Chapter 2
Objective Functions

Abstract The optimal design of a canal consists of minimization of an objective function which is subjected to certain constraints. The known parameters are flow discharge, longitudinal bed slope of canal, and the canal surface roughness. There are various objective functions such as flow area, earthwork cost, lining cost, seepage loss, evaporation loss, and their combinations. This chapter describes geometric properties and seepage loss functions of commonly used channel sections as well as computation of lining cost, earthwork cost, cost of water lost as seepage and evaporation loss, and capitalized cost. A unification of all these costs results in cost function of rigid boundary canals. A natural channel is a stream in equilibrium, which is neither silting nor scouring over a period of time. Such a stable channel develops a cross-sectional area of flow through natural processes of deposition and scour. Using Lacey's equations for stable channel geometry and using geometric programming, an objective function for stable alluvial channels can be synthesized. Thus, this chapter formulates objective functions for rigid boundary canals and mobile boundary (natural) canals.

Keywords Natural channel • Stable channel • Cost function • Objective function • Annuity • Capitalization • Seepage loss • Evaporation loss • Geometric properties • Lining cost • Earthwork cost • Cost of water lost

A design problem, like a canal design, is an optimization problem where an objective function is minimized subject to various constraints. In water resource projects, this function is cost-benefit ratio. Here, as benefits are constant, it is sufficient to minimize cost. The cost consists of capital costs and recurring costs. Costs of land, excavation, and lining are capital costs, whereas cost maintenance and water loss on account of seepage and evaporation are recurring costs. In this book, the cost of land is not considered as it involves alignment problem involving topography. Thus, the design of canal section is considered with known entities being flow discharge, longitudinal canal bed slope, and the canal surface roughness. As the general problem is an involved one, sometimes it is simplified by considering the flow area being the objective function. Such a canal is a minimum area or maximum velocity canal or best hydraulic section. The best hydraulic section has the minimum flow area and flow perimeter for a given discharge but not necessarily

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the most economical section. A network of canals represents a major cost item in an irrigation project, and the economy of the canal network is vital. The maximum economy is achieved by minimizing the cost of the canals. The design of minimum cost irrigation canals involves minimization of the sum of earthwork cost which varies with canal depth, cost of lining, and cost of water lost as seepage and evaporation subject to uniform flow condition in the canal. Such minimum cost canal design problem results in nonlinear objective function and nonlinear equality constraint, making the problem hard to solve analytically. In the book, the earthwork and the lining costs have been considered for the flow section only.

2.1 Flow Area

Figure 2.1 depicts commonly used canal sections. Triangular sections are generally constructed for carrying small discharges. For a triangular section (Fig. 2.1a), the flow area A is given by

$$A = my_n^2 \tag{2.1.1}$$

where $m =$ side slope and $y_n =$ normal depth (m). Rectangular sections are generally constructed for moderate discharges. For a rectangular section of bed width b (m) (see Fig. 2.1b), the flow area is

$$A = by_n \tag{2.1.2}$$

For carrying large discharges, rectangular sections are not preferred. This is on account of stability of side slopes. Vertical side walls require large thickness to resist the earth pressure. On the other hand, sloping side walls require less thickness. For a trapezoidal section of bed width b and side slope m (see Fig. 2.1c), the flow area is

$$A = y_n \left(b + my_n\right) \tag{2.1.3}$$

For small discharges, semicircular sections are often adopted for irrigation canals. For a circular section of diameter D (m) (see Fig. 2.1d), the flow area is

$$A = 0.25D^2 \left[\cos^{-1}\left(1 - 2\eta_n\right) - 2\left(1 - 2\eta_n\right)\sqrt{\eta_n\left(1 - \eta_n\right)}\right] \tag{2.1.4}$$

where $\eta_n = y_n/D$. A power law section is described by

$$Y = \left|k_p X\right|^p \tag{2.1.5}$$

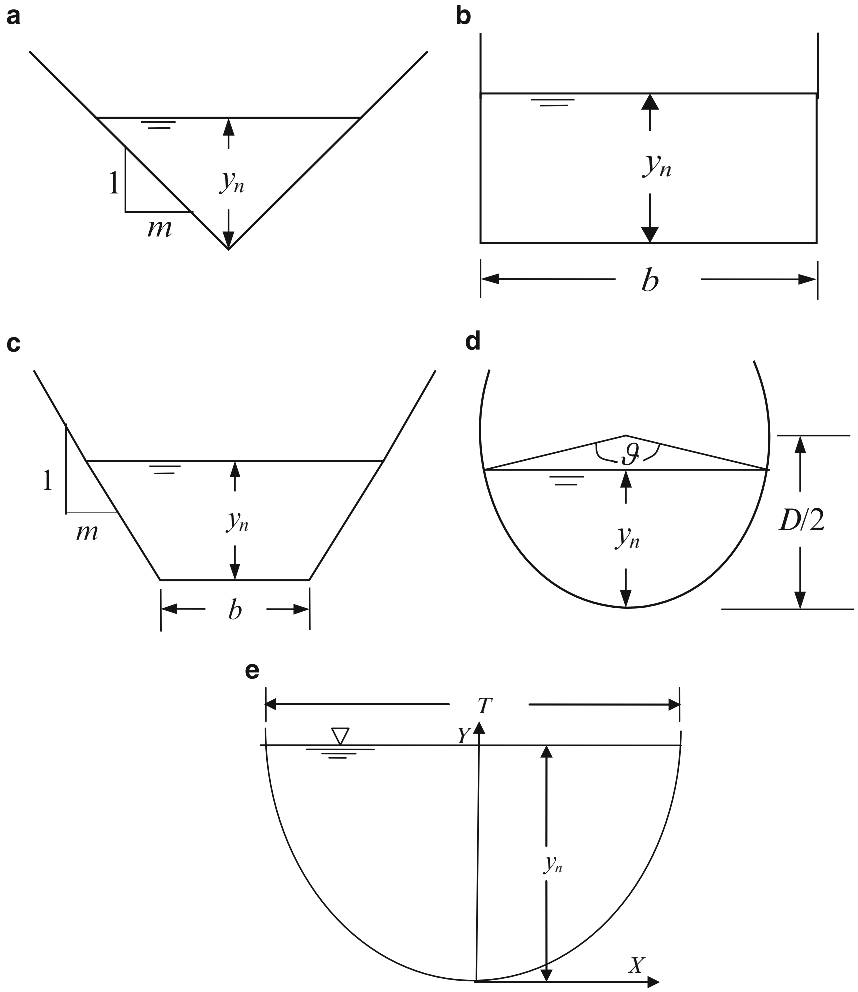

Fig. 2.1 Canal sections: (**a**) triangular, (**b**) rectangular, (**c**) trapezoidal, (**d**) circular, and (**e**) parabolic

where X and Y = horizontal and vertical coordinates, respectively, as shown in Fig. 2.1e, k_p = coefficient, and p = exponent. For $p = 1$, the exponential section is a triangle of side slope $m = 1/k_p$; for $p = 2$, it is a parabola of latus rectum $1/k_p^2$. For $p = \infty$, $Y = 0$ when $\left|k_p X < 1\right|$ and $Y = \infty$ when $|k_p X|$ just exceeds unity. Thus, a rectangle of bed width $2/k_p$ is obtained. The width of water surface for the power law section is given by

$$T = \frac{2}{k_p} y_n^{1/p} \tag{2.1.6}$$

The area of the section for depth y_n is given by

$$A = \frac{2py_n^{(p+1)/p}}{k_p\,(p+1)} \tag{2.1.7}$$

For $p = 2$ in Eq. (2.1.5), the power law section becomes a parabola having the equation

$$Y = k_p^2 X^2 \tag{2.1.8}$$

This parabola has latus rectum $a = 1/k_p^2$. The area of this section is given by

$$A = \frac{4y_n^{3/2}}{3k_p} = \frac{4}{3} y_n \sqrt{a y_n} \tag{2.1.9}$$

For $p = 0.5$ in Eq. (2.1.5), the power law section becomes an inverse parabola having the equation

$$Y = \sqrt{k_p X} \tag{2.1.10}$$

This parabola has latus rectum $a = k_p$. The area of this section is given by

$$A = \frac{2y_n^3}{3k_p} = \frac{2y_n^3}{3a} \tag{2.1.11}$$

On the other hand, for $p = \infty$, the section gets converted into a rectangle of bed width $2/k_p$. The area of this section is $2y_n/k_p$.

2.2 Lining Cost

The lining cost depends on the extent of canal surface area to be lined and the type of lining material to be used. Considering unit cost of lining (cost per unit surface area covered) to be independent of depth of placement, the cost of lining C_L (monetary unit per unit length of canal, e.g., ₹/m) is expressed as

$$C_L = c_L P \tag{2.2.1}$$

where c_L = unit cost of lining (monetary unit per unit area, e.g., ₹/m^2) and P = channel perimeter (m). Table 2.1 lists the perimeter of various shapes of a canal section. Once the perimeter is known, the following are the lining costs for various sections:

Table 2.1 Geometrical properties of canal sections

Section shape	Geometric elements		
	Flow perimeter	Area of flow	Depth of centroid of area
	P	A	$\overline{y}$
(1)	(2)	(3)	(4)
Triangular	$2y_n\sqrt{1+m^2}$	my_n^2	$\frac{y_n}{3}$
Rectangular	$b+2y_n$	by_n	$\frac{y_n}{2}$
Trapezoidal	$b+2y_n\sqrt{1+m^2}$	$(b+my_n)\,y_n$	$\frac{y_n}{6}\left(\frac{3b+2my_n}{b+my_n}\right)$
Circular	$0.5D\vartheta$	$\frac{D^2}{8}(\vartheta-\sin\vartheta)$	$\frac{D}{6}\left(\frac{6\,\sin\frac{\vartheta}{2}-3\,\cos\frac{\vartheta}{2}-2\,\sin^3\frac{\vartheta}{2}}{\vartheta-\sin\,\vartheta}\right)$
Parabolic	$\frac{a}{2}\left[2\sqrt{\eta_n\,(1+4\eta_n)} + \ln\left(2\sqrt{\eta_n} + \sqrt{1+4\eta_n}\right)\right]$	$\frac{4}{3}y_n\sqrt{ay_n}$	$\frac{2}{5}y_n$

where $\vartheta = 2\cos^{-1}(1-2\eta_n)$, wherein $\eta_n = y_n/D$ for circle and $\eta_n = y_n/a$ for parabola

$$C_L = 2c_L y_n\sqrt{1+m^2} \quad \text{triangular section} \tag{2.2.2}$$

$$C_L = c_L\,(b+2y_n) \quad \text{rectangular section} \tag{2.2.3}$$

$$C_L = c_L\left(b+2y_n\sqrt{1+m^2}\right) \quad \text{trapezoidal section} \tag{2.2.4}$$

$$C_L = c_L D\cos^{-1}(1-2\eta_n) \quad \text{circular section} \tag{2.2.5}$$

$$C_L = \frac{c_L a}{2}\left[2\sqrt{\eta_n\,(1+4\eta_n)} + \ln\left(2\sqrt{\eta_n}+\sqrt{1+4\eta_n}\right)\right] \quad \text{parabolic section} \tag{2.2.6}$$

where $\eta_n = y_n/D$ for circular section and y_n/a for parabolic section.

2.3 Earthwork Cost

Earthwork in the form of cutting and/or filling along the canal alignment is required for providing canal flow area. Earthwork cost is the major cost item for a canal passing through hard/firm strata, where lining may not be required. Sometimes canals are lined with low-cost lining materials, in which case the cost of the earthwork is more significant than the cost of lining. The cost of earthwork depends on the volume and depth of cut and fill. It also depends on the strata to be excavated and the distance of haulage if required in transporting the soil materials. The cost

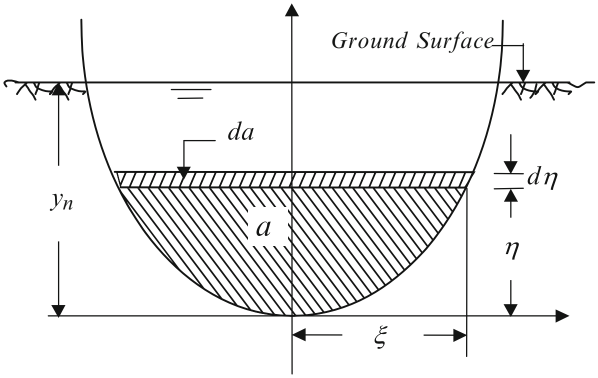

Fig. 2.2 Definition sketch for earthwork cost

function consists of earthwork cost of unit length of the canal. For a canal section with the normal water surface at the average ground level as shown in Fig. 2.2, the earthwork cost C_e (monetary unit per unit length of canal, e.g., ₹/m) is given by

$$C_e = c_e A + c_r \int_{\text{Area}} (y_n - \eta)\, da = c_e A + c_r \int_0^{y_n} (y_n - \eta)\, 2\xi \; d\eta \qquad (2.3.1)$$

where c_e = cost per unit volume of earthwork at ground level (₹/m^3), c_r = the additional cost per unit volume of excavation per unit depth (₹/m^4), η = vertical ordinate, ξ = half the width of excavation at the ordinate η, $d\eta$ = incremental vertical ordinate, a = excavation area up to height η, and da = incremental excavation area. It was assumed in Eq. (2.3.1) that the cost per unit volume of excavation is linear function of the depth of excavation. Integrating Eq. (2.3.1) by parts resulted in

$$C_e = c_e A + c_r |a\,(y_n - \eta)|_0^{y_n} + c_r \int_0^{y_n} a\,d\eta = c_e A + c_r A\overline{y} \qquad (2.3.2)$$

where $\overline{y}$ = depth (m) of the centroid of the area of excavation from the ground surface. Table 2.1 lists A and $\overline{y}$ for triangular, rectangular, trapezoidal, circular, and parabolic canal sections.

2.4 Annual Water Loss Cost

Annual cost of water loss per unit length of canal consists of costs of annual seepage loss and annual evaporation loss from one meter length of the canal. In Sects. 2.4.1 and 2.4.2 given below, the water loss per second is described.

2.4.1 Seepage Loss

The seepage loss from irrigation canals constitutes a substantial part of usable water. According to the Indian Bureau of Standards (1980), the loss of water by seepage from unlined canals in India generally varies from 0.3 to 7.0 m^3/s per 10^6 m^2 of wetted surface. Providing perfect lining can prevent seepage loss from canals, but cracks in lining develop due to several reasons and performance of canal lining deteriorates with time. An examination of canals by Wachyan and Rushton (1987) indicated that even with the greatest care, the lining does not remain perfect. A well-maintained canal with 99 % perfect lining reduces seepage about 30–40 % only (Wachyan and Rushton 1987). Thus, significant seepage losses occur from a canal even if it is lined. The seepage loss from canals is primarily governed by hydraulic conductivity of the subsoil, canal geometry, and potential difference between the canal and the aquifer underneath which in turn depends on the initial and boundary conditions. Seepage losses are also influenced by clogging of the canal surfaces depending on the suspended sediment content of the water and on the grain-size distribution of the suspended sediment particles. The clogging process can decrease the seepage discharge both through bottom and slopes. Thus, the seepage loss can change within time, and under certain conditions, it can diminish. Therefore, the seepage loss can be higher at the beginning of the canal operation and can be lower after a few years of operation.

The steady seepage loss from an unlined or a cracked lined canal complies with Darcy's law. Swamee et al. (2000) expressed the analytic solutions of seepage loss in the following simple form:

$$q_s = k y_n F_s \tag{2.4.1}$$

where q_s = seepage loss per unit length of canal (m^2/s), k = hydraulic conductivity of porous medium (m/s), and F_s = seepage function, which is a function of channel geometry and boundary condition. Depending upon the geometry of the flow domain, one of the following boundary conditions may exist: (i) porous medium underlain by an impermeable layer at a finite depth; (ii) porous medium underlain by a drainage layer at a finite depth, and water table is above the top of the drainage layer; (iii) porous medium underlain by a drainage layer at finite depth, and water table is below the top of the drainage layer; (iv) water table at a finite depth in a porous medium of infinite depth; and (v) porous medium of infinite depth in which water table is at infinite depth or a drainage layer and water table are both at infinite depth. Case (iii) and case (v) are important in canal design considering seepage.

Porous medium having infinite depth: Analytic solutions for the seepage function for case (v) are available for known canal dimensions (Harr 1962; Morel-Seytoux 1964; Polubarinova-Kochina 1962). The analytical form of these solutions, which contain improper integrals and unknown implicit state variables, is not convenient in estimating seepage from the existing canals and in designing canals considering seepage loss. These methods have been simplified using numerical methods for easy

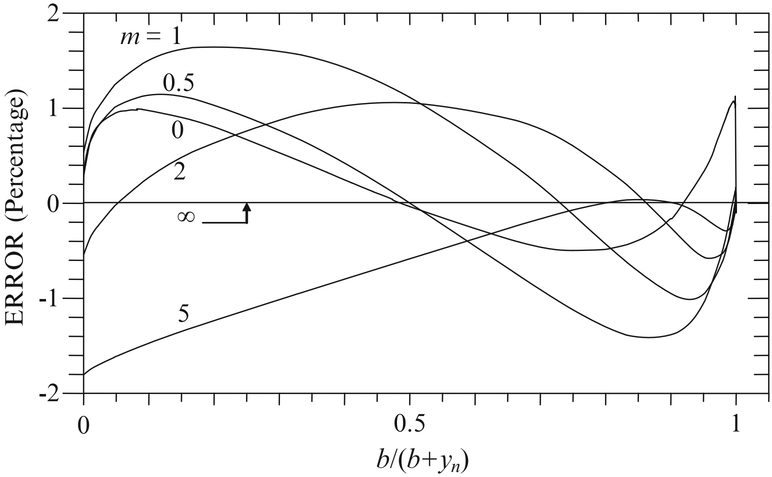

Fig. 2.3 Error diagram – Eq. (2.4.4)

computation of seepage function by Swamee et al. (2000). The simplified equations are as follows:

$$F_s = \left\{[\pi\,(4-\pi)]^{1.3} + (2m)^{1.3}\right\}^{0.77} \quad \text{triangular section} \tag{2.4.2}$$

$$F_s = \left\{[\pi\,(4-\pi)]^{0.77} + \left(\frac{b}{y_n}\right)^{0.77}\right\}^{1.3} \quad \text{rectangular section} \tag{2.4.3}$$

$$F_s = \left(\left\{[\pi\,(4-\pi)]^{1.3} + (2m)^{1.3}\right\}^{\frac{0.77+0.462m}{1.3+0.6m}} + \left(\frac{b}{y_n}\right)^{\frac{1+0.6m}{1.3+0.6m}}\right)^{\frac{1.3+0.6m}{1+0.6m}} \quad \text{trapezoidal section} \tag{2.4.4}$$

Figure 2.3 depicts the errors involved in Eq. (2.4.4) for computation of seepage from a trapezoidal canal. A perusal of Fig. 2.3 shows the maximum error as 1.8 % for the triangular section ($b = 0$). For the rectangular section ($m = 0$), the maximum error is within 1 %. The error in the practical range is less than 0.9 % for the triangular section ($0.5 \le m \le 2.5$), 0.5 % for the rectangular section ($0.5 \le b/y_n \le 10$), and 1.4 % for the trapezoidal section ($0.5 \le m \le 5$ and $0.5 \le b/y_n \le 10$).

A very narrow and deep channel is called a *slit*. Solution for seepage loss from a slit forms a particular case of the solutions given for triangular, rectangular, and trapezoidal canals. For a slit, the width T at water surface approaches zero, i.e., the ratio $T/y_n \to 0$, which means $m \to 0$ for triangular section, $b/y_n \to 0$ for rectangular channel, and both $m \to 0$ and $b/y_n \to 0$ for a trapezoidal canal. With these conditions Eqs. (2.4.2), (2.4.3), and (2.4.4) give seepage function for a slit as

$$F_s = \pi\,(4 - \pi) \tag{2.4.5}$$

which is near exact as the error is within 0.12 %, since the exact seepage function for a slit (Chahar 2000; Swamee et al. 2001b) is

$$F_s = \pi^2/(4G) \tag{2.4.6}$$

where $G =$ Catalan's constant. A *strip* is a reverse case of a slit, i.e., for a strip $b/y_n \to \infty$. This happens for a very wide and shallow channel. The seepage function for a strip channel is given by (Swamee et al. 2001b)

$$F_s = \frac{b}{y_n} + \frac{16G}{\pi^2} \tag{2.4.7}$$

Since b/y_n is very large in comparison to $16G/\pi^2$, Eq. (2.4.7) reduces to

$$F_s = b/y_n \tag{2.4.8}$$

Solution with limit $m \to \infty$ in Eq. (2.4.2) or $b/y \to \infty$ in Eq. (2.4.3) gives the seepage function for a strip as given by Eq. (2.4.8).

The analytic solutions for seepage discharge from canals pertain to triangular, rectangular, and trapezoidal sections which are polygonal sections. Apart from the usual assumptions of homogeneity and isotropy of the conducting porous medium, these solutions are mostly based on the assumption of an infinite depth drainage layer. At infinite depth, the streamlines become vertical; hence, the seepage width attains its potential (maximum) value, and seepage occurs under unit hydraulic gradient.

For a circular section, Swamee and Kashyap (2001) gave the following equation for seepage function:

$$\begin{aligned} F_s = \eta_n^{-1}\Bigg\{&\left[2\sqrt{\left(\eta_n^{-1}-1\right)} + 6.24\eta_n^{-1}\left(\eta_n^{-1}-1\right)^{-1.65}\right]^{-0.5} \\ &+0.584\left[\eta_n^{-1} + 3.55\eta_n^{-1}\left(\eta_n^{-1}-1\right)^{0.8}\right]^{-0.5}\Bigg\}^{-2} \end{aligned} \tag{2.4.9}$$

where $\eta_n = y_n/D$. For power law section with $p \geq 1$, the following equation is obtained (Swamee and Kashyap 2001):

$$F_s = \left\{ [\pi\,(4-\pi)]^{\frac{0.7p+0.3}{0.91p-0.14}} + \left(\frac{2}{k_p y_n^{(p-1)/p}} \right)^{\frac{0.7p+0.3}{0.91p-0.14}} \right\}^{\frac{0.91p-0.14}{0.7p+0.3}} \tag{2.4.10}$$

Similarly, for the exponent range $0.25 \le p \le 0.75$, the following equation for F_s was obtained:

$$F_s = \frac{2}{k_p} y_n^{(1-p)/p} \left\{ \left(1 + \frac{250}{1+0.8p} \phi^{-\frac{1}{1+8p}} \right)^{-\frac{2.5}{p}} + \left[1 + (0.1 + 1.6p)\, \phi^{-0.25p^{-2}} \right]^{-\frac{2.5}{p}} \right\}^{-0.4p} \tag{2.4.11}$$

where $\phi = y_n / k_p^{p/(1-p)}$. It can be seen that for $p = 1$, Eq. (2.4.10) is converted to

$$F_s = \left\{ [\pi\,(4-\pi)]^{1.3} + \left(\frac{2}{k_p} \right)^{1.3} \right\}^{0.77} \tag{2.4.12}$$

which is the same as Eq. (2.4.2). On the other hand, for $p = \infty$, Eq. (2.4.10) is changed to

$$F_s = \left\{ [\pi\,(4-\pi)]^{0.77} + \left(\frac{2}{k_p y_n} \right)^{0.77} \right\}^{1.3} \tag{2.4.13}$$

which is the same as Eq. (2.4.3). Setting $p = 2$ in Eq. (2.4.10), the parabolic section yielded the following equation:

$$F_s = \pi\,(4-\pi) + 2\sqrt{\frac{a}{y_n}} \tag{2.4.14}$$

Letting $p = 0.5$ in Eq. (2.4.11), the inverse parabolic section yielded the following equation:

$$F_s = \frac{2y_n}{a} \left\{ \left(1 + 179 \left(\frac{a}{y_n} \right)^3 \right)^{-5} + \left[1 + 0.9 \left(\frac{a}{y_n} \right)^{0.5} \right]^{-5} \right\}^{-0.2} \tag{2.4.15}$$

Drainage layer at a finite depth: Eqs. (2.4.2), (2.4.3), (2.4.4), (2.4.5), (2.4.6), (2.4.7), (2.4.8), (2.4.9), (2.4.10), (2.4.11), (2.4.12), (2.4.13), (2.4.14), and (2.4.15) are applicable for canals passing through a homogeneous and isotropic porous medium of infinite extent. However, in most alluvial plains, the soil is stratified. In many cases, highly permeable layers of sand and gravel underlie the top low permeable layer of finite depth. In that case, the lower layer of sand and gravel acts as a free

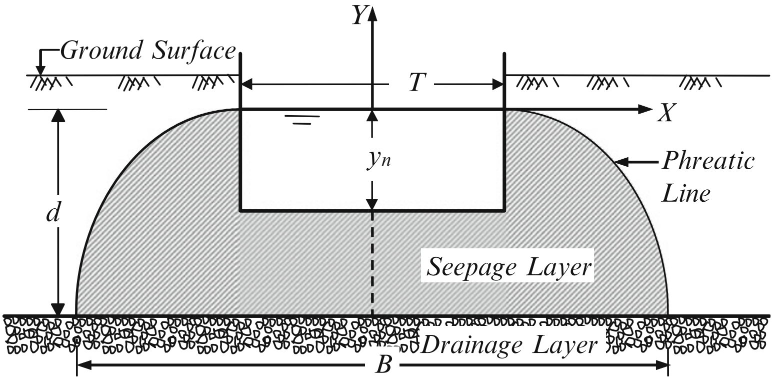

Fig. 2.4 Location of drainage layer

drainage layer for the top seepage layer. The seepage from a canal running through such stratified strata is much more than that in homogeneous medium of very large depth. The difference in quantity of seepage becomes appreciable when the drainage layer lies at a depth less than twice the depth of water in the canal. Swamee et al. (2001a) gave the following simple algebraic equations for the seepage function for polygon canal sections underlain by a drainage layer at depth d (see Fig. 2.4):

$$F_s = \left\{ \left(\frac{1.81m^{1.18} + 2.1}{(d/y_n - 1)^{0.26}} \right)^{9.35} + \left(\left(4\pi - \pi^2\right)^{1.3} + (2m)^{1.3} \right)^{7.2} \right\}^{0.107}$$

triangular section (2.4.16)

$$F_s = \left\{ \left(\frac{2.5(b/y_n)^{0.84} + 0.45}{(d/y_n - 1)^{0.69}} \right)^{2.38} + \left[\left(4\pi - \pi^2\right)^{0.77} + (b/y_n)^{0.77} \right]^{3.094} \right\}^{0.42}$$

rectangular section (2.4.17)

$$F_s = \left\{ \left(1.81 \left[m^{1.3} + 1.432 \left(\frac{b}{y_n} \right)^{0.93} \right]^{0.9} + \frac{b + 100 m y_n}{2.22b + 47.62 m y_n + 1.57 b m^5} \right)^{p_1} \right.$$
$$\left. \left(\frac{d}{y_n} - 1 \right)^{-p_1 p_2} + \left\{ \left[\left(4\pi - \pi^2\right)^{1.3} + (2m)^{1.3} \right]^{0.77 p_3} + \left(\frac{b}{y_n} \right)^{p_3} \right\}^{\frac{p_1}{p_3}} \right\}^{\frac{1}{p_1}}$$

trapezoidal section (2.4.18)

where

$$p_1 = \frac{2.38b + 7.48my_n}{b + 0.8my_n}; \quad p_2 = \frac{0.318b + 0.26my_n}{0.461b + my_n}; \quad \text{and} \quad p_3 = \frac{1 + 0.6m}{1.3 + 0.6m} \tag{2.4.19}$$

On the other end, for shallow depth of drainage layer, Swamee and Kashyap (2004) obtained the following equation for a circular channel:

$$F_s = \eta_n^{-1} \left((\eta_n F_{s\infty})^{6.25} + \left\{ \frac{0.87\eta_n^{-0.08} + 1}{[(d/y_n) - 1]^{0.5}} \right\}^{6.25} \right)^{0.16} \tag{2.4.20}$$

where $F_{s\infty}$ = seepage function for circular section with infinite depth of drainage layer given by Eq. (2.4.9). Swamee and Kashyap (2004) gave the following equation for seepage function for a parabolic section:

$$F_s = \frac{y_n^4 B_0 + (d - y_n)^4 B_\infty}{y_n \left[y_n^4 + (d - y_n)^4 \right]} \tag{2.4.21}$$

where

$$B_0 = \frac{1.27a^{1.17} + 2.7y_n^{1.17}}{(d - y_n)^{0.17}} \tag{2.4.22}$$

$$B_\infty = \pi (4 - \pi) y_n + 2\sqrt{ay_n} \tag{2.4.23}$$

As $d \to \infty$, Eqs. (2.4.16), (2.4.17), (2.4.18), (2.4.20), and (2.4.21) become functions of canal geometry only and reduce to the seepage functions for canals passing through a homogeneous porous medium of infinite depth. The drainage layer can be assumed at an infinite depth when $d \geq T + 3y_n$.

2.4.2 Evaporation Loss

Evaporation loss depends on (1) the supply of energy to provide the latent heat of vaporization and (2) the ability to transport the vapor away from the evaporating surface, which in turn depends on the wind velocity over the surface and the specific humidity gradient in the air above the water surface. A large number of equations for estimating evaporative rate are available in the literature. A review indicated that these equations fall into the following categories: (a) energy balance equations, (b) mass transfer equations, and (c) combination of the two. The energy balance equations require a variety of climatological data. The need of sophisticated equipment for direct measurement of radiation, frequent temperature surveys for heat storage, etc., make the method unattractive. On the other hand, the mass transfer equations are most convenient and useful for determining evaporation from flowing canals. The mass transport-type equations are expressed as

$$E = (e_s - e_d)\, f_w \tag{2.4.24}$$

where E = evaporation discharge per unit free surface area, e_s = saturation vapor pressure of the air at the temperature of the water surface, e_d = saturation vapor pressure of the air at the dew point, and f_w = wind function. The difference between the saturation vapor pressure of the air at the temperature of water surface and at the dew point ($e_s - e_d$) was given by Cuenca (1989) as

$$e_s - e_d = 610.78\left[\exp\left(\frac{17.27\theta_w}{237.3+\theta_w}\right) - R_h\ \exp\left(\frac{17.27\theta_a}{237.3+\theta_a}\right)\right] \tag{2.4.25}$$

where θ_w = water surface temperature in °C, θ_a = mean air temperature in °C, and R_h = relative humidity expressed as fraction. The wind function for a flowing channel in m/s per Pa was given by Fulford and Sturm (1984) as

$$f_w = 3.704 \times 10^{-11}\,(1 + 0.25u_2) \tag{2.4.26}$$

where u_2 = wind velocity in m/s at 2 m above the water surface. Combining Eqs. (2.4.24), (2.4.25), and (2.4.26), E in m/s is obtained as

$$E = 2.262 \times 10^{-8}\,(1 + 0.25u_2)\ \left[\exp\left(\frac{17.27\theta_w}{237.3+\theta_w}\right) - R_h\ \exp\left(\frac{17.27\theta_a}{237.3+\theta_a}\right)\right] \tag{2.4.27}$$

Equation (2.4.27) shows that in the simplest form of mass transfer approach, E is a function of the wind velocity over the evaporating surface, the water surface temperature, the air temperature, and relative humidity of the air above the water surface, though it may be affected by many other factors. Once E is known, the evaporation loss from a canal can be expressed as

$$q_e = ET \tag{2.4.28}$$

where q_e = evaporation discharge per unit length of canal (m^2/s).

2.4.3 Annual Cost of Total Water Loss

Adding Eqs. (2.4.1) and (2.4.28), the water loss per unit length of canal q_w (m^2/s) becomes

$$q_w = ky_nF_s + ET \tag{2.4.29}$$

The water loss m^3/m/year is $60 \times 60 \times 24 \times 365 q_w = 3.1536 \times 10^7 q_w$. Considering c_w as the cost of 1 m^3 of water and using Eq. (2.4.29), the annual cost of water loss (A_w) is $3.1536 \times 10^7 c_w q_w$, thus

$$A_w = 3.1536 \times 10^7 c_w \left(k y_n F_s + ET\right) \tag{2.4.30}$$

It can be seen that whereas C_L and C_e are capital costs, A_w is an annual cost.

2.5 Unification of Costs

The costs of earthwork and lining are incurred at the time of construction of a canal project, whereas the costs of the repair and maintenance of the canal have to be incurred every year. A canal lining may last about 50–60 years. After the life of a component, for example, canal lining, is over, it has to be replaced. The replacement cost has also to be considered as an additional recurring cost. Thus, there are two types of costs: (1) capital cost or the initial investment which has to be incurred for commissioning of the project and (2) the recurring cost which has to be incurred continuously for keeping the project in the operating condition.

These two types of costs cannot be simply added to find the overall cost. These costs have to be brought to same units before they can be added. For combining these costs, the methods generally used are the capitalization method and the annuity method. These methods are described below.

2.5.1 Capitalization Method

In this method, the recurring costs are converted to capital costs. This method finds out the amount of money to be kept in a bank yielding an annual interest equal to the annual recurring cost. If an amount C_A is kept in a bank with an annual interest rate of landing r per unit of money, the annual interest on the amount will be rC_A. Equating the annual interest to the annual recurring cost A_r, the capitalized cost C_A is obtained as

$$C_A = \frac{A_r}{r} \tag{2.5.1}$$

A component of a canal network has a finite life T_L. The replacement cost C_R has to be kept in a bank for T_L year so that its interest is sufficient to get the new component. If the original cost of a component is C_0, by selling the component after T_L year as a scrap, an amount αC_0 is recovered, where $\alpha =$ salvage factor. Thus, the net liability after T_L year is

$$C_N = (1-\alpha)\,C_0 \tag{2.5.2}$$

On the other hand, the amount C_R with interest rate r yields the compound interest I_R given by

$$I_R = \left\{(1+r)^{T_L} - 1\right\}\, C_R \tag{2.5.3}$$

Equating I_R and C_N, the replacement cost is obtained as

$$C_R = \frac{(1-\alpha)\,C_0}{(1+r)^{T_L} - 1} \tag{2.5.4}$$

Denoting the annual maintenance factor as β, the annual maintenance cost is given by βC_0. Using Eq. (2.5.1), the capitalized cost of maintenance C_{ma} works out to be

$$C_{ma} = \frac{\beta C_0}{r} \tag{2.5.5}$$

Adding C_0, C_R, and C_{ma}, the overall capitalized cost C_c is obtained as

$$C_c = C_0\left[1 + \frac{1-\alpha}{(1+r)^{T_L} - 1} + \frac{\beta}{r}\right] \tag{2.5.6}$$

Using Eqs. (2.5.1) and (2.5.6), all types of costs can be capitalized to get the overall cost of the project.

Example 2.1 Find the overall cost of canal lining considering scrap factor $\alpha = 0.05$, maintenance factor $\beta = 0.01$, interest rate $r = 0.05$ ₹/₹/year, and life of the lining $T_L = 60$ years.

Solution Using Eq. (2.5.6), the overall cost of lining is

$$C_L = C_{L0}\left[1 + \frac{1-0.05}{(1+0.05)^{60} - 1} + \frac{0.01}{0.05}\right] = C_{L0}\,(1{+}0.0537{+}0.2) = 1.254 C_{L0}$$

That is, the overall cost of lining is 1.254 times the initial cost.

2.5.2 *Annuity Method*

This method converts the capital costs into recurring costs. The capital investment is assumed to be incurred by borrowing the money that has to be repaid in equal annual installments throughout the life of the component. These installments are paid along with the other recurring costs. The annual installments (called annuity) can be combined with the recurring costs to find the overall annual investment.

If annual installments A_r for the system replacement are deposited in a bank up to T_L year, the first installment grows to $A_r(1+r)^{T_L-1}$, the second installment to $A_r(1+r)^{T_L-2}$, and so on. Thus, all the installments after T_L year add to C_N given by

$$C_N = A_r\left[1 + (1+r) + (1+r)^2 + \cdots + (1+r)^{T_L-1}\right] \tag{2.5.7}$$

Summing up the geometric series, one gets

$$C_N = A_r\frac{(1+r)^{T_L}-1}{r} \tag{2.5.8}$$

Using Eqs. (2.5.2) and (2.5.8), A_r is obtained as

$$A_r = \frac{(1-\alpha)\,r}{(1+r)^{T_L}-1}C_0 \tag{2.5.9}$$

The annuity A_0 for the initial investment is given by

$$A_0 = rC_0 \tag{2.5.10}$$

Adding up A_0, A_r, and the annual maintenance cost βC_0, the annuity A is

$$A = rC_0\left[1 + \frac{1-\alpha}{(1+r)^{T_L}-1} + \frac{\beta}{r}\right] \tag{2.5.11}$$

Comparing Eqs. (2.5.6) and (2.5.10), it can be seen that the annuity is r times the capitalized cost. Thus, one can use either annuity or the capitalization method.

2.5.3 Cost Function

Using Eq. (2.5.1), the annual cost of water loss is converted to the capitalized cost of water loss C_w (₹/m) as

$$C_w = \frac{3.1536\times 10^7 c_w}{r}\,(ky_nF_s + ET) \tag{2.5.12}$$

Adding Eqs. (2.2.1), (2.3.1), and (2.5.12), the cost of canal per unit length C (₹/m) is obtained as

$$C = C_e + C_L + C_w = c_eA + c_rA\overline{y} + c_LP + 3.1536\times 10^7 c_w\,(ky_nF_s + ET)\,/\,r \tag{2.5.13}$$

The following terms are further defined:

$$c_{ws} = 3.1536 \times 10^7 k c_w / r \tag{2.5.14}$$

$$c_{wE} = 3.1536 \times 10^7 E c_w / r \tag{2.5.15}$$

Therefore, Eq. (2.5.13) becomes

$$C = c_e A + c_r \bar{A}_y + c_L P + c_{ws} F_s Y_n + c_{wE} T \tag{2.5.16}$$

and Eq. (2.2.12) converts to

$$C_w = C_{ws} y_n F + C_{wE} T \tag{2.5.17}$$

As c_L/c_e, c_e/c_r, and c_w/c_e have length dimension, they remain unaffected by the monetary unit chosen. These ratios can be obtained for various types of linings, soil strata, and climatic condition by using appropriate unit rates. Using "Schedule" (1997) and "UP" (1992), the c_L/c_e and c_e/c_r ratios were obtained for various types of linings and soil strata. The ratios are listed in Table 2.2.

Table 2.2 Lining and earthwork cost coefficient

Types of strata	c_L/c_e (m)									c_e/c_r (m)
	Type of lining									
	Concrete tile			Brick tile			Brunt clay tile			
	With LDPE film		Without film	With LDPE film		Without film	With LDPE film		Without film	
	100 μ	200 μ		100 μ	200 μ		100 μ	200 μ		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Ordinary soil	12.75	13.02	12.24	6.39	6.67	5.88	6.08	6.35	5.57	6.96
Hard soil	10.00	10.22	9.60	5.01	5.23	4.62	4.77	4.99	3.37	8.86
Impure lime nodules	8.90	9.10	8.55	4.47	4.66	4.11	4.25	4.44	3.89	9.96
Dry shoal with shingle	6.56	6.71	6.30	3.29	3.43	3.03	3.13	3.27	2.86	13.50
Slush and lahel	6.40	6.54	6.14	3.21	3.35	2.95	3.05	3.19	2.79	13.86

LDPE low density polyethylene

2.6 Stable Channel Objective Function

A stable channel is a stream in equilibrium that is neither silting nor scouring over a period of time. Obviously, such a stream has developed a cross-sectional area of flow through natural processes of deposition and scour. Lacey (1930) gave a set of empirical equations for flow area, flow perimeter, and bed slope. There has to be an objective function whose minimization yields Lacey's equations. Using Lacey's equations with geometric programming, Swamee (2000) synthesized the following objective function F_E for stable alluvial channels, which is energy spent per unit length of canal:

$$F_E = \frac{\rho g \nu^{1/9} d^{1/3} Q^{8/3}}{3[(s-1)g]^{8/9} A^{5/3} R^{4/3}} + 5.64 \times 10^{-4} \rho g \left\{ \frac{[(s-1)g]^2}{\nu} \right\}^{\frac{1}{3}} A d + 5.27 \times 10^{-4} \rho g \left\{ \frac{[(s-1)g]^{2.3}}{\nu^{1.6}} \right\}^{\frac{1}{3}} d^{1.1} A^{0.6} R \tag{2.6.1}$$

where $Q =$ discharge in channel, $\rho =$ mass density of water, $g =$ gravitational acceleration, $s =$ specific gravity of bed material, $R = A/P =$ hydraulic radius, and $d =$ bed material size expressed in m and $\nu =$ kinematic viscosity of fluid. The kinematic viscosity depends on the temperature of the fluid, which can be obtained using the equation given by Swamee (2004):

$$\nu = 1.792 \times 10^{-6} \left[1 + \left(\frac{T}{25} \right)^{1.165} \right]^{-1} \tag{2.6.2}$$

where $T =$ the water temperature in degrees Celsius. The minimum energy per unit length F_E^* corresponding to Eq. (2.6.1) is (Swamee 2000)

$$F_E^* = 0.009023 \rho g \left\{ \frac{[(s-1)\,g]^4}{\nu^5} \right\}^{\frac{1}{18}} d^{5/6} Q^{5/6} \tag{2.6.3}$$

For non-alluvial streams, the first term of Eq. (2.6.1), responsible for the flow maintenance, will depend upon the corresponding resistance law. Thus, the general form of the stable channel objective function is expressed as (Swamee 2000)

$$F_E = \rho g Q S_o + 5.64 \times 10^{-4} \rho g \left\{ \frac{[(s-1)\,g]^2}{\nu} \right\}^{\frac{1}{3}} A d + 5.27 \times 10^{-4} \rho g \left\{ \frac{[(s-1)\,g]^{2.3}}{\nu^{1.6}} \right\}^{\frac{1}{3}} d^{1.1} A^{0.6} R \tag{2.6.4}$$

where S_o = stream bed slope. For river Brahmaputra, F_E^* is found to be (Swamee et al. 2008)

$$F_E^* = 0.01011\rho g \left\{ \frac{[(s-1)\,g]^4}{\nu^5} \right\}^{\frac{1}{18}} d^{5/6} Q^{5/6} \tag{2.6.5}$$

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