

# Dealing with Uncertainty: From Rough Sets to Interactive Rough-Granular Computing

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*If you thought that science was certain-  
well, that is just an error on your part.*

—Richard P. Feynman,  
The Nobel Prize in Physics (1965)

**Abstract** We discuss an approach for dealing with uncertainty in complex intelligent systems. The approach is based on interactive computations over complex objects called here complex granules (c-granules, for short). C-granules are defined relative to a given agent. Any c-granule of a given agent specifies a perceived structure of local environment of physical objects, called hunks. There are three kinds of such hunks: (i) hunks in the agent external environment creating the hard\_suit of c-granule, (ii) internal hunks of agent, creating the soft\_suit of c-granule, some of which can be represented by agent as infogranules, and (iii) hunks creating the link\_suit of c-granule and playing the role of links between hunks from the hard\_suit and soft\_suit. This structure is used for recording by means of infogranules the results of interactions of hunks from the local environment. We begin from the discussion on dealing with uncertainty in the rough set approach and next we move toward interactive computations on c-granules. In particular, from our considerations it follows that the fundamental issues of intelligent systems based on interactive computations concern the efficiency management in controlling of computations performed by such systems. Our approach is a step toward realization of the Wisdom

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Technology (WisTech) program. The approach was developed over years of work, based on the work on different real-life projects.

**Keywords** Information granule · Physical object · Interaction · Complex granule · Granular computing · Rough set · Complex vague concept approximation · Adaptive judgment · Efficiency management

## 1 Introduction

There are quite many well-known different approaches for dealing with uncertainty (e.g., [13, 16, 17, 23, 24, 43, 44]). We emphasize some basic issues related to uncertainty in: (i) object perception, (ii) concept perception as well as (iii) reasoning about concepts. In real-life applications, the objects and concepts we are dealing with are complex. Moreover, they are often vague what causes additional problems in coping with them.

We start from the rough set approach proposed by Professor Pawlak [23, 24, 27] as a tool for dealing with imperfect knowledge, in particular with vague concepts. Rough set theory has attracted the attention of many researchers and practitioners all over the world. We discuss uncertainty issues in object and concept perception in the rough set framework.

Granular Computing (GC) is now an active area of research [29]. Objects we are dealing with in GC are *information granules* (or *infogranules*, for short). Such granules are obtained as the result of information granulation [47]:

Information granulation can be viewed as a human way of achieving data compression and it plays a key role in implementation of the strategy of divide-and-conquer in human problem-solving.

The concept of granulation is rooted in the concept of a linguistic variable introduced by Professor Lotfi Zadeh in 1973. Information granules are constructed starting from some elementary ones. More compound granules are composed of finer granules that are drawn together by distinguishability, similarity, and functionality [45].

Understanding of interactions of objects on which are performed computations is fundamental for modeling of complex systems [3]. For example, in [21] this is expressed in the following way:

[...] interaction is a critical issue in the understanding of complex systems of any sorts: as such, it has emerged in several well-established scientific areas other than computer science, like biology, physics, social and organizational sciences.

When we move to dealing with perception of interacting complex objects in observed situations one should consider that due to resource bounds only some parts of complex objects may be perceived at a given moment of time. These parts are perceived as values of compound attributes computed on the basis of the delivered (e.g., by control of the agent) parameters of sensors and recorded in relevant information (decision) systems as the results of sensory measurements. Hence, uncertainty in identification of the environment state often causes that results of interactions with and within the environment cannot be predicted with certainty. As a consequence, e.g., results of performed actions may be different than the predicted ones.

In this paper, we outline an extension of Interactive Rough-Granular Computing (IRGC) approach (see, e.g., [29, 38, 41, 42]) by introducing *complex granules* (*c-granules*, for short) making it possible to model interactive computations performed by an agent. In such computations, interactions among physical objects and interactions of these physical objects with information granules possessed by the agent are represented.

In IRGC, the rough set approach in combination with other soft computing approaches are used for inducing approximations of complex vague concepts.

Different problems related to dealing with uncertainty in IRGC are outlined in the paper.

Let us mention here that our discussion on IRGC based on *c-granules* is strongly related to the following sentences:

As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality. (Albert Einstein, [2])

Constructing the physical part of the theory and unifying it with the mathematical part should be considered as one of the main goals of statistical learning theory. (Vladimir Vapnik, [43] p. 721)

This paper covers some issues presented in the invited talk at ICFUA 2013.

In Sect. 2, we discuss some basic problems related to dealing with uncertainty in the rough set approach. Section 3 outlines the approach to IRGC based on *c-granules* and reports some issues concerning uncertainty in IRGC. In particular, due to uncertainty, e.g., in identification of the global environment state, development of the efficiency management techniques for controlling by agent computations performed over *c-granules* for achieving goals is crucial for intelligent systems based on IRGC.

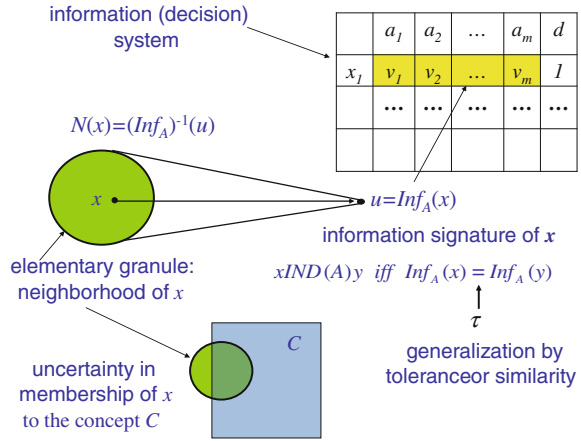
## 2 Rough Sets and Uncertainty

### 2.1 Uncertainty in Object Perception

The rough set philosophy [23, 24, 27] is founded on the assumption that with every object of the universe of discourse, we associate some information (data, knowledge) called the object signature. Objects characterized by the same information are indiscernible (similar) in view of the available information about them. The *indiscernibility relation* generated in this way is the mathematical basis of rough set theory. This understanding of indiscernibility is related to the idea of Gottfried Wilhelm Leibniz that objects are indiscernible if and only if all available functionals take on them identical values [15]. However, in the rough set approach, indiscernibility is defined relative to a given set of functionals (attributes).

Any set of all indiscernible (similar) objects is called an *elementary granule*, and forms a basic granule (atom) of knowledge about the universe. In Fig. 1, we illustrate

**Fig. 1** Elementary granules in rough sets defined by signatures of objects and their partial inclusion in concepts (sets)



the elementary granules defined by the *indiscernibility relation*  $IND(A)$  on the basis of the *object signatures*  $Inf_A(x)$ , where

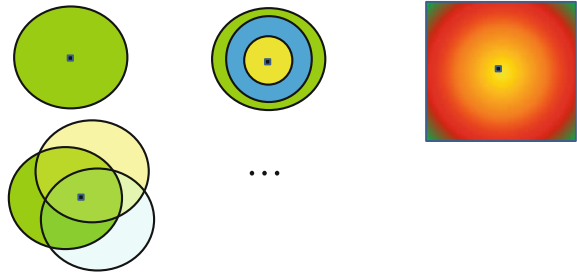
$$Inf_A(x) = \{(a, a(x)) : a \in A\} \quad (1)$$

for  $x \in U$  [27].

If, e.g., the results of measurements are uncertain, one should consider more compound elementary granules (see, e.g., Fig. 2).

Many research papers on rough sets are dedicated to different issues related to *uncertainty of object perception* [39], e.g., to uncertainty caused by: (i) missing attribute values, (ii) imperfect measurement of attribute values, (iii) noise, (iv) unknown relevant structure or context of objects in hierarchical learning, and (v) unknown relevant attributes for approximation (feature selection and constructive induction problems). All the above aspects concerning uncertainty in object perception have an impact on definitions of indiscernibility (discernibility) relations, elementary granule, granules constructed from them, and approximations of concepts.

**Fig. 2** Examples of elementary granules with centers and different uncertainty in membership measurement: binary case, monotonic discrete multivalued case, monotonic continuous fuzzy case, non-monotonic discrete case



## 2.2 Uncertainty in Concept Perception

In this section, we start from an explanation how uncertainty in object perception influences the concept perception. Next, we consider the impact of imperfect information about concepts of their perception.

Any union of elementary granules is referred to as *crisp* (precise) set. If a set is not crisp then it is called *rough* (imprecise, vague).

Note that due to the computational complexity of searching for relevant crisp sets in solving problems related to concept approximation, the searching is usually restricted to a feasible subfamily of the family of all possible unions of elementary sets, e.g., consisting of conjunctions of descriptors only [25, 27].

Each rough set has *borderline cases*, i.e., objects which cannot be classified with certainty as members of either the set or its complement. Obviously, crisp sets have no borderline elements at all. This means that borderline cases cannot be properly classified by employing available knowledge.

Thus, the assumption that objects can be “seen” only through the information available about them leads to the view that knowledge has granular structure. Due to the granularity of knowledge, some objects of interest cannot be discerned and appear as the same (or similar). As a consequence, vague concepts in contrast to precise concepts, cannot be characterized in terms of information about their elements. Therefore, in the proposed approach, we assume that any vague concept is replaced by a pair of precise concepts—called the *lower and the upper approximation of the vague concept*. The lower approximation consists of all objects which definitely belong to the concept and the upper approximation contains all objects which possibly belong to the concept. The difference between the upper and the lower approximation constitutes the *boundary region* of the vague concept. Approximation operations are two basic operations in rough set theory. Hence, rough set theory expresses vagueness not by means of membership, but by employing a boundary region of a set. If the boundary region of a set is empty it means that the set is crisp, otherwise the set is rough (inexact). A nonempty boundary region of a set means that our knowledge about the set is not sufficient to define the set precisely.

In the literature, one can find more details on different aspects of rough set approximations of *vague concepts*. In particular, discussion on vague (imprecise) concepts in philosophy includes the following characteristic features of them [12]: (i) the presence of borderline cases, (ii) boundary regions of vague concepts are not crisp, and (iii) vague concepts are susceptible to sorites paradoxes. The rough set approach is consistent with this view. For example, one should consider that the set of attributes and the set of objects and/or attributes are changing. Hence, the boundary region is drifting and it is only possible to construct temporary crisp definitions of boundary region.

The original approach by Pawlak [23, 24, 27] was based on indiscernibility defined by equivalence relations. Any such indiscernibility relation defines a partition of the universe of objects. Over the years many generalizations of this approach were introduced and the most of them are based on coverings rather than partitions. In particular, one can consider similarity (tolerance)-based rough set approach, binary

relation-based rough sets, neighborhood and covering rough sets, dominance-based rough set approach, hybridization of rough sets and fuzzy sets, and many others [26, 39]. One should note that dealing with coverings requires solving several new algorithmic problems such as selection of family of definable sets or resolving problems with selection of relevant definition of approximation of sets among many possible ones. For a given problem (e.g., classification problem), it is necessary to discover the relevant covering (or partial covering) for the target classification task. In the literature, there are numerous papers dedicated to theoretical aspects of the *covering rough set approach*. However, still more work should be done on algorithmic problems concerning discovery of the relevant covering.

Another issue investigated in the rough set approach concerns (rough) *inclusion measures* [26]. In particular, approximation spaces with rough inclusion measures have been investigated [26, 35]. This approach was further extended to rough mereological approach [30, 31]. More general cases of approximation spaces with rough inclusion were also discussed in the literature including approximation spaces in GC [38]. It is worthwhile mentioning here the approach for ontology approximation used in hierarchical learning of complex vague concepts (see, e.g., [1, 39]). Different rough inclusion measures and based on them quality measures are used for inducing from dataset decision rules, dependencies of attributes, concept description, clusters, or classifiers. They are based, e.g., on the positive region, different kinds of entropy or relative entropy. In the case of rough set-based classifiers, often are used different versions of the *minimum length description principle* [32, 33]. In searching for the high quality classifiers, quality measures aggregating two components are used. The first one is related to the data model quality and the second one to the model description length. The aggregation of such uncertainty measures is optimized for obtaining the high quality classifiers [27, 39]. Let us also note that many known similarity indices can be defined by rough inclusion measures [4].

Due to uncertainty in perception of concepts, the rough set approach is used for developing methods for inducing approximations of concepts in the form of classifiers or clusters. This direction is strongly related to *inductive reasoning* and also to more general reasoning called *adaptive judgment* [6–9, 11]. The general idea is as follows. From a given decision table, a set of granules in the form of decision rules is induced together with arguments *for* and *against* for each decision rule and decision class. For any new object with known signature, one can select rules matching this object. Note that the left-hand sides of decision rules are described by formulas making it possible to check for new objects if they satisfy them assuming that the signatures of these objects are known. In this way, one can consider two semantics of formulas: on a sample of objects  $U$  and on its extension  $U^* \supseteq U$ . Definitely, one should consider a risk of such generalization in the decision rule inducing. Next, a conflict resolution should be applied for resolving conflicts between rules matching the new object and voting for different decisions. The whole procedure can be generalized for the case of approximation of more compound information granules than concepts.

It is worthwhile mentioning that in the rough set approach were also developed approaches for inducing approximate reasoning schemes [36, 39].

### 3 IRGC and Uncertainty

Solving under uncertainty problems concerning interactive computations require to consider issues such as [6]: (i) changing attention in time relative to parts of complex objects which cannot be perceived as the whole at a given moment of time, (ii) values of compound attributes are computed on the basis of the delivered (e.g., by the agent control) parameters of sensors and recorded (in relevant information systems) results of sensory measurements, and (iii) interaction with the environment may cause different results of actions than the predicted ones.

#### 3.1 Complex Granules and Computations Over Complex Granules

Any c-granule of a given agent specifies a perceived structure of local environment of portions of matter (physical objects), called hunks [5]. There are three kinds of such hunks: (i) hunks in the agent external environment creating the *hard\_suit* of c-granule, (ii) internal hunks of agent, creating the *soft\_suit* of c-granule, some of which can be represented by agent as *information granules* (*infogranules*, for short), and (iii) hunks creating the *link\_suit* of c-granule and playing the role of links between hunks from the *hard\_suit* and *soft\_suit*. This structure is used in recording by means of infogranules, the results of interactions of hunks from the local environment [6, 10].

Any atomic infogranule  $g$  of c-granule can be treated as a hunk  $h_g$  with states encoded by objects such as numbers or words. Objects encoding the states of  $h_g$  are possible values of  $g$  (or  $h_g$ ). More formally, one can treat  $h_g$  as a collection of hunks consisting of values of  $h_g$ . In the interaction of  $h_g$  with the local environment of c-granule, the hunk encoding the relevant state is selected from  $h_g$ . More compound infogranules are obtained by relevant aggregation of already defined infogranules. This is a generalization of a notion of infogranules considered e.g., in [29, 36], where the values are assumed to be given while here they are obtained as the result of interaction processes in the local environment of c-granule.

One can distinguish two functionalities of each c-granule.

The first one, corresponding to the c-granule syntax frame, consists of the specification of the local environment structure. This structure, defined in the *soft\_suit* part of c-granule and represented by infogranules, consists of representations of hunks and links, defined in *link\_suit* part of c-granule, between the representations of hunks and corresponding to them parts of the physical world—creating the *hard\_suit* of c-granule. Roughly speaking, the structure of the local environment of c-granule describes structure of glasses of c-granule through which interactions of physical objects in the local environment of c-granule may be perceived. We assume that any c-granule has the ability to check if its local environment has the required structure.

The second one, corresponding to the c-granule semantics, is making it possible to record properties of processes such as degrees of satisfiability of features or values of

compound attributes. The processes are running in the perceived local environment with the structure of interacting hunks predefined for the c-granule by agent. Roughly speaking, the second part of c-granule is making it possible to record and process the relevant results of the perceived interactions, observed through the glasses of c-granule, in the local environment and next to provide them to other c-granules.

The above two functionalities of c-granule are making it possible to perceive by c-granule of interactions in its local environment of hunks.

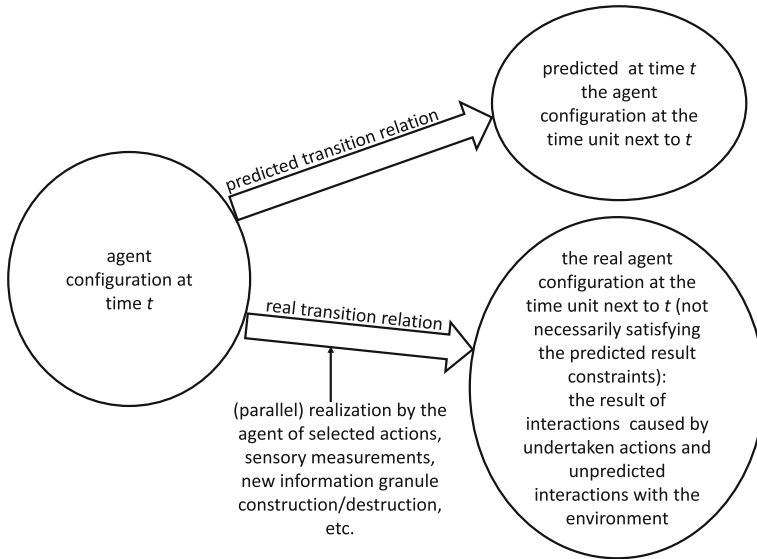
We also assume that to any c-granule may be assigned pair(s) of the form (pre-condition, post-condition). It is assumed that if c-granule is perceiving a structure satisfying the precondition (including interaction initiation), then the results of interactions of physical objects (perceived by means of properties of hunks from the local environment of c-granule) recorded by infogranules from the c-granule are expected (by control of agent) to satisfy the post-condition of the c-granule. One should remember that due to uncertainty, e.g., unpredictable interactions in the environment, the real results of interactions (which may be recorded only a posteriori) may differ from the expected ones. Roughly speaking, the pre-condition and the post-condition of c-granule are used to describe the expected changes of properties of the local environment of c-granule caused by interactions in this environment. The changes may be interpreted as the result of performing an action realized by c-granule. Actions (sensors or plans) represented by an agent in *link\_suits* of c-granules are used by the agent for exploration and/or exploitation of the environment on the way to achieve the targets. Again, due to the bounds of the agent perception abilities, usually only a partial information about the interactions in the physical world may be available for agents. Hence, in particular, the results of performed actions by agents cannot be predicted with certainty. This causes the necessity of adaptation of preconditions and post-conditions by c-granule or agents.

The above-described extension of c-granules by preconditions and post-conditions may be interpreted as a delegation by an agent of some control functionalities to c-granules and may take more advanced forms. For example, more compound c-granules may have more pairs (precondition, post-condition) which lead to a possibility of realization by c-granule packages or plans of actions together with some autonomy embedded by agent into c-granules concerning some control functionalities, e.g., in the process of selection-relevant actions for realization.

Any agent operates in its local world of c-granules by generating (or elimination) some c-granules and measuring the results of perceived interactions. The agent is aiming to control computations performed by c-granules from this local world for achieving the target goals.

The *transition relation* is usually defined between configurations of agent *ag* at succeeding moments of time. Any configuration of *ag* at time *t* (relative to the time clock of *ag*) consists of all c-granules being at time *t* under the agent control. It is worthwhile mentioning that the configuration at the time next to *t* cannot be defined *a priori* (at time *t*). Due to uncertainty, in particular, unpredictable interactions with the environment, the agent *ag* can only predict the next configuration and the real one, resulting due to interactions, can be perceived by *ag* at time  $t' > t$  when the results of interactions can be perceived by the agent using the relevant c-granules.





**Fig. 3** Two transition relations

Hence, we obtain two transition relations: the *predicted transition relation* and the *real transition relation* (see Fig. 3).

Note that the introduced model of interactive computations based on c-granules differs from the Turing model of computations. The results of computations based on c-granules depend on interactions of physical objects and linked to them information granules (also represented by means of physical objects).

Agents and societies of agents may also be represented as (generalized) c-granules. For more details on IRGC based on c-granules also on the agent architecture as well as on societies of agents and communication languages, the reader is referred to [6].

### 3.2 Agent Interactions and Communication Languages

Languages of agents consist of special c-granules called (*agent*) *expressions*. A *soft\_suit* of expression (treated as a c-granule) includes an infogranule corresponding to syntax of the expression as well as the specification of the local environment for the expression. The expression value (e.g., the satisfiability degree when expression is a formula) is computed using the functionalities of the expression (treated as a c-granule) concerning perception of interactions of hunks in its local environment. Note that some hunks of expression may belong to its *soft\_suit*, e.g., may belong to the agent memory or its “brain”.

The agents can create new names or expressions, e.g., for new structured objects or their indiscernibility (similarity) classes. Expressions from languages of agents consist of partial descriptions of situations (or their indiscernibility or similarity classes) perceived by agents using c-granules as well as description of approximate reasoning schemes about the situations and their changes caused by actions and/or plans. The situations may be represented in hierarchical modeling by structured objects (e.g., relational structures over attribute value vectors and/or indiscernibility (similarity classes) of such structures [40]).

From the point of view of dealing with uncertainty, it is important to observe that any expression usually represents classes of hunks [5] rather than a single hunk. This follows from the fact that the agents have bounded abilities on discerning of perceived objects. Also more compound expressions, e.g., representing different behavioral patterns may be indiscernible relative to the set of attributes used by the agent. Hence, it follows that the agents perceive in the same way objects belonging to the same indiscernibility and/or similarity class. This is an important feature allowing agents to use generalization. For example, a new, unseen so far, situation may be matched and classified to the perceived indiscernibility classes what allows agents to use strategies of generalization.

In reasoning about the situation changes [37], one should take into account that the predicted actions and/or plans may depend not only on the changes recognized in the past situations but also on the performed actions and plans in the past. This is strongly related to the idea of perception pointed out in [19]:

The main idea of this book is that perceiving is a way of acting. It is something we do. Think of a blind person tap-tapping his or her way around a cluttered space, perceiving that space by touch, not all at once, but through time, by skillful probing and movement. This is or ought to be, our paradigm of what perceiving is.

Many challenging issues are related to the origin and evolution of communication languages of agents (see, e.g., [20]). Here, we present only a few preliminary comments on these issues. We assume that the agents can perceive behavioral patterns of other agents or their groups, and based on this they can try to exchange some messages [18]. It is worthwhile mentioning that at the beginning, agents do not have common understanding of the meaning of such messages. In the consequence, this leads to misunderstanding, uncomfortable situation for agents. However, after series of trials in a dialogues they have a chance to set up common meaning of some behavioral patterns. In other words, they start to create common c-granules which use fixed in dialogues links to other hunks or infogranules. For example, at the beginning the messages could be linked to warning situations or to identifications of some sources required for satisfiability of some agent needs. This kind of simple messages could be passed by very simple behavioral pattern. Next, based on these very simple behavioral patterns the agents can develop more compound messages related to c-granules corresponding to common plans of cooperation of group of agents or/and competition with other groups of agents. This very general framework could be implemented in many ways using different AI paradigms. Especially, many models from Natural Computing could be quite helpful (e.g., modification of cellular automata or evolutionary

programming). However, our proposal is to implement this general scheme by agents (using c-granules) built up on the hierarchies of interactive information (decision) systems linked to configurations of hunks. The approach based on rough sets is quite convenient for implementation by computers well prepared for manipulation of data tables.

Let us consider a simple example illustrating how the names may originate in the environment where agents are interacting. Let us assume that an agent possesses a metaphorically understood “brain” with the states represented by configurations of hunks. The brain of agent *ag* is involved in interaction processes (IP) with the local environment and is perceiving a hunk *h* in this environment. In effect, the brain of agent *ag* launches an interaction process IP’ with the environment. IP’ introduces a hunk *h’* which is an image of *h* and constitutes a compressed form of *h*. The hunk *h’* can be considered as a name for the hunk *h* in language of agent *ag*. The frequent perception by another agent of hunk structures constituting of co-occurrence of agent *ag* and hunks *h*, *h’* may lead to accepting *h’* as the name for *h* by this agent (see Fig.4).

The perception by an agent of hunk structures constituting expressions in its language and their aggregations leads to the creation of grammar rules. The agent learns the usage rules of its language through interaction with the environment.

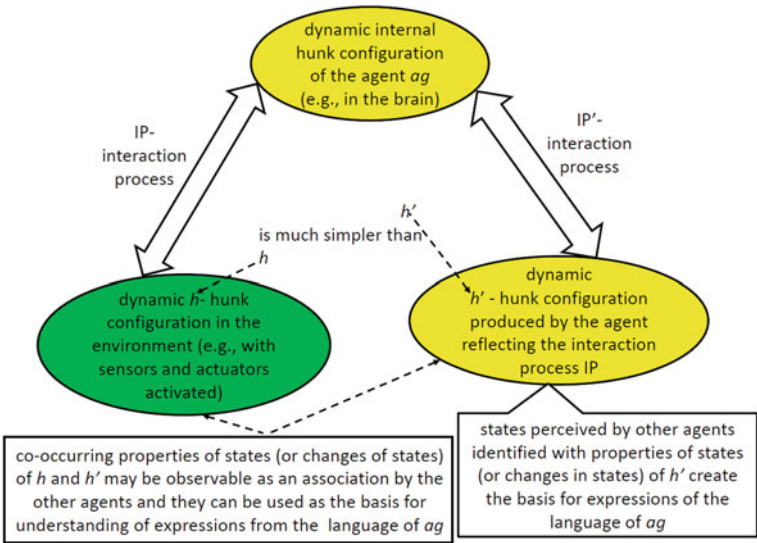


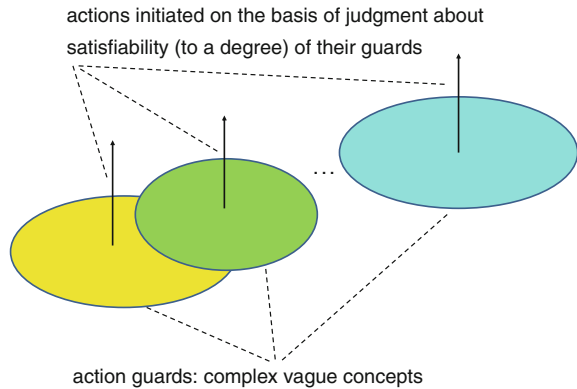
Fig. 4 Creating names

## 4 Adaptive Judgment: Toward Efficiency Management in Interactive Computations Over Complex Granules

The reasoning making it possible to derive relevant c-granules used for obtaining solutions of the target tasks is called *adaptive judgment*. *Intuitive judgment* and *rational judgment* are distinguished as different kinds of judgment [11]. Deduction and induction as well as abduction or analogy-based reasoning as well as reasoning for efficiency management are involved in adaptive judgment. Among the tasks for adaptive judgment are the following ones supporting reasoning under uncertainty toward: searching for relevant approximation spaces, discovery of new features, selection of relevant features, rule induction, discovery of inclusion measures, strategies for conflict resolution, adaptation of measures based on the minimum description length principle, reasoning about changes, perception (action and sensory) attributes selection, adaptation of quality measures over computations relative to agents, adaptation of object structures, discovery of relevant context, strategies for knowledge representation and interaction with knowledge bases, ontology acquisition and approximation, learning in dialogue of inclusion measures between information granules from different languages (e.g., the formal language of the system and the user natural language), strategies for adaptation of existing models, strategies for development and evolution of communication language among agents in distributed environments, strategies for efficiency management, e.g., risk management in distributed computational systems. Definitely, in the language used by agents for dealing with adaptive judgment (i.e., intuitive and rational) some deductive systems known from logic may be applied for reasoning about knowledge relative to closed worlds. This may happen, e.g., if the agent languages are based on classical mathematical logic. However, if we move to interactions in open worlds then new specific rules or patterns relative to a given agent or group of agents in such worlds should be discovered. The process of inducing such rules or patterns is influenced by uncertainty because they are induced by agents under uncertain and/or imperfect knowledge about the environment. Hence, considering only the absolute truth becomes unsatisfactory.

It is worthwhile mentioning that we propose to base adaptive judgment about interactive computations on complex granules not only on risk management (in particular, on risk assessment) but on a more general approach based on efficiency management using properly adopted well-known techniques such as SWOT analysis, Cost–Benefit Analysis (CBA) and others [6]. In the efficiency analysis, one should consider a variety of complex vague concepts and relations between them as well as reasoning schemes over such concepts and relations related, e.g., to the bow-tie diagram well known in the risk management area. To make such objects as vague concepts, relations among them as well as reasoning schemes over vague concepts “understandable” for agent control language, the relevant adaptive approximate methods for such objects should be developed. For example, the ontology approximation methodology was applied successfully in different real-life projects (see [1] and also the references in this paper).

**Fig. 5** Games based on complex vague concepts



One can consider the above-mentioned tasks of approximation of vague complex concepts initiating actions as the complex game discovery task (see Fig. 5) from data and domain knowledge. The agents are using the discovered games for achieving their targets in the environment. The discovery process can be based on hierarchical learning supported by domain knowledge [1, 6]. Such games are evolving in time (drifting in time) together with data and knowledge about the approximated concepts. The relevant adaptive strategies for adapting the games to changes perceived by agents are required. These adaptive strategies are used to control the behavior of agents toward achieving by them targets. Note that also these strategies should be learned from data and domain knowledge.

One can observe that some of the discussed tasks such as conflict resolution among classifiers initiating actions voting for decisions or efficiency management require an extension beyond the approaches based on ontologies. This extension requires usage of relevant fragments of natural language. In such fragments, one can express reasoning performed by humans based on concepts and relations from a given ontology. The challenge is how artificial agents can learn to perform approximate reasoning consistent to a satisfactory degree with reasoning performed by humans in those fragments of natural language. This challenge is related to the following sentences formulated by Pearl [28]:

Traditional statistics is strong in devising ways of describing data and inferring distributional parameters from sample. Causal inference requires two additional ingredients:

1. a science-friendly language for articulating causal knowledge, and
2. a mathematical machinery for processing that knowledge, combining it with data and drawing new causal conclusions about a phenomenon.

The analogous idea was also formulated by Lotfi Zadeh in the framework of computing with words (see, e.g., [46–50]).

Issues related to interactions among objects in the physical and mental worlds as well as adaptive judgment belong to the fundamental issues in Wisdom Technology (WisTech) [6–9] based on the following meta-equation:

$$\text{WISDOM} = \text{INTERACTIONS} + \text{ADAPTIVE JUDGEMENT} + \text{KNOWLEDGE.} \quad (2)$$

## 5 Conclusions

We discussed some basic issues related to dealing with uncertainty in the rough set approach and in IRGC. The outlined research on the nature of interactive computations is crucial for understanding complex systems. Our approach is based on complex granules (c-granules) on which agents are performing interactive computations. More compound granules represent agents and societies of agents. Computations over c-granules are controlled by the agent control. We emphasized the role of risk management and other techniques from management theory in IRGC. In our research, we plan to further develop the foundations of interactive computations based on c-granules toward tools for modeling and analysis of computations in Natural Computing [34], Wisdom Web of Things [51], or Cyber-Physical Systems [14].

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