

Preface

The book is designed for researchers and students working in the field of fuzzy set, rough set, soft set, and their hybrid models. In our real-life problems, there are situations with the uncertain data that may not be successfully modelled by the classical mathematics. There are some mathematical tools for dealing with uncertainties: fuzzy set theory, rough set theory, and soft set theory. The book is written in order to accumulate all the contents of generalised fuzzy theory and all hybrid structures of fuzzy set, rough set, and soft set, so that the researchers get all the information at one place. The primary goal of this book is to help bridge the gap to provide a textbook on the hybrid structures in fuzzy mathematics and their applications in social science.

The concept of ‘fuzzy set theory’ was first introduced by Lotfi A. Zadeh in 1965 (*Information and Control*, vol. 8, pp. 338–353) and thereafter by C.L. Chang (in Fuzzy topological spaces, *J. Math. Anal. Appl.*, vol. 24, pp. 182–190), paved the way of subsequent development of numerous fuzzy topological concepts. In 1983, Atanassov introduced the concept of ‘intuitionistic fuzzy set’ as a generalisation of the notion of a fuzzy set. Intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership, which are independent of each other. The only requirement is that the sum of these two degrees is not greater than 1. Using intuitionistic fuzzy sets, not only vagueness but also uncertainty is modelled. The concept of ‘rough set theory’, which was first introduced by Z. Pawlak in 1981/1982, deals with the approximation of sets that are difficult to describe with the available information. Rough set introduced by Z. Pawlak is expressed by a boundary region of a set. It is also an approach to vagueness. Thus, fuzzy sets and rough sets are two different approaches to vagueness or impreciseness of the real-life problems. The ‘soft set theory’, which was introduced by Molodtsov in 1999, takes care of the problem that involves such vagueness. In 2001, Maji et al. introduced the idea of intuitionistic fuzzy soft set theory and established some results on them. Theories of fuzzy sets and rough sets are powerful mathematical tools for modelling various types of uncertainty. Molodtsov [2] initiated a novel concept called soft sets, a new mathematical tool for dealing with uncertainties. It has been found that fuzzy set, rough set, and soft set are closely related

concepts [1]. Research works on soft sets are very active and progressing rapidly in these years.

The book introduces the concept and basic properties of ‘generalised interval-valued intuitionistic fuzzy soft sets’. It also discusses the application of generalised interval-valued intuitionistic fuzzy soft sets in decision making with respect to interval of degree of preference. The book also introduces the concept of ‘soft rough intuitionistic fuzzy sets’ and ‘interval-valued intuitionistic fuzzy soft rough sets’. The concept of interval-valued intuitionistic fuzzy soft rough set-based multi-criteria group decision-making scheme is presented, which refines the primary evaluation of the whole expert group and enables us to select the optimal object in a most reliable manner. The proposed scheme is to be illustrated by an example regarding the candidate selection problem.

The book also introduces the concept of ‘interval-valued intuitionistic fuzzy soft topological space (IVIFS topological space)’ together with intuitionistic fuzzy soft open sets (IVIFS open sets) and intuitionistic fuzzy soft closed sets (IVIFS closed sets). Then, we define neighbourhood of an IVIFS set, interior IVIFS set, interior of an IVIFS set, exterior IVIFS set, exterior of an IVIFS set, closure of an IVIFS set, IVIF soft basis, and IVIFS subspace. Some examples and theorems regarding these concepts are presented. The concepts of ‘interval-valued intuitionistic fuzzy soft multi-sets’ and that of ‘interval-valued intuitionistic neutrosophic soft sets’ are introduced and their applications are studied.

The book is organised in ten chapters. In Chap. 1, the basic concepts of fuzzy set, rough set, soft set, fuzzy topology, and intuitionistic fuzzy topology are given for ready reference. Also relation among fuzzy set, rough set, and soft set are shown with suitable examples. Here, soft multi-set and the concept of topological structure formed by soft multi-sets are also discussed. One of the areas in which fuzzy sets, rough sets, and soft sets have been applied most extensively is in modelling for managerial decision making. Thus, this area has been selected by us for consideration. In Chap. 2, the concept of generalised interval-valued intuitionistic fuzzy soft sets together with their basic properties is introduced. Besides, an application of generalised interval-valued intuitionistic fuzzy soft sets in decision making is also presented. In Chap. 3, soft rough intuitionistic fuzzy sets (SRIF sets) are also discussed. Finally, an example that demonstrated that this method can be successfully work is also provided here. It can be applied to problems of many fields that contain uncertainty. The aim of Chap. 4 is to introduce a new concept—interval-valued intuitionistic fuzzy soft rough sets. Also, interval-valued intuitionistic fuzzy soft rough set-based multi-criteria group decision-making scheme is presented. In Chap. 5, the concept of interval-valued intuitionistic fuzzy soft topology is introduced and their basic properties are studied. Chapter 6 introduces the concept of interval-valued intuitionistic fuzzy soft multi-sets and study of its properties and operations. The concept of interval-valued intuitionistic fuzzy soft multi-set relations (IVIFSMS relations for short) is also proposed. Besides, the basic properties of the IVIFSMS relations and various types of IVIFSMS relations are presented in this chapter. Chapter 7 introduces the concept of interval-valued neutrosophic set (IVNS), interval-valued neutrosophic soft set (IVNSS), and

interval-valued neutrosophic soft set (IVNSS) relations. Besides, the basic properties of IVNSS and IVNSS relations and various types of IVNSS relations are studied and also discussed in this chapter. In Chap. 8, the concept of fuzzy soft multi-topology is introduced and their basic properties are studied. The notion of fuzzy soft multi-points, fuzzy soft multi-open sets, fuzzy soft multi-closed sets, fuzzy soft multi-basis, fuzzy soft multi-sub basis, neighbourhoods and neighbourhood systems, and interior and closure of a fuzzy soft multi-set are introduced and their basic properties are investigated. It is shown that a fuzzy soft multi-topological space gives a parameterised family of fuzzy topological spaces. Here, the concepts of fuzzy soft multi-topological subspaces and some basic properties of these concepts are also explored. Lastly, the concept of fuzzy soft multi-compactness and fuzzy soft multi-separation axioms is introduced. The aim of Chap. 9 was to introduce the concept of soft interval-valued intuitionistic fuzzy rough sets. Also, some properties based on soft interval-valued intuitionistic fuzzy rough sets are presented here. Finally, a soft interval-valued intuitionistic fuzzy rough set-based multi-criteria group decision-making scheme is presented. The proposed scheme is illustrated by an example regarding the car selection problem. The aim of the Chap. 10 is to construct IF parameterised intuitionistic fuzzy soft set (IFPIFS set) decision-making problem and to solve the problem with IF parameterised intuitionistic fuzzy soft set theories.

In our real-life problems, there are situations with the uncertain data that may not be successfully modelled by the classical mathematics. There are some mathematical tools for dealing with uncertainties; they are fuzzy set theory introduced by Prof. Zadeh [4], rough set theory introduced by Pawlak [3], and soft set theory initiated by Molodtsov [2] that are related to our work.

I gratefully acknowledge the support provided by Springer for publishing this book.

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Generalized Rough Sets

Hybrid Structure and Applications

Mukherjee, A.

2015, XIII, 160 p. 1 illus., Hardcover

ISBN: 978-81-322-2457-0