Chapter 2
Round Squares Are No Contradictions (Tutorial on Negation Contradiction and Opposition)

Jean-Yves Beziau

Abstract We investigate the notion of contradiction taking as a central point the idea of a round square. After discussing the question of images of contradiction, related to the contest *Picturing Contradiction*, we explain why from the point of view of the theory of opposition, a round square is not a contradiction. We then draw a parallel between different kinds of oppositions and different kinds of negations. We explain why from this perspective, when we have a paraconsistent negation $\neg$, the formulas p and $\neg p$ cannot be considered as forming a contradiction. We finally introduce the notions of paranormal negation and opposition which may catch the concept of a round square.

Keywords Contradiction · Opposition · Negation · Paraconsistency · Round square

Mathematics Subject Classification (2000) Primary 03B53 · Secondary 03A05 · 03B45 · 03B50 · 03B60 · 03B05

Paraconsistent logic helps to clarify the concepts of negation and contradiction. On the one hand there are authors for whom contradictions play a quasi-mystical role, used to explain nearly everything in the universe, on the other hand excellent specialists think that contradiction is something unintelligible. Paraconsistent logic not only is useful to demystify contradiction but contributes to calm anyone who is afraid of it. Newton da Costa [33].

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Fig. 2.1 Round square (J)

2.1 Picturing Contradiction

What is a contradiction? Contradiction is a famous notion. But do we have an idea, an image, or a definition of a contradiction? And what is the reality of contradiction, if any? In this paper we will investigate this notion considering the round square as a platform for developing a discussion about the trinity negation, contradiction, opposition.

What is a round square? A simple reply to this answer is the following Fig. 2.1

But this is not very satisfactory because this image is just a juxtaposition of a circle and a square. One may want to develop a logic of imagination considering $\bigcirc$ as a modal operator of imagination and taking as an axiom:

$$\bigcirc A \wedge \bigcirc B \rightarrow \bigcirc (A \wedge B)$$

But according to this logic of imagination, we can imagine lots of things.[1] It is not the same to imagine a man and a horse and a centaur, just compare classical mythology and modern mythology (Fig. 2.2).

Maybe the following image is a better representation of a round square, closer to the centaur construct, result of a blending (Fig. 2.3).

But according to standard plane geometry, this is indeed neither a square nor a circle. At the 5th World Congress on Paraconsistency in Kolkata, India, February 13–17, 2014, we organized the contest *Picturing Contradiction*. We asked people from all over the world to send us an image picturing contradiction. It was on the one hand a way to promote the participation of all the people, even those who were not able to come to Kolkata, and on the other hand a way to check if contradiction is not just a mere *flatus vocis*, if there is really something behind this word.

We received few interesting images. At the end the one which won the prize was entitled "Bridge to Nowhere," submitted by Daniel Strack, Associate Professor of

[1] The logic of imagination is still a quite new and open field. A starting point was a paper by Ilkka Niiniluoto in 1985 [44]; for a critical account of this paper see [30].

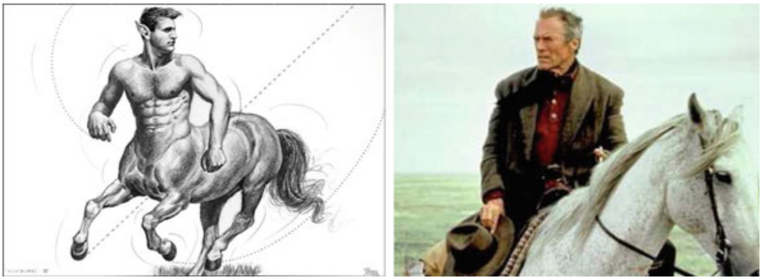


Fig. 2.2 Centaur versus man on a horse

Fig. 2.3 Round square (B)

the University of Kitakyushu, Japan (Fig. 2.4).[2] This is a juxtaposition of two objects representing two opposite ideas which are melting in some way, closer therefore to the second image of a round square above (but there the melting is purely material) rather than the first one.

One of the main themes of this 5th edition of the World Congress on Paraconsistency was quantum physics and we had chosen the Fig. 2.5 as a key image for the event (see, in particular, the web site http://www.paraconsistency.org/). This a poetic representation of the duality wave/particle. For the contest itself we chose the image Fig. 2.6, representing this duality in a still metaphoric but more conceptual way.

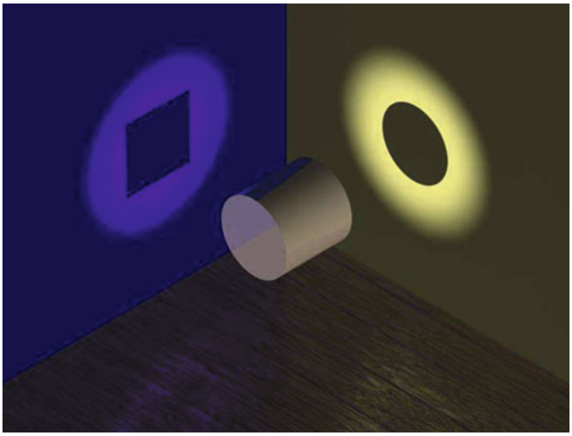
According to Fig. 2.7, the same object appears both as a circle and as a square. One could say that it is both a circle and as a square, from the point of view of 2-dimensional space. This figure corresponds to the spirit of the philosophy of David Bohm who has used the distinction between 2-dimensional and 3-dimensional space in various ways (see his book [26]), in particular to explain inseparability: a 3-dimensional fish is projected into two 2-dimensional fishes whose interaction seems difficult to understand at a the flat level.

[2]The president of the jury was Kuntal Ghosh, from the Indian Statistical Institute in Kolkata where the event was taking place.

Fig. 2.4 Bridge to nowhere

Fig. 2.5 Yemanjá playing with particles

Fig. 2.6 A geometrical metaphor for the duality wave particle



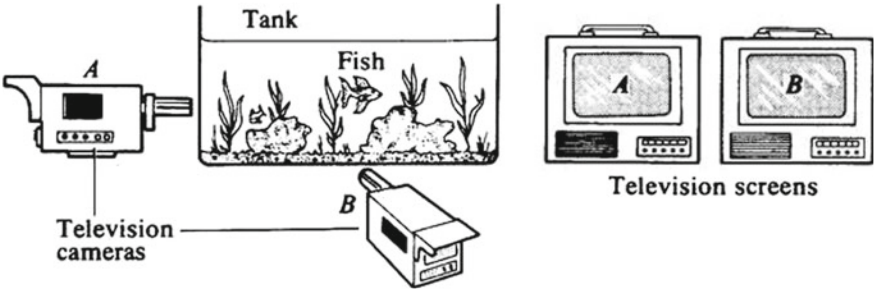


Fig. 2.7 David Bohm's metaphor for inseparability

Niels Bohr also had some ideas corresponding to Fig. 2.6. He wrote: "A complete elucidation of one and the same object may require diverse points of view which defy a unique description [27]." For both Bohm and Bohr the duality wave/particle can be interpreted as showing that reality is beyond wave and particle which are just appearances of it. This can be developed either in a Platonic perspective or in a Kantian perspective. The Kantian perspective has been emphasized by Bernard d'Espagnat (see, e.g., [34]), winner of the Templeton prize in 2009, under which I wrote a dissertation [3] at the Sorbonne in 1986 comparing Bohr, Heisenberg, and Bohm's views.[3]

In quantum physics we have a conceptual theory explaining reality but we do not have images of this reality. From this perspective one can argue that reality is beyond imagination, but that maybe our reason can catch it in some way. After developing the so-called Bohr's atom, inspired by the Rutherford's atom, a figure of microscopic reality establishing a parallel with macroscopic reality, Bohr rejected this approach and developed complementarity. He liked to wear on his jacket a picture of the Tao symbol. For him, this was not a picture of reality, but the symbol of his theory of complementarity (Fig. 2.8).

On the other hand Fig. 2.9 represents a more cosmic vision of the Tao, related with new age philosophy. It is not exactly clear what it means. The Tao can be interpreted as an intrinsic link between two contradictory notions, metaphorically represented by black and white. In Maoist philosophy, a blend of Marxism and Taosim, everything is inherently contradictory. Contradiction is understood as the unity and struggle of opposites and the law of contradiction is considered as the fundamental law governing nature and society. The unity and identity of all things is viewed as temporary and relative, while the struggle between opposites is considered as ceaseless and absolute (cf. Mao's 1937 essay *On contradiction* [43]). Such kind of theory, like the theory of evolution, can easily be used to justify war and conflict. First it is important to distinguish contradiction from conflict. Second we can consider that the world is always changing without seeing contradiction or/and conflict as a driving force. For

[3] I had the opportunity at this time to meet and discuss with David Bohm in London. After that I wrote a dissertation on Plato's cave [4] and later on I developed the paraconsistent logic Z inspired by Bohm's ideas. About this logic, see [9], and about how it was conceived, see [10].

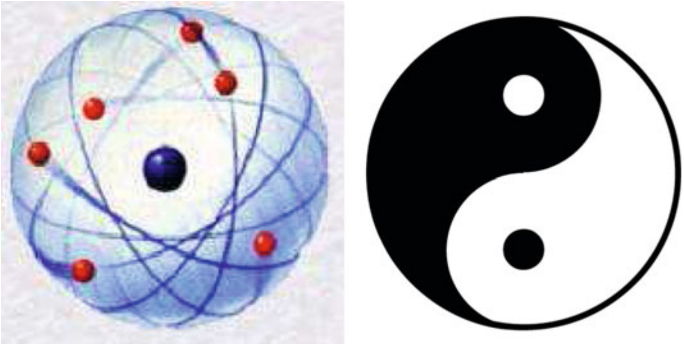


Fig. 2.8 Rutherford's atom versus the Tao of complementarity

Fig. 2.9 Taoist version of the universe

someone like Bergson contradiction is not the essence of reality but the result of the incapacity of our thought to catch the flux of reality, see e.g. [2].

2.2 Contradiction and the Square of Opposition

A standard and traditional definition of contradiction can be found in the square of opposition. Before entering into the details let us point out that we are using here the expression *square of opposition* as a name for the theory of opposition. This theory can be traced back to Aristotle, a no-square stage,[4] and is continuing to develop up to now, important stages in the development of this theory being the design of a square

[4]Larry Horn has, however, pointed out that even if we do not have a picture of the square of opposition by Aristotle, the Stagyrite suggested such a picture—see [39].

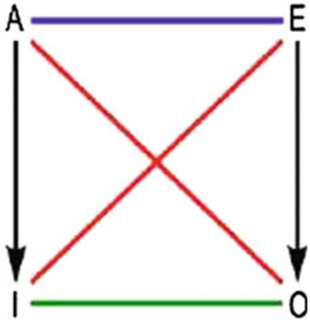


Fig. 2.10 Basic square of opposition

by Apuleius and Boethius, and the hexagon of Blanché.[5] This theory is not limited to a particular instantiation of the square figure, nor to the figure of the square itself. The backbone of this theory comprises three notions of opposition: contradiction, contrariety, and subcontrariety.[6]

These three notions can be defined as follows: two propositions are said to be *contradictories* iff they can neither be false, nor true together, *contraries*, iff they can be false together, but not true together, *subcontraries*, iff they can be true together, but not false together. These three notions of opposition can be applied directly or indirectly to concepts and properties in an intensional or extensional way. We can say that two concepts C and D are contradictories iff an object o cannot be at the same time C and D but has to be C or D. Putting this into propositions: "o is C" and "o is D" can neither be true nor false together. Extensionally speaking we can say that the sets of C-objects and D-objects are complement sets, a binary partition of the universe of objects. We can similarly adapt the two other notions of opposition, i.e. contrariety and subcontrariety, to concepts.

One of the basic figures presenting some relations between these three notions of opposition is the square represented by (Fig. 2.10). We have kept here the traditional names for the four corners, but these corners can be interpreted in many different ways: a variety of propositions and concepts, logical, metalogical, and of any field. Since 2003 [7] we have introduced colors for the three oppositions: red for contradiction, blue for contrariety, green for subcontrariety.[7] In black appears, besides the three notions of opposition, the notion of subalternation.

When we have a pair of contradictory concepts, we can talk about a contradiction. For example, in plane standard geometry a curved straight line is a contradiction. In other words: an object cannot be both a curve and a straight line.

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[5]The work of Blanché has been published in [23–25], about the hexagon see [12].

[6]Since 2007 we are organizing a world congress on the square of opposition. The first edition happened in Montreux, the second in Corsica in 2010, the third in Beirut in 2013, the fourth in the Vatican in 2014, the next one is projected to happen in Easter Island in 2016—see http://www.square-of-opposition.org. Related publications are [11, 14, 18–22].

[7]These are the three primitive colors. The theory of opposition can also be applied to the theory of colors, see in particular the hexagon of colors of Dany Jaspers [40].

Figure 2.11 is not more a curved line than Fig. 2.1 is a round circle, but it is a juxtaposition of two images showing the contrast between the two concepts. The figure shows that something cannot be at the same time a curve and a straight line.

In the school of Pythagoras, there was the idea to explain everything by a series of pairs of concepts, considered as contradictory, listed in the Fig. 2.12.

What is interesting in this table is that the two sides of each pair are rather positive. One is not explicitly thought as the negation of the other, linguistically, and/or conceptually (excepted finite/infinite). One could argue that the idea of "classical" negation arose from that and not vice versa. Classical negation is perhaps an abstraction from a series of concrete contradictions. Plato, who had strongly been influenced by the Pythagoreans, developed the method of dichotomy, a way of thinking dividing everything in two. This method is strikingly presented in the dialogue *The Sophist*, where it is used to catch the animal of the same name. Pythagoreans were considering mathematics as the most important science. For Plato there was a further step. It

Fig. 2.11 A curved *straight line* is a contradiction

Fig. 2.12 Pythagoras' table of opposites

Odd	Even
Finite	Infinite
Straight	Crooked
Square	Oblonge
Right	Left
One	Many
In	Out
Happy	Sad
Close	Open
Rest	Motion
Good	Evil
Light	Darkness

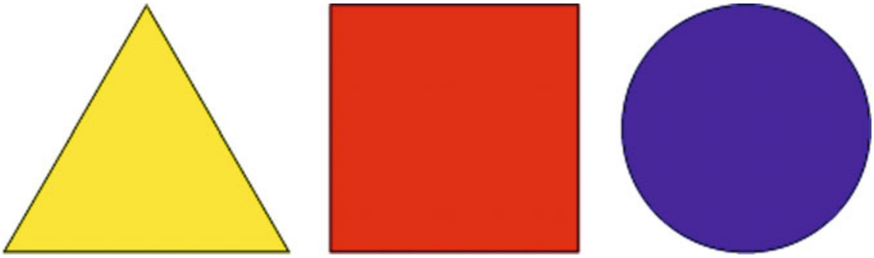

Fig. 2.13 A plane geometrical figure can be neither a *square* nor a *circle*

was *Dialectics*, a general methodology to think, reason and understand reality, the dichotomic procedure being a typical example of this methodology. Funny enough the word "dialectics" has been used later on by Hegel to denote something contrary (we use this word here in the technical sense defined above) to Plato's dialectics, the idea being that beyond the thesis and the antithesis, there is the synthesis. The table of opposites of Pythagoras is known in particular through Aristotle, but Aristotle went beyond the Pythagoro-Platonico dichotomy, long before Hegel and in a different way. He promoted the notion of contrariety (see, e.g., [1]). This is why he is considered as the father of the square of opposition. What is interesting in the square theory is that the dichotomy truth and falsity generates a trichotomy of oppositions.

Let us now come back to our mascot, the round square. Is its status the same as the curved line? No. From the point of view of standard plane geometry,[8] a figure can be neither a square nor a circle, for example, a triangle. Square and circle are not contradictory concepts, but contrary concepts: something cannot be at the same time a square and circle. A curved line is a true contradiction, a round square a fake contradiction (Fig. 2.13).

One may find two ways to explain the semantical sliding justifying naming a round square a contradiction, or qualifying it as such. The first justification is that contradiction and contrariety are both of the same family which can be labeled the *incompatibility* family: two propositions are incompatible if they cannot be true together, two concepts are incompatible if they have nothing in common. Maybe someone by saying that a round square is a contradiction has just in mind the notion of incompatibility. The second justification would be that a circle is considered as a typical representative of non-angular figures and a square as a typical representative of angular figures. But if non-angular is understood as with no angles at all, this would not work unless we define angular figures as figures having at least one angle. Angular and non-angular are in fact rather considered as a contrary pair of opposites of the same type as the famous pair which is at the the top of the square of quantification

[8] This context is important, not only to rule out other geometries—one may claim that a point is both a straight and a curved line, so that a curved line is not a contradiction, but in standard geometry a point is not a line—but also objects out of the scope of geometry, like an abstract concept such as beauty. It is possible to say that beauty is neither a square nor a circle, but this is not necessarily a convincing example to sustain that square and circle are not contradictory.

(all vs. none). In this case the "non" of "non-angular" is understood as a contrary negation (see next section).

These kinds of semantical slidings are quite common. One may consider that there are part of the semantical process which is based on variations of meanings leading sometimes to the situation where a word has at some stage a meaning opposite to a previous one. These semantical slidings can be explained in different ways, for example, by describing their mechanisms, a work which has been initiated by Bréal himself in his original 1897 book *Essai de sémantique—science des significations* [28], coining the word "sémantique" which has been later on increasingly popular. But a description of a phenomenon does mean that the phenomenon is right even if it is real. On the one hand one may want to justify some semantical variations with a theory of meaning explaining that they are coherent, this is for example the line of work developed by Larry Horn with the neo-Gricean notion of scalar implicature [38]. Some people may also argue that these slidings have a interesting creative aspect.[9] But such slidings can be consciously or unconsciously used in a dangerous way promoting confusion, this is common in advertisement and politics, part of the most monstrous creatures of the zoo of fallacies.

2.3 Negation and Contradiction

The notion of contradiction according to the square of opposition does not directly depend on the notion of negation, but only on the notions of truth and falsity. And we can define negation from the notion of contradiction, saying that two contradictory propositions or concepts are the negations of each other.

On the other hand it is also possible to define contradiction from negation, saying that the two propositions p and $\neg p$ form a contradiction. If we consider that $\neg$ is classical negation then this definition is equivalent to the square notion of contradiction. One of the most classical definition of classical negation is based on truth and falsity: p is true iff $\neg p$ is false. In this definition truth and falsity are considered as forming a dichotomy, the same dichotomy used to define the three notions of opposition of the square of opposition.

In the same way that this dichotomy can be used to define three types of oppositions, it can also be used to define three kinds of negations:

1. p is true iff $\neg p$ is false
2. if p is true then $\neg p$ is false, but not the converse
3. if p is false then $\neg p$ is true, but not the converse

[9] André Breton promoted as a key feature of surrealist writing the idea of "carambolage sémantique" [29]. But this is not the same as a "dérapage sémantique." The idea is to create a poetic effect by putting together opposed notions, leading to a sense of absurdity. Flaubert used systematically in his masterpiece *Bouvard et Pécuchet* [35] a process qualified as "antithetic juxtaposition" consisting of putting side by side two different opinions or theories. This was to show that human knowledge is not really coherent.

Note that these definitions are equivalent to the three following:

1. p and $\neg p$ cannot be true together, cannot be false together
2. p and $\neg p$ can be false together, but cannot be true together
3. p and $\neg p$ can be true together, but cannot be false together

And this second formulation clearly shows that there is a one-to-one correspondence between these three negations and the three oppositions of the square theory. To emphasize this connection and also to avoid words proliferation we can call these three negations:

1. contradictory negation
2. contrary negation
3. subcontrary negation

Let us apply these definitions to our mascot, the round square. If we have a contradictory negation, we cannot say that a square is a non-circle, we need a contrary negation and, yes, from this point of view a square can be considered as a non-circle and a circle as a non-square.

Someone may want to defend the idea that a "real" or "true" negation must be a contradictory negation. But what is the reality of negation, if any? One can claim that the word "negation" is, or, has been, used in correspondence with an operator behaving like a contradictory negation. This is ambiguous. Does this mean that the contradictory negation of classical logic is a good description of the way we use the word or that we should reason on the basis of such a negation? The ambiguity is also present the other way round. If someone rejects the classical position, does this mean that classical negation is not a good description of the way we are using negation in natural language and thought or does this mean that we shall use another negation?

Let us emphasize that it is a bit artificial to claim that classical logic is natural. Take the example of a classical non-cat. It is an abstract entity of which we do not have a positive idea or image, because the objects which are non-cats is a class of heterogeneous objects (ranging from dogs to cars through numbers). At the end we can produce an image only incorporating the abstract symbol of the cross (Fig. 2.14).[10]

On the other hand to say that classical negation is wrong, like Richard Routley, who liked to claim that every morning before breakfast, seems exaggerated.[11] Contradictory negation is the product of abstraction and abstraction is a fundamental power of human mind. The full strength of contradictory negation has to be recognized, this negation is not something which has to be rejected, but which has to be used with moderation. We do not support the idea that classical negation is the only negation and that we cannot use the word "negation" for other operators. This does not mean that we can use this word in an arbitrary way. We believe it is important to give the right name to the right thing, not based on a purely descriptive perspective, but by developing a theory which is, as any theory, relatively normative, keeping an

[10] For more discussion about the variety of symbolism, see [17].

[11] This was reported to me by Newton da Costa. He faced this phenomenon when visiting the Australopithecus in his own country in the 1970s.

Fig. 2.14 A non-cat is an abstract entity

equilibrium with description, the way the concept and the word are used. We defend the idea that the three above negations deserve to be qualified as negations. This is in particular coherent with the theory of the square of opposition. This is a also coherent with the development of modern logic where intuitionistic negation, which is a specific example of contrary negation, is called a "negation."

And to use the same symbol, "$\neg$," for different negations corresponds to a natural procedure of "abus de language" common in mathematics where the same symbol, "0", is used for different numbers having different properties, to keep trace to their common properties. The idea of a perfect unambiguous language in science promoted by Frege (see [36]) and some neopositivists seems absurd to us nowadays.

If we want to put in the same bag contradictory and contrary negations, we can talk about *incompatible negations* or *negations of incompatibility*, the definition being that p and $\neg p$ cannot be true together. Someone may claim that a negation should be an incompatible negation that we have to exclude subcontrary negations. This is a kind of neo-Aristotelian position, because the Stagyrite rejected subcontrariety as an opposition. But there is a strong symmetry and duality between contrariety and subcontrariety that is clearly revealed by the picture of the square. In modern logic, if one admits a contrary negation, like intuitionistic negation, there is no good reasons to reject its dual, which is a subcontrary negation, part of the family of paraconsistent negations.

There are different ways of dualizing intuitionistic negation. I. Urbas presented a dualization based on sequent calculus considering restriction of one formula on the left instead as on the right [51]. I have myself worked on a dualization based on modal interpretation which can be extended to other contrary negations, defined

Fig. 2.15 Duality between contrary and subcontrary negations in modal logic

as "not possible," $\neg\Diamond$, where $\neg$ is classical negation, following the interpretation of intuitionistic negation in S4 by Gödel [37]. The dualization of $\neg\Diamond$ is $\neg\Box$, which is a subcontrary negation as illustrated by Fig. 2.15.[12]

2.4 Paraconsistent Logic and Contradiction

The starting point of paraconsistent logic is to reject the so-called law of explosion.[13] It means that we have a negation $\neg$ and propositions p and q such that:

$$p, \neg p \nvdash q$$

Considering a basic general Tarskian framework for consequence relation this is equivalent as to say that there is a proposition p, such that p and $\neg p$ can be true together—see [13, 41].

According to the theory of opposition, p and $\neg p$ do not therefore form a contradictory pair. They are at best a subcontrary pair, and paraconsistent negation at best a subcontrary negation. The place where there are contradictions is a logic with a classical negation. If there are contradictions in a paraconsistent logic it is because it is possible to define a classical negation within it, like in the paraconsistent logic $C1$ of Newton da Costa [31].

If someone says that given a paraconsistent negation $\neg$, p and $\neg p$ form a contradiction, she is changing the meaning of the word "contradiction," giving it a meaning opposite to the one it has in the theory of the square of opposition. The square is not a sacred cow and we do not necessarily need to be very strict with the use of the words, but bilateral exchange of meanings certainly leads to confusion: if someone calls a

[12] As explained in [11], not satisfied with this octagon, I split it in three stars that I put together in a three-dimensional polyhedron of opposition which also perfectly reflects the duality and symmetry between these two negations. The multidimensional theory of opposition has been further developed by Moretti [42], Smessaert [49] and Pélissier [45].

[13] For a detailed discussion about how to define a paraconsistent negation, see [5, 6].

square a circle and a circle a square, she will be able to claim that a circle has four corners and so on. Such claim may attract the attention, like many "tours de passe passe," but it is just a trick. G. Priest has gone somewhat in this direction, apparently not aware himself at first of the confusion, because he has even used the standard definitions of the square of opposition to claim that the negation of his system LP was a real negation, by contrast to the negation of da Costa system $C1$ (see [8, 46–48]). He has also introduced the word "dialetheia" to talk about a proposition p such that p and $\neg p$ can be true together. A dialetheia p is therefore not a contradiction considering that p and $\neg p$ do not form a contradictory pair.

To avoid any ambiguity it is better to call "paraconsistent" a formula such that p and $\neg p$ can be true together. A paraconsistent formula p and its paraconsistent negation $\neg p$ do not form a contradictory pair. And a paraconsistent formula is not a *trivial* formula, a formula from which everything follows. On the contrary it is a non trivial formula. From the point of view of a Tarskian consequence relation this definition of trivial formula is the same as the definition of a formula having no models, being always false.

Wittgenstein in the *Tractatus* [52] calls a trivial formula, a contradiction, by contrast to a tautology, a formula which is always true. In some sense it seems better to use the word "antilogy" to talk about a trivial formula, because the abstract idea of triviality does not depend on contradictory pair of formulas or/and on contradictory negation.[14] However, there is a relation and for Wittgenstein a typical example of a trivial formula is the formula of classical logic $p \wedge \neg p$, which can be seen as a pair of contradictory propositions. Tarski was at some point considering as an additional axiom of the consequence operator theory, the existence of at least a trivial formula (cf. Axiom 5 of [50]). Such kind of a formula is nowadays often singled out using the symbol $\perp$. What we know is that a trivial formula is related to negation. If we have a classical implication $\rightarrow$, the formula $p \rightarrow \perp$ has the bevahiour of a classical negation. And if we have an intuitionistic implication $\rightarrow$, the formula $p \rightarrow \perp$ has the bevahior of an intuitionistic negation. But we may have a logic with a negation and without a trivial formula, without contradiction, it is the case of the logic LP which has a subcontrary negation.

To finish let us explain why there is a good reason not to identify paraconsistent negation with subcontrary negation. This is because it is possible to have paraconsistent negations which are paranormal negations. A *paranormal* negation $\neg$ is a negation such that p and $\neg p$ can be true together and can be false together. Can we really still talk about negation for such an operator? A positive reply to this question is given by De Morgan logic, logic in which the four De Morgan laws hold as well as double negation, but where we do not have explosion, nor the validity of the law

[14] At the metalevel, tautology and antilogy form a contrary pair, see the metalogical hexagon presented in [16].

of excluded middle. A De Morgan negation seems to have enough properties to be called a negation.[15]

Now can we say that two propositions *p* and *q* are opposite if *p* and *q* can both be true and also can both be false? Yes if we put some additional properties corresponding to De Morgan laws and double negation. Adopting this "loose" perspective, we can defend the idea that a round square is a paranormal object. Because on the one hand, as we have pointed out, something can be neither a square, nor a circle, for example, a triangle and one the other hand something can appear as both a square and a circle, as illustrated by Fig. 2.6. At the end this figure is not a good metaphor for quantum physics, because a quanton may appear as a wave and as a particle, but may not be something else, so a quanton is rather a subcontrary object.

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[15] The expression "paranormal logic" was used in the paper [32] where a paranormal logic different from De Morgan logic was introduced. De Morgan logic is derived from De Morgan algebra, for details about this, see [15].

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