Preface to the Second Edition

Most of the material discussed in the present revised, enlarged edition has appeared in the first edition of the book, *An Introduction to Ultrametric Summability Theory* (Springer, 2013). The first three chapters of the first edition have been retained in the second edition. In Chaps. 4–9 of the present edition, we present a survey of the literature on "ultrametric summability theory". We have supplemented substantially to the survey in the current edition. Our survey starts with a paper by Andree and Petersen of 1956 (it is the earliest known paper on the topic).

In Chap. 4, the Silverman–Toeplitz theorem is proved by using the "sliding hump method". Schur's theorem and Steinhaus theorem also find a mention here. It is proved that certain Steinhaus-type theorems fail to hold. An interesting characterization of infinite matrices in $(\ell_\alpha, \ell_\alpha)$, $\alpha > 0$ is proved. There is, as such, no classical analogue for this result. The core of a sequence and Knopp's core theorem are discussed. Towards the end of the chapter, an important result on Cauchy multiplication of series is proved. In Chap. 5, we introduce the Nörlund and Weighted mean methods in the ultrametric set-up, and their properties are elaborately discussed. We also show that the Mazur–Orlicz theorem and Brudno's theorem fail to hold in the ultrametric case. In Chap. 6, we introduce the Euler and Taylor methods and discuss their properties extensively. In Chap. 7, we prove Tauberian theorems for the Nörlund, the weighted mean and the Euler methods. In Chap. 8, we introduce double sequences and double series in ultrametric analysis. We prove Silverman–Toeplitz theorem for four-dimensional infinite matrices. We also prove Schur's and Steinhaus theorems for four-dimensional infinite matrices. Towards the end of the chapter, we obtain some interesting characterizations of two-dimensional Schur matrices. Finally, in Chap. 9, we introduce the Nörlund and the weighted mean methods for double sequences and make a detailed study of their properties. The author thanks E. Boopal for typing the manuscript.

Errors in the first edition, both typographical and conceptual, have been corrected in the present revised, enlarged edition.

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Preface to the First Edition

The purpose of the present monograph is to discuss briefly what summability theory is like when the underlying field is not $\mathbb{R}$ (the field of real numbers) or $\mathbb{C}$ (the field of complex numbers) but a field K with a non-Archimedean or ultrametric valuation, i.e., a mapping $|\cdot|: K \to \mathbb{R}$ satisfying the ultrametric inequality $|x+y| \leq \max(|x|, |y|)$ instead of the usual triangle inequality $|x+y| \leq |x| + |y|$, $x, y \in K$.

To make the monograph really useful to those who wish to take up the study of ultrametric summability theory and do some original work therein, some knowledge of real and complex analysis, functional analysis and summability theory over $\mathbb{R}$ or $\mathbb{C}$ is assumed.

Some of the basic properties of ultrametric fields—their topological structure and geometry—are discussed in Chap. 1. In this chapter, we introduce the p-adic valuation, p being prime and prove that any valuation of $\mathbb{Q}$ (the field of rational numbers) is either the trivial valuation, a p-adic valuation or a power of the usual absolute value $|\cdot|_\infty$ on $\mathbb{R}$, i.e., $|\cdot|_\infty^\alpha$, where $0 < \alpha \leq 1$. We discuss equivalent valuations too. In Chap. 2, we discuss some arithmetic and analysis in $\mathbb{Q}_p$, the p-adic field for a prime p. In Chap. 2, we also introduce the concepts of differentiability and derivatives in ultrametric analysis and very briefly indicate how ultrametric calculus is different from our usual calculus.

In Chap. 3, we speak of ultrametric Banach space, and also mention the many results of the classical Banach space theory, viz., the closed graph, the open mapping and the Banach-Steinhaus theorems carry over in the ultrametric set-up. However, the Hahn-Banach theorem fails to hold. To salvage the Hahn-Banach theorem, the concept of a "spherically complete field" is introduced and Ingleton's version of the Hahn-Banach theorem is proved. The lack of ordering in an ultrametric field K makes it quite difficult to find a substitute for classical "convexity". However, classical convexity is replaced, in the ultrametric setting, by a notion called "K-convexity", which is briefly discussed towards the end of the chapter.

In the main Chap. 4, our survey of the literature on "Ultrametric Summability Theory", starts with the paper of Andree and Petersen of 1956 (it was the earliest known paper on the topic) to the present. As far as the author of the present monograph knows, most of the material discussed in the survey has not appeared in book form earlier. Almost all of Chap. 4 consists entirely of the work of the author of the present monograph. Suitable references have been provided at appropriate places indicating where further developments may be found.

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