

Preface

Over the last three decades, a variety of qualitative research methods have emerged within mathematics education. In 2003, two volumes of ZDM were dedicated to such methods in mathematics education with a focus on interpretative research, to promote a discussion about qualitative methods. Those two volumes necessarily presented only a selection of the range of research available. This book provides a different selection, including chapters based on research since 2003 and research approaches not included in the ZDM volumes, with some overlap in areas of particular importance. It continues the discussion, bringing additional depth and variety and including the close relationship between theory and methodology.

In his book on doing qualitative research, Roth (2005) describes how participating in research practice helps students to understand methodologies in a much better way than general how-to-do descriptions are able to achieve (see also Roth 2006). Given that doing research is more than can be written down as a procedure or as a description, how can a book offer an in-depth insight into such a methodologically enriched process?

In handbooks on research in science and mathematics education (English 2002; Kelley et al. 2008; Lester 2007; Kelly and Lesh 2000), we find chapters on methodological considerations (Cobb 2007; Cobb and Gravemeijer 2008; Lesh 2000, 2002; Schoenfeld 2002, 2007; Silver and Herbst 2007), but detailed descriptions on how methodologies are substantiated in a specific project, how they are implemented to investigate a research question, and how they are used to capture the research objects are normally missing. One exception is a monograph edited by Teppo (1998). Therein scholars have outlined general descriptions of methodologies that they illustrated by examples from their own research. For example, Goldin (1998) described task-based interviews on problem solving this way, and Pirie (1998) exhibited her search for a methodology and her decision-making process in research, concluding as follows:

...this choice of methodology, should not be undertaken hastily. We must review imaginatively the range of possible approaches to answering our research questions. One approach may at first sight appear seductive, but it is in the details that the connections between questions and successful explorations lie. (p. 96)

The editors of this book share Pirie's and Roth's views and have looked for a way to make such a search for adequate methodologies easier to accomplish by documenting and offering insight into a variety of different methodologies and how each of them can be used in research. However, this was not the only reason for publishing this book. We also felt that like every research discipline, scholars in mathematics education also should communicate their new developments in research methodologies and make them accessible to others in order to sustain a critical debate about methodologies in our field. This is especially demanding for qualitative methodologies because they are deeply intertwined with the respective research objects and research goals. To solve this problem, we have chosen a format that devotes to each research methodology one part of the book. Each part includes both a description of the theoretical and methodological underpinnings of the research approach and a concrete research example of how the approach is used in practice. Some parts describe the underpinnings and the example in two separate chapters, while others take an integrated approach. This structure means the reader can use the book also as an actual guide for the selection of an appropriate methodology, on the basis of both methodological depth and practical implications. The methods and examples presented are not intended as procedures to imitate, but rather they illustrate how different methodologies come to life when applied to a specific question in a specific context.

The exception to this structure is Part **XI** which presents three alternate approaches to design-based research. It illustrates how cultural and institutional contexts may not only require distinctive and sophisticated methodical adaptations, but also can imply fundamentally different methodological and theoretical underpinnings. Design-based research in the tradition of Realistic Mathematics Education in the Netherlands, didactical engineering in the French didactical culture, and conducting educational design research in the US context to support system-wide instructional improvement demonstrate substantially distinct understandings of design-based research. The theoretical underpinnings described and the examples of the three contributions in this part illustrate this.

Many of the methodologies presented in this book are also used outside mathematics education, but the examples provided are chosen so as to situate the approach in a mathematical and educational context. Some of the methodologies are well known in mathematics education, while others provide innovative approaches to research that readers may not have encountered previously. The contributors come from a wide range of backgrounds within and outside mathematics education, including both experienced and new researchers.

In the first part, Anne R. Teppo provides an introduction to grounded theory as a methodology, beginning with Glaser and Strauss's seminal work in 1967. A clear layout of basic ideas and methodical principles allows the reader to establish a fundamental understanding of essential methods of grounded theory. Teppo's further discussion of variations of this approach by second-generation researchers then insightfully reveals the underlying, and sometimes diverging, methodological perspectives of grounded theory approaches. In the second chapter of Part **I**, Maike Vollstedt illustrates in the context of mathematics education how such a perspective

can shape the way a grounded theory is developed methodically. Based on Strauss and Corbin's (1990/1996) outlines of grounded theory, she constitutes the concept of personal meaning from interview data, collected in Germany and Hong Kong in an intercultural study. Through a pragmatic interpretation of theoretical sampling, comparing codes, and using a coding paradigm, Vollstedt identifies different types of personal meanings and describes conditions of their emergence, which constitute significant elements of a theory of personal meaning in mathematics learning.

In the second part, Götz Krummheuer, Christine Knipping, and David Reid offer two different perspectives on reconstructing social interaction and argumentation in mathematics classrooms, both following Toulmin's theory of argumentation. For Krummheuer, argumentative learning is the essential research agenda, and so methodical analyses of students' participation in collective argumentation are central in his approach. Goffman's idea of decomposition of the speaker's role is a key element in this. While Krummheuer focuses on elementary classrooms and locally developed arguments, Knipping and Reid contribute a "global" model of argumentation, based on empirical research in secondary classrooms. As their focus is reconstructing entire proving processes in the mathematics classroom in this context, comparative methods that allow description of the "gross, anatomical structure" and rationale of the emerging global arguments are essential. Both chapters provide examples to illustrate the methodologies.

In the third part, Angelika Bikner-Ahsbabs shows how the construction of ideal types can be used as a methodological principle of theory construction. She first explains the underlying idea of ideal types, different kinds of these, and their role in theory development. She then illustrates methodical principles of ideal type construction and demonstrates how different heuristics for generating these can ground an emerging theory in empirical contexts. In her second contribution to this part, Bikner-Ahsbabs discusses an example of the ideal type reconstruction of epistemic processes in so-called interest-dense-situations. Key features of structures of these situations are singled out using an approach divided into four steps. Based on these characteristics, several ideal types are construed, providing theoretical insights into the dynamics of epistemic processes.

In Part IV, Luis Radford and Cristina Sabena present a methodology based on a Vygotskian perspective on semiotics. They describe the Vygotskian semiotic approach in terms of an interrelated triplet of principles, methodology, and research questions and refer in particular to two methodological constructs: the semiotic node and the semiotic bundle. In the second half of their chapter, they illustrate the semiotic approach with an example of the analysis of pattern generalization in classroom activity. The research reported in the example, concerning the role of words, gestures, and rhythm in the process of becoming aware of mathematical relationships, contributed to the development of the semiotic approach when unexpected data required the transformation of the theory, methods, and research questions.

In Part V, Tommy Dreyfus, Rina Hershkowitz, and Baruch Schwarz present Abstraction in Context (AiC) as a theoretical-methodological approach for researching students' knowledge constructions. Emergence of constructs that are new to students is investigated, taking into account the particular learning environments

and their specific mathematical, curricular, and social components. The authors are especially interested in an integral approach that allows the study of learners' processes of constructing abstract mathematical knowledge, within a methodology based on the AiC theoretical framework. The main methodological tools of AiC are three observable epistemic actions: Recognizing, Building-with, and Construction. A specific example illustrates how these actions and AiC as a theoretical-methodological approach can be applied in a methodical way in research.

In Part VI, the networking of theories is proposed as a methodology by Ivy Kidron and Angelika Bikner-Ahsbahr. Both authors discuss and demonstrate how engaging different theoretical frameworks and models can allow for a more comprehensive understanding of concepts and phenomena on the one hand and the theories involved on the other. The authors argue that this can be done in a strategic, methodological way. Networking strategies and cross-methodologies are presented and illustrated briefly by research examples in the first chapter of this part. In the second chapter, one research example on combining the theory of Abstraction in Context, a cognitive approach, with the theory of interest-dense-situations, a social approach, pictures how the networking process is accomplished. The authors demonstrate how bringing these two perspectives together offers methodologically new ground for gaining insights into students' epistemic processes when learning mathematics.

Part VII offers a methodology for studying classroom processes of teaching and learning over significant spans of time. In the first sections of their chapter, Geoffrey B. Saxe, Kenton de Kirby, Marie Le, Yasmin Sitabkhan, and Bona Kang present a conceptual framework for understanding the reproduction and alteration of a common ground in classroom communities through time. This framework incorporates analyses at collective and individual levels, looking for collective norms and the function of individuals' use of representations. Later in the chapter, a 19-lesson sequence on integers and fractions is introduced as an example of design research based on the conceptual framework presented earlier. The organization of empirical analysis based on this framework is described. The empirical methods and techniques presented illustrate the innovative potential of this multilevel analytic approach, which is further discussed in the conclusion of the chapter.

Qualitative methodologies are the main focus of this book; however, in Part VIII, Udo Kelle and Nils Buchholtz point to the limitations of a purely qualitative approach. They critically review the continuing dispute about qualitative and quantitative research methods that overshadows research in mathematics education. Both authors question the restriction to either quantitative or qualitative methods, which they find particularly striking in research on teacher knowledge. They argue how a "mixed methods design" can enrich educational research in this domain. Based on data and results from an empirical study on a teacher training program in mathematics, they demonstrate how a mixed methods approach can mutually validate qualitative and quantitative findings.

In Part IX, a mixed methods approach to text analysis, "Qualitative Content Analysis," is introduced. This approach is well established within the social sciences, but it has only recently been applied within mathematics education. In the first

chapter of this part, Philipp Mayring describes the theoretical background and methodical procedures of this approach to text analysis. He concludes by comparing these procedures with similar techniques in other methodological approaches, reflecting on strengths and weaknesses of each approach. In the second chapter of this part, Björn Schwarz addresses an example for applying qualitative content analytic methods in a study on professional competence for future mathematics teachers. First, he substantiates why this methodology was implemented into the study and then describes how this was done, while demonstrating the added value of involving inductive and deductive procedures of this methodology.

The idea of validation is critically reflected on in Part X. Ida Ah Chee Mok and David Clarke argue that methodologies required by cross-cultural comparative research are poorly served by the use of triangulation as a mechanism of convergence, but benefit from a wider understanding of triangulation that involves complementary accounts instead. Their argument is illustrated by examples taken from the Learner's Perspective Study (LPS), which examined patterns of participation in the mathematics classroom in 18 countries. Mok and Clarke offer a more in-depth look into how different forms of triangulation, including what they call cultural triangulation, are able to portray the variation of real class activities, by means of the description of two studies, namely, a study on lesson structures of classrooms in Hong Kong and Shanghai and a cross-cultural comparison of learning tasks.

Part XI, on design research as a research methodology, is divided into three chapters. In the first chapter, Arthur Bakker and Dolly van Eerde offer an introduction to design-based research as a specific kind of this methodological approach in realistic mathematics education, reviewing key features of it and how validity and reliability are interpreted. Illustrating and reflecting the pivotal methodical steps and the role of the theory of design-based research, they close with an example from statistics education.

In the second chapter of this part, Michèle Artigue considers didactical engineering in the French tradition as a case of design research. She describes the evolution of didactical engineering, its characteristics as a research methodology, and its close connections with the development of the theory of didactical situations. Current developments within this design culture are described, in particular the integration of a design element into the anthropological theory of didactics, and second-generation didactical engineering. Specific examples are used to illustrate methodological principles.

Educational design research can be employed at different levels, from the design of a single task to longer sequences of classroom activity and beyond. In the third chapter of Part XI, Erin Henrick, Paul Cobb, and Kara Jackson describe educational design research in the context of school system-wide instructional improvement. They expound the theoretical framework for design research at this level and its research focus on wide-scale instructional improvement. The authors discern that design studies at this level are interventionist in nature, and they describe how researchers address both the complexity of educational settings and the problems that various participants in those settings encounter as they endeavor to make improvements. Examples drawn from the MIST study (study on Middle-School

Mathematics and the Institutional Setting of Teaching), as one of the few design studies conducted at this level, are used to illustrate these points.

Taken together, these 11 parts provide a systematic account of a variety of directions in which qualitative research in mathematics education is moving, through an analysis of the essential interaction between theoretical and methodological aspects of this research. In each case, a description of a pragmatic example in which the methodology has been used brings these considerations to life, thus adumbrating ways in which certain methodologies bring certain issues to the fore. We summarize the connections between the parts in Part XII. The account is of necessity incomplete: As research in mathematics education continues to evolve, so do the tools with which researchers investigate their questions. Even as a snapshot of current research, the account is incomplete, because we have chosen to highlight developments in qualitative methodologies, with only a small glimpse into their interactions with their quantitative counterparts. However, the usefulness of this book lies in the juxtaposition, with practical examples, of accounts of theoretical and methodological aspects of qualitative mathematics education research that, taken together, illustrate the current state of the art.

Bremen, Germany

Angelika Bikner-Ahsbahr
Christine Knipping
Norma Presmeg

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Approaches to Qualitative Research in Mathematics
Education

Examples of Methodology and Methods

Bikner-Ahsbahr, A.; Knipping, C.; Presmeg, N.C. (Eds.)

2015, XV, 592 p. 112 illus., 18 illus. in color.,

ISBN: 978-94-017-9181-6