugate. Note that cross-correlation is not commutative. $w(0)$ is the correlation between $x(t)$ and $y(t)$.

2.1.3 Autocorrelation

Autocorrelation is the cross-correlation of a signal with itself. Informally, it is the similarity between observations as a function of the time separation between them. It is a mathematical tool for finding repeating patterns such as the presence of a periodic signal obscured by noise, or identifying the missing fundamental frequency in a signal implied by its harmonic frequencies. It is often used in signal processing for analyzing functions or series of values, such as time domain signals.

Autocorrelation is a method which is frequently used for the extraction of fundamental frequency, f_0: if a copy of the signal is shifted in phase, the distance between correlation peaks is taken to be the fundamental period of the signal (directly related to the fundamental frequency). The method may be combined with the simple smoothing operations of peak and center clipping, or with other low-pass filter operations.

Given a signal $x(t)$, the continuous autocorrelation $\psi_x(t)$ is most often defined as the continuous cross-correlation integral of $x(t)$ with itself, at lag τ

$$\psi_x(t) = \int_{-\infty}^{\infty} x^*(\tau)x(t+\tau)\mathrm{d}k \qquad \Psi_x(f) = |X(f)|^2 \geq 0 \quad \forall f \tag{2.7}$$

Parseval's theorem for energy signals says that

$$E_x = \int_{-\infty}^{\infty} |x|^2 \mathrm{d}t = \int_{-\infty}^{\infty} \Psi_x \mathrm{d}f \tag{2.8}$$

When $x(t)$ is a stationary random signal, $\Psi_x(t)$ is used to indicate its autocorrelation function, that is,

$$\psi_x(t) = E[x^*(k)x(k+t)] \tag{2.9}$$

and $\Psi_x(f)$ is used to indicate its power spectral density.

Properties of one-dimensional autocorrelations function:

1. A fundamental property of the autocorrelation is symmetry, $R(i) = R(-i)$, which is easy to prove from the definition. In the continuous case, the autocorrelation is an even function, i.e., $R_f(-\tau) = R_f(\tau)$, when f is a real function and and the autocorrelation is a Hermitian function, i.e., $R_f(-\tau) = R_f{}^*(\tau)$, when f is a complex function
2. The continuous autocorrelation function reaches its peak at the origin, where it takes a real value, i.e., for any delay τ, $|R_f(\tau) \leq R_f(0)|$. This is a consequence of the Rearrangement inequality. The same result holds in the discrete case.
3. The autocorrelation of a periodic function is, itself, periodic with the same period.
4. The autocorrelation of the sum of two completely uncorrelated functions (the cross-correlation is zero for all τ is the sum of the autocorrelations of each function separately.

5. Since autocorrelation is a specific type of cross-correlation, it maintains all the properties of cross-correlation.
6. The autocorrelation of a continuous-time white noise signal will have a strong peak (represented by a Dirac delta function) at $\tau = 0$ and will be absolutely 0 for all other τ.
7. The WienerKhinchin theorem relates the autocorrelation function to the power spectral density via the Fourier transform:

$$
\begin{aligned}
R(\tau) &= \int_{-\infty}^{\infty} S(f) \exp j2\pi f \tau \mathrm{d}f \\
S(f) &= \int_{-\infty}^{\infty} R(\tau) \exp -j2\pi f \tau \mathrm{d}\tau
\end{aligned} \tag{2.10}
$$

8. For real-valued functions, the symmetric autocorrelation function has a real symmetric transform, so the WienerKhinchin theorem can be re-expressed in terms of real cosines only:

$$
\begin{aligned}
R(\tau) &= \int_{-\infty}^{\infty} S(f) \cos 2\pi f \tau \mathrm{d}f \\
S(f) &= \int_{-\infty}^{\infty} R(\tau) \cos 2\pi f \tau \mathrm{d}\tau
\end{aligned} \tag{2.11}
$$

2.1.4 Power Spectral Density

Power spectral density function (PSD) shows the strength of the variations (energy) as a function of frequency. In other words, it shows at which frequencies variations are strong and at which frequencies variations are weak. The unit of PSD is energy per frequency (width) and you can obtain energy within a specific frequency range by integrating PSD within that frequency range. The goal of spectral density estimation is to estimate the spectral density of a random signal from a sequence of time samples. Depending on what is known about the signal, estimation techniques can involve parametric or nonparametric approaches, and may be based on time-domain or frequency-domain analysis. The spectral density is usually estimated using Fourier transform methods.Computation of PSD is done directly by the method called FFT or computing autocorrelation function and then transforming it. PSD is a very useful tool if you want to identify oscillatory signals in your time series data and want to know their amplitude. Before explain Power spectral density function (PSD) mathematically lets define a few related notions which we have not discussed earlier.

A signal is said to be **Energy Signal** if the signal $x(t)$ has $0 < E < \infty$ for average energy and it signal has zero average power. Where the average energy E is given by

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, \mathrm{d}t \tag{2.12}$$

On the other hand the **Power Signal**'s have infinite average energy ($E = \infty$) or of finite mean signal power ($P < \infty$) respectively. Power signals are generally not integrable so dont necessarily have a Fourier transform. A power signal $x(t)$ has $0 < P < \infty$ for average power

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, \mathrm{d}t \tag{2.13}$$

Energy Spectral Density (*ESD*) describes how the energy of a signal or a time series is distributed with frequency. Here, the term energy is used in the generalized sense of signal processing; that is, the energy of a signal $x(t)$ is $\int_{-\infty}^{\infty} |x(t)|^2 \, \mathrm{d}t$. Energy spectral density is most suitable for transients, i.e., pulse-like signals, for which the Fourier transforms of the signals exist. In this case, Parseval's theorem gives us an alternate expression for the energy of the signal in terms of its Fourier transform:

$$X(f) = \int_{-\infty}^{\infty} e^{-2\pi f t} x(t) \mathrm{d}t \tag{2.14}$$

$$\int_{-\infty}^{\infty} |x(t)|^2 \, \mathrm{d}t = \int_{-\infty}^{\infty} |X(f)|^2 \, \mathrm{d}f \tag{2.15}$$

Here f is the frequency in Hz, i.e., cycles per second. Often used is the angular frequency $\omega = 2\pi f$. Since the integral on the right-hand side is the energy of the signal, the integrand $|X(f)|^2$ can be interpreted as a density function describing the energy per unit frequency contained in the signal at the frequency f. In light of this, the energy spectral density of a signal $x(t)$ is defined as

$$S_{xx} = |x(t)|^2 \tag{2.16}$$

Since power signals have infinite energy, fourier transform and energy spectral density of such signal may not exist. So, we need a alternate spectral density definition with similar properties as ESD. What we can do is to obtain ESD for a power signal $x(t)$ that is time windowed with window size $2T$. So we define Power Spectral Density (PSD) as the normalized limit of the ESD for the windowed signal $x_T(t)$:

$$S_x(f) = \lim_{T\to\infty} \frac{1}{2T} |X_T(f)|^2 \tag{2.17}$$

PSD measures the distribution of signal power $P = \lim_{T\to\infty} 1/2T \int_{-T}^{T} |x(t)|^2\,\mathrm{d}t = \int S_x(f)\mathrm{d}f$ over frequency domain. PSD of a signal is always positive, i.e., $S_x(f) \geq 0$ and PSD of a signal for positive frequencies is the mirror image of its corresponding negative frequencies, i.e., $S_x(-f) = S_x(f)$.

2.1.5 Basic Definitions Related to Baseband and Passband Signals

Baseband Signals. A time-continuous signal is said to be baseband when all its power (energy) spectral density is nonzero at $f = 0$ and it is zero at and at $|f| > f_{\max}$, with $f_{\max} < \infty$.

Passband Signals. A time-continuous signal is said to be passband when all its power (energy) spectral density lies in the frequencies f such that

$$0 < f_{\min} \leq |f| \leq f_{\max} < \infty \tag{2.18}$$

Real Signals. Signals from the real world are real functions of time. The Fourier transform of a real signal x(t) is Hermitian, that is,

$$X^*(-f) = X(f) \tag{2.19}$$

Since the spectrum on the negative frequencies is obtained from the spectrum on the positive frequencies, all the properties of the signal can be expressed by looking to the positive frequencies only.

Bandwidth of Real Signals. The bandwidth is a notion referred to the positive frequencies only. The bandwidth of a real baseband signal is the maximum frequency contained in the spectrum:

$$B = f_{\max} \tag{2.20}$$

The bandwidth of a real passband signal is

$$B = f_{\max} - f_{\min} \tag{2.21}$$

A passband signal is said to be passband around f_0 provided that

$$\frac{f_{\max} + f_{\min}}{2} + \min\left(\frac{B}{2}, f_{\min}\right) \geq f_0 \geq \frac{f_{\max} + f_{\min}}{2} - \min\left(\frac{B}{2}, f_{\min}\right) \tag{2.22}$$

where B is the bandwidth of the passband signal. Although not strictly necessary, often $(f_{\max} + f_{\min})/2$ is selected.

2.1.6 Upconversion Theorem in Time Domain

Theorem *A passband signal of bandwidth B around f_0 is a real signal that can be written in time domain as*

$$s_{\mathrm{pb}}(t) = \sqrt{2}s_c(t)\cos(2\pi f_0 t) - \sqrt{2}s_s(t)\sin(2\pi f_0 t) \tag{2.23}$$

where $s_c(t)$ and $s_s(t)$ are real signals. If the spectrum of the passband signal contains energy (power) at $f = f_0$, then at least one among $s_c(t)$ and $s_s(t)$ is a baseband signal.

2.1.7 Upconversion Theorem in Frequency Domain

From the property of modulation we see that a passband signal around f_0 can be written in frequency domain as

$$S_{\mathrm{pb}}(f) = \frac{1}{\sqrt{2}}\left[S_c(f - f_0) + S_c(f + f_0) + jS_s(f - f_0) - jS_s(f + f0)\right] \tag{2.24}$$

where $j = \sqrt{-1}$, and $S_{\mathrm{pb}}(f)$ is Hermitian because $s_{\mathrm{pb}}(t)$ is real. Since $S_c(f)$ and $S_s(f)$ are Hermitian, we also have

$$S_{\mathrm{pb}}(f) = \frac{1}{\sqrt{2}}(S_c(f - f_0) + S_c{}^*(f + f_0) + jS_s(f - f_0) - jS_s{}^*(f + f0)) \tag{2.25}$$

Equation (2.25) guarantees that $S_{\mathrm{pb}}(f) = 0$ at $f = 0$.

2.1.8 Complex Envelope

The baseband equivalent model of a passband signal, also called complex envelope, is a mathematical model of a passband signal. It is a complex function of time defined as

$$s(t) = s_c(t) + js_s(t) \tag{2.26}$$

The Fourier transform of the complex envelope is

$$S(f) = S_c(f) + jS_s(f) \tag{2.27}$$

Note that since, in the general case, $s(t)$ is not a real signal, $S(f)$ is not Hermitian, therefore the spectrum on the negative frequencies cannot be obtained from the spectrum on the positive frequencies. In other words, both the negative and the positive frequencies contribute to defne the complex envelope in a nontrivial manner.

2.1.9 Complex Upconversion in Time Domain

In time domain, the upconversion theorem can be rephrased by using the complex envelope and the complex exponential:

In the technical language, complex upconversion in time domain is called I/Q modulation. Recalling the second form of the upconversion theorem in frequency domain we can write

$$S_{\mathrm{pb}}(f) = \frac{1}{\sqrt{2}}(S_c(f-f_0)+S_c{}^*(-f-f_0)+jS_s(f-f_0)-jS_s{}^*(-f-f_0)); \quad (2.28)$$

since

$$\begin{aligned} S_c{}^*(-f-f_0) - jS_s{}^*(-f-f_0)) &= (S_c(-f-f_0) + jS_s(-f-f_0) \\ &= S^*(-f-f_0) \end{aligned} \quad (2.29)$$

one gets

$$S_{\mathrm{pb}}(f) = \frac{1}{\sqrt{2}}\left(S(f-f_0) + S^*(-f-f_0)\right); \quad (2.30)$$

which is immediately recognized as the Fourier transform of the rightmost term of Eq. (2.26).

2.1.10 Complex Downconversion

Note that $S(f)$ is obtained by translating around $f = 0$ the portion of $S_{\mathrm{pb}}(f)$ that insists on the positive frequencies:

$$S(f) = \sqrt{2}S_{\mathrm{pb}}(f+f_0); \qquad f_{\mathrm{min}} - f_0 \le f \le f_{\mathrm{max}} - f_0 \quad (2.31)$$

In other words, $S(f)$ is the Fourier transform of the lowpass portion of $\sqrt{2}S_{\mathrm{pb}}(t)$ $e^{-j2\pi f_0 t}$, (complex downconversion)

$$\begin{aligned} s(t) &= \text{lowpass part of } \left(\sqrt{2}s_{\mathrm{pb}}(t)e^{-j2\pi f_0 t}\right) \\ s_c(t) &= \text{lowpass part of } \left(\sqrt{2}s_{\mathrm{pb}}(t)\cos 2\pi f_0 t\right) \\ s_s(t) &= \text{lowpass part of } \left(-\sqrt{2}s_{\mathrm{pb}}(t)\sin 2\pi f_0 t\right) \end{aligned}$$

Complex downconversion is the inverse of complex upconversion:

$$\begin{aligned} s(t) &= DC(UC(s(t)), \\ s_{\mathrm{pb}}(t) &= UC(DC(s_{\mathrm{pb}}(t)) \end{aligned}$$

Multiplying the passband signal around f_0 by a sinusoid at frequency f_0, one obtains a signal whose spectrum can be seen as the sum of two portions: one portion of the spectrum is around $f = 0$, another portion of the spectrum is around $f = 2f_0$. Equation (2.22) guarantees that, after multiplication, the portion of spectrum around $f = 0$ does not overlap the portion of spectrum around $f = 2f_0$. The spectral portion at $2f_0$ can be therefore eliminated by a low-pass filter, the remaining portion being just the complex envelope. In the technical language, complex downconversion in time domain is called I/Q demodulation.

2.1.11 Power and Energy Spectra

The power of the complex envelope is equal to the power of the passband signal:

$$P_{\mathrm{pb}} = P = P_c + P_s \tag{2.32}$$

For the power spectral density we have

$$\Psi_{s_{\mathrm{pb}}}(f) = \frac{1}{2}\Psi_s(f - f_0) + \frac{1}{2}\Psi_s(f - f_0) \tag{2.33}$$

$$\Psi_s(f) = 2\Psi_{s_{\mathrm{pb}}}(f + f_0), \qquad f_{\mathrm{min}} - f_0 \le f \le f_{\mathrm{max}} - f_0 \tag{2.34}$$

We can say that half of the power spectral density of the complex envelope is translated to the positive frequencies, half is reversed and translated to the negative frequencies. Note that, since $s_{\mathrm{pb}}(t)$ is a real signal, its power spectral density is real and even, while the power spectral density of the complex envelope is real but not necessarily even. Energy spectra are treated in a similar way. To elaborate the facts, let us consider two examples:

Passband Signal and Complex Envelope

Consider the sinusoid

$$s_{\mathrm{pb}}(t) = \sqrt{2P}\cos(2\pi f_0 t + \phi) = \sqrt{2P}\mathbb{R}\left(e^{j\phi}e^{j2\pi f_0 t}\right) \tag{2.35}$$

The complex envelope is the phasor

$$s(t) = \sqrt{P}e^{j\phi} \tag{2.36}$$

For the power spectral density one has

$$\Psi_{s_{\mathrm{pb}}}(f) = \frac{P}{2}\delta(f - f_0) + \frac{P}{2}\delta(f + f_0)$$

$$\Psi_s(f) = P\delta(f)$$

Note that

$$P_s = P \int_{-\infty}^{\infty} \delta(f)df = P \tag{2.37}$$

Passband Signal and Complex Envelope
Consider the sum of two sinusoids
$s_{\mathrm{pb}}(t) = \sqrt{2P_1}\cos(2\pi(f_0 + f_1)t + \phi_1) + \sqrt{2P_2}\cos(2\pi(f_0 - f_2)t + \phi_2)$
with $f_1 \geq 0$, $f_1 \geq 0$

The complex envelope is the sum of two rotating phasors:

$$s(t) = \sqrt{P_1}e^{j(2\pi f_1 t + \phi_1)} + \sqrt{P_2}e^{j(-2\pi f_2 t + \phi_2)} \tag{2.38}$$

This example is useful to catch the meaning of negative frequencies in the Fourier transform of the complex envelope: they represent the frequencies below f_0 of the passband signal.

2.1.12 Complex Envelope of a Data Signal

The complex envelope of a linearly modulated data signal is

$$s(t) = \sum_k a_k h(t - kT) \tag{2.39}$$

where T is the symbol repetition interval. We assume that data a_k are complex i.i.d. random variables with zero mean and variance σ^2. The power spectral density and the power of $s(t)$ are

$$\Psi_s(f) = \frac{{\sigma_a}^2}{T}|H(f)|^2, \qquad P_s = \int_{-\infty}^{\infty} \Psi_s(f)\mathrm{d}f = \frac{{\sigma_a}^2}{T}E_h \tag{2.40}$$

The power spectral density of the passband data signal $s_{\mathrm{pb}}(t)$ around f_0 is

$$\Psi_{s_{\mathrm{pb}}}(f) = \frac{{\sigma_a}^2}{2T}|H(f - f_0)|^2 + \frac{{\sigma_a}^2}{2T}\left|H^*(-f - f_0)\right|^2 \tag{2.41}$$

2.1.13 Passband White Noise

In a passband white noise process with two-sided power spectral density $N_0 = 2$

$$n_{\mathrm{pb}}(t) = \sqrt{2}n_c(t)\cos(2\pi f_0 t) - \sqrt{2}n_s(t)\sin(2\pi f_0 t) \tag{2.42}$$

the baseband components $n_c(t)$ and $n_s(t)$ are uncorrelated white random processes with power spectral density

$$\Psi_{n_c}(f) = \Psi_{n_s}(f) = \frac{N_0}{2} \tag{2.43}$$

The power spectral density of the complex envelope of the noise is

$$\Psi_n(f) = N_0 = \Psi_{n_{\mathrm{pb}}}(f + f_0) \qquad f_{\min} - f_0 \leq f \leq f_{\max} - f_0 \tag{2.44}$$

Given passband white noise with power spectral density $N_0 = 2$ around f_0, its power in a bandwidth B around f_0 is $N_0 B$.

2.1.14 Thermal Noise

The common case of thermal noise is obtained with

$$N_0 = kT_{\mathrm{noise}} \tag{2.45}$$

where k is Boltzmann's constant, and T_{noise} is the noise temperature. Thermal noise is a Gaussian process. Its acronym is WGN. Often it is added to the wanted signal. In this case, it is called AWGN.

2.2 Baseband Representation of the Passband Signals

Communication on a wireless channel is inherently different from that on a wireline channel. The main difference is that unlike wireline channel, wireless is a shared medium. The medium is considered as a federal resource and is federally regulated. The entire spectrum is split into many licensed and unlicensed bands. An example of the point-to-point communication in the licensed band is the cellular phone communication, whereas wi-fi, cordless phones, and bluetooth are some of the examples of communication in the unlicensed band.

The transmission over a wireless channel is restricted to a range of frequencies $(f_c - \frac{w}{2}, f_c + \frac{W}{2})$ around the central carrier frequency f_c. The wire is a low pass filter and hence the carrier frequency for the wireline channel is $f_c = 0$.

This restriction immediately posses some questions about the design of the wireless communication systems. The foremost question being how is reliable communication related to the carrier frequency? Is the communication strategy and hence the transmitter-receiver design particular to the specific carrier frequency? Do we have to design the system based on f_c?

It turns out that we can always work in with the baseband signal (i.e., the signal with $f_c = 0$) even for the wireless communication and then convert the baseband signal to the passband signal (a signal that is centered around some nonzero carrier frequency) with the desired carrier frequency. This makes the design of the transmitter and receiver transparent to the carrier frequency. Thus, only the front end of of the system needs to be changed if we change f_c. Also, since the bandwidth of the signal W (typically in KHz) is much smaller than the carrier frequency f_c (typically in MHz), the design of DAC and ADC becomes much easier and modular.

The focus of this section will be on the conversion of the baseband signal to the passband signal and vice versa. Also, the actual wireless channel affects the passband signal. How do these effects translate in the baseband domain, i.e., is there a baseband equivalent of the wireless channel? We will also address this question.

As mentioned before, most of the processing such as coding/decoding, modulation/demodulation, etc., is done at the baseband. At the transmitter, the last stage of the operation is to "up-convert" or "mix" the signal with the carrier frequency and transmit it via the antenna. Similarly, the first step at the receiver is to "down-convert" the RF signal to the baseband before processing. Therefore, it is most important to have a baseband equivalent representation of signals.

Let us begin with the real baseband signal $x_b(t)$ (of double-sided bandwidth W) that we want to transmit over the wireless channel in a band centered around f_c. In wireline channel, $x_b(t)$ would be the signal at the output of the DAC. We know that we can up-convert this signal by multiplying it by $\cos(2\pi f_c t)$.

$$x(t) = x_b(t)\sqrt{2}\cos 2\pi f_c t \tag{2.46}$$

The resulting signal $x(t)$ has spectrum centered around f_c and $-f_c$. Figure 2.1 shows this transformation diagramatically. We scale the carrier by $\sqrt{2}$ as $\cos(2\pi f_c t)$ has

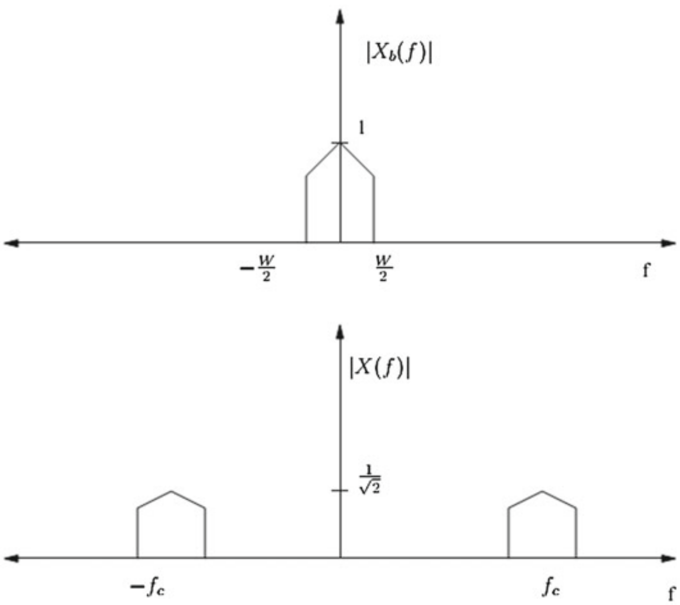

Fig. 2.1 Magnitude spectrum of the real baseband signal and its passband signal

power $1/2$. Thus, by scaling, we are keeping the power in $x_b(t)$ and $x(t)$ same. Note that since $x_b(t)$ is real, the magnitude of its Fourier transform, $X_b(f)$ is symmetric in f and hence the magnitude of the spectrum of the RF signal, $X(f)$ is symmetric around f_c and $-f_c$. We note that to get real $x(t)$, we need not have $X(f)$ symmetric around f_c and $-f_c$. This is a consequence of $x_b(t)$ being real.

To get back the baseband signal, we multiply $x(t)$ again by $\sqrt{2}\cos(2\pi f_c t)$ and then pass the signal through a low pass filter with bandwidth W.

$$\begin{aligned} x(t) &= x_b(t)\sqrt{2}\cos 2\pi f_c t x_b(t) \\ &= (1 + \cos 4\pi f_c t)x_b(t) \end{aligned}$$

The low-pass filter will discard the signal $x_b(t)\cos 4\pi f_c t$ as it is the bandpass signal centered around $2f_c$. Figure 2.2 shows this transformation diagramatically.

One can see that if we multiply $x(t)$ by $\sin\sqrt{2} f_c t$ instead of $\sqrt{2}\cos 2\pi f_c t$, we get $x_b(t)\sin 4\pi f_c t$ and low pass filter will discard this signal completely. There will be a similar outcome had we modulated the baseband signal on $\sqrt{2}\sin 2\pi f_c t$ and try to recover it by using $\sqrt{2}\cos 2\pi f_c t$. Thus,

1. Since the only difference in $\sqrt{2}\cos 2\pi f_c t$ and $\sqrt{2}\sin 2\pi f_c t$ is the phase lag of $\frac{\pi}{4}$, synchronization of carrier phase is crucial in up-conversion and down-conversion.
2. We also note that the signals modulated on $\sqrt{2}\cos 2\pi f_c t$ and $\sqrt{2}\sin 2\pi f_c t$ never get mixed up in the process of down-conversion. Though both the signals share te same frequency band, they are orthogonal to each other. Thus, we could have transmitted two real baseband signals in the same frequency band and doubled the data rate. This is possible as now we are using total double sided bandwidth of $2W$ instead of W as in wireline channel. The resulting RF signal is still real. However, the magnitude of the spectrum of the RF signal need not be symmetric around f_c and $-f_c$.

$$\sqrt{2}\cos 2\pi f_c t - \sqrt{2}\sin 2\pi f_c t \tag{2.47}$$

The baseband signals $x_{b_1}(t)$ and $x_{b_2}(t)$ are obtained at the receiver by multiplying $x(t)$ by $\sqrt{2}\cos 2\pi f_c t$ and $\sqrt{2}\sin 2\pi f_c t$ separately and then passing both the outputs through the low pass filters. Here we are modulating the amplitude of the carrier by the baseband data. Such a scheme is called amplitude modulation. When we modulate both sin and cos parts of the carrier by two undependent baseband signals, the scheme is called Quadrature Amplitude Modulation (QAM). The baseband signal

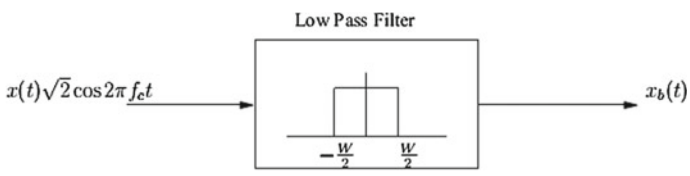

Fig. 2.2 Down-conversion at the receiver

$x_b(t)$ is now defined in terms of the pair $(x_{b_1}(t); x_{b_2}(t))$. In the literature, this pair is denoted as $(x_b{}^I(t); x_b{}^Q(t))$, where I stands for "in phase" signal and Q stands for "quadrature phase" signal. To make the notation compact we can think of $x_b(t)$ as a complex signal defined as follows:

$$x_b(t) \stackrel{\text{def}}{=} x_b{}^I(t) + x_b{}^Q(t) \tag{2.48}$$

We will follow this notation hereafter.

If the wireless channel is just the AWGN channel, then we know how to recover the baseband signal from the RF signal at the receiver and we are done. However, wireless channel is not AWGN channel. If $h(t)$ denote the impulse response of the (time-invarient) wireless channel, the received RF signal is

$$y(t) = h(t) * x(t) + w(t) \tag{2.49}$$

where $w(t)$ is the RF noise. We will ignore the noise for the time being. Then, $y(t) = h(t) * x(t)$, where $x(t)$ is obtained by up-converting the baseband signal $x_b(t)$. We obtain the baseband signal $y_b(t)$ at the receiver by down-converting the received RF signal $y(t)$.

The question we want to address now is: How does the channel impulse response manifests itself in baseband? How are the baseband signals $y_b(t)$ and $x_b(t)$ related?

It turns out that there is a baseband equivalent filter $h_b(t)$ of the channel filter $h(t)$. The transmitted baseband signal $x_b(t)$ is filtered through the baseband channel filter $h_b(t)$ to give the received baseband signal $y_b(t)$.

$$y_b(t) = h_b(t) * x_b(t) \tag{2.50}$$

To understand the relation between $h(t)$ and $h_b(t)$, let us consider a few examples.

1. Let us take the simple case when $h(t) = \delta(t)$. Then, $y(t) = x(t)$ and hence $y_b(t) = xb(t)$. Hence, $h_b(t) = \delta(t)$.
2. Let's consider $h(t) = \delta(t - t_0)$. In this case,

$$\begin{aligned} y(t) = x(t - t_0) &= x_b{}^I(t - t_0)\sqrt{2}\cos 2\pi f_c t(t - t_0) \\ &- x_b{}^Q(t - t_0)\sqrt{2}\sin 2\pi f_c t(t - t_0) \end{aligned} \tag{2.51}$$

We obtain the baseband signal $y_b{}^I(t)$ as

$$\begin{aligned} y_b{}^I(t) &= LPF\left(y(t)\sqrt{2}\cos 2\pi f_c t\right) \\ &= LPF\Big[(2\cos 2\pi f_c(t - t_0)\cos 2\pi f_c t)x_b{}^I(t - t_0) \\ &\qquad -(2\sin 2\pi f_c(t - t_0)\cos 2\pi f_c t)x_b{}^Q(t - t_0)\Big] \end{aligned}$$

$$= LPF\Big[(2\cos 2\pi f_c(2t - t_0) + \cos 2\pi f_c t_0){x_b}^I(t - t_0)$$
$$-(2\sin 2\pi f_c(2t - t_0) - \sin 2\pi f_c t_0){x_b}^Q(t - t_0)\Big]$$
$$= {x_b}^I(t - t_0)\cos 2\pi f_c t_0 + {x_b}^Q(t - t_0)\sin 2\pi f_c t_0$$
$$= \mathbb{R}\left[x_b(t - t_0)e^{-j2\pi f_c t_0}\right]$$

Similarly, we obtain the baseband signal ${y_b}^Q(t)$ as

$${y_b}^I(t) = LPF\left(-y(t)\sqrt{2}\sin 2\pi f_c t\right) \tag{2.52}$$

$$= \mathbb{I}\left[x_b(t - t_0)e^{-j2\pi f_c t_0}\right] \tag{2.53}$$

Thus,

$$y_b(t) = x_b(t - t_0)e^{-j2\pi f_c t_0} \tag{2.54}$$

hence,

$$h_b(t) = e^{-j2\pi f_c t_0}\delta(t - t_0) \tag{2.55}$$

Thus, the baseband signal also gets delayed by the same amount as the passband signal. However, its phase also changes. This phase lag depends on the delay t_0 as well as on the carrier frequency f_c.

We can generalize the second example to obtain the baseband equivalent representation of a generalized channel. Suppose the wireless channel is given by:

$$h(t) = \sum_{l=0}^{L-1} a_l\delta(t - t_1) \tag{2.56}$$

Then, the baseband equivalent of the channel will be

$$h(t) = \sum_{l=0}^{L-1} a_l e^{-j2\pi f_c t_l}\delta(t - t_1) \tag{2.57}$$

2.3 Conclusion

In this chapter, we have seen how we represent the radio frequency signal in baseband. Writing baseband signal as a complex number simplifies the notation a lot. Analyzing a high frequency passband communication system is a cumbersome job. To address this issue, we learned how to work with the baseband equivalent of the passband signal and then convert the baseband signal back into the passband signal. We have also discussed some related signal processing tools available to analyze the passband signals.

Further Reading

1. Haykin S (2008) Communication systems, 4th edn. Wiley, New York
2. Proakis JG, Saleh M (2007) Digital communications, 5th edn. Tata McGraw-Hill, New Delhi
3. Lapidoth A (2009) A foundation in digital communications. Cambridge University Press, Cambridge
4. Proakis JG, Manolakis DG (2007) Digital signal processing: principles, algorithms and applications, 4th edn. Pearson LPE, London
5. Couch LW (2000) Digital and analog communication systems, 6th edn. Prentice-Hall Inc, New Jersey

http://www.springer.com/978-81-322-2040-4

A Digital Phase Locked Loop based Signal and Symbol Recovery System for Wireless Channel

Purkayastha, B.B.; Sarma, K.K.

2015, XXI, 244 p. 118 illus., Hardcover
ISBN: 978-81-322-2040-4