Chapter 2
Mathematics Education in Secondary Schools and Didactics of Mathematics in the Period Between the Two World Wars

> *Om mijn verhaal goed in te kaderen, moet ik nog veel verder teruggaan en dan tot 1924—jaren voor ik in Nederland kwam—het soort annus mirabilis van de wiskundedidactiek in Nederland. (To embed my story well, I have to go back much further than 1924—years before I came to the Netherlands—a kind of annus mirabilis in the didactics of mathematics in the Netherlands.)*
> Hans Freudenthal on the Dutch mathematics education in *Schrijf dat op, Hans,* 1987 [1]

Hans Freudenthal's first introduction to the Dutch educational system took place at the end of 1930 when, at the age of 26, he was employed by the University of Amsterdam. His image of the Dutch student was that of a diligent, not particularly self-reliant pupil who on top of that was also slow-witted. It was remarkable that Freudenthal blamed the (rote) learning of many foreign languages for the passive attitude of the students. He wrote in a letter in 1931:

> Das Niveau des hollaendischen Studenten ist geringer als das des deutschen; das haengt mit dem etwas schwerfaelligen, traetigen Volkskarakter zusammen. Der hollaendische Student arbeitet sicher fleissich, begreift aber sehr langsam. Selbstaendigen produktives Arbeiten findet man bei ihm sehr selten. Schuld daran ist vielleicht auch das Viellernenmuessen, das von dem Zwang Sprachen zu beherrschen, der bei einem kleinen Volk verstaendlich ist, herkommt […] Leider habe ich mit den Studenten noch nicht viel Kontakt bekommen und sehe sicherlich manche Dinge noch sehr vom Gesichtspunkt des Dozenten. Die Studenten sind sehr schuechtern, stellen weder in der Vorlesung noch nachher Fragen, man kommt sich waehrend des Vortrages wie ein Prediger in der Wueste vor. (The level of education of the Dutch student is lower than that of the German student; that is related to the somewhat ponderous, obtuse character of the people. The Dutch student certainly works very diligently, but understands very slowly. You will rarely find him working independently and productively. Perhaps having to learn such a great deal, because of the obligation to learn foreign languages, as is understandable for a small nation, is to blame. Unfortunately, I have not yet been much in contact with students and it is certain that there are a number of things I only know from the viewpoint of a teacher. The students are very shy, do not ask questions during or after lectures, and as a teacher you feel like a preacher in the wilderness when lecturing.) [2]

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Freudenthal was doing nothing more than expressing his astonishment about the differences between Dutch and German students. In the 1930s he spent most of his time working as the assistant of the mathematician L.E.J. Brouwer. During the Second World War his interest was aroused by the teaching of arithmetic and the accompanying didactics of arithmetic at primary schools and he was engaged in a literature study on the subject [3].

It was only after the war that Freudenthal became actively involved in mathematics education and didactics of mathematics in the Netherlands and would go public with his ideas. In the 1940s and 1950s he particularly focussed on mathematics education at secondary schools (VHMO, Voorbereidend Hoger en Middelbaar Onderwijs, Prepatory Higher and Secondary Education). It was not the frontiers of knowledge he was studying. Quite a lot of discussions had already been held in that field before Freudenthal became involved.

To get an idea of how Freudenthal fitted into this mathematical-didactical landscape, a rough outline of mathematics education in the period between the two World Wars is needed first. What did secondary schools (VHMO) and the way mathematics was taught look like at that time? Which discussions were held? And who were the leading participants in these discussions? [4].

2.1 Secondary Education in the Period Between the Two World Wars

2.1.1 The Origin of the School Types in Secondary Education

Around 1918 secondary education in the Netherlands was governed by the 1863 law on secondary education, the MO or *Middelbaar Onderwijs* law, the first concerning secondary education. Various types of school such as the ULO—*Uitgebreid Lager Onderwijs*—for extended primary education, and the MULO—*Meer Uitgebreid Lager Onderwijs*—for further extended primary education, were in the primary education category. However, the Gymnasium (comparable to Grammar school in the UK) was part of higher education. The author of the MO-law, liberal Johan Rudolf Thorbecke (1798–1872), clearly had a dual function of education, directly related to the hierarchy of the class society, in view.

First of all there was the kind of school type that would offer a training course directly aimed at the needs of society. In Thorbecke's view this education was meant for the 'core of the society', the large middle class (consisting of the 'humble middle class' and the 'upper middle class'). This group needed a modern, non-vocational yet practice-oriented type of education. This would prepare the children from the middle classes for their future tasks in society, for example working in trade and industry and offices. To this aim a new school type was designed for the middle class (HBS: Hogere Burger School, Grammar school but without the Classic

languages). The Polytechnische School (Polytechnic School) in Delft was defined as an extension for some of the HBS students. Along with the secondary schools for girls (MMS) these school types formed the secondary education. The HBS and the Polytechnische School would prove to be the most successful types [5].

In addition, Thorbecke distinguished higher education as comprising the Gymnasium and the University. The Gymnasium was the only school that offered the Classic languages, education that was reserved for the children of the learned classes, and the only possible path to a university study [6]. Thorbecke, advocate of a strict distinction between 'mind' (higher education) and 'matter' (secondary education), thus maintained the Gymnasium as a separate institute with a higher status.

2.1.2 Some School Types

2.1.2.1 The HBS

One of the characteristics of the Dutch educational system of that time was the freedom of choice that the schools had, at least at first. The MO-law did not lay down requirements for the final examination, neither for the subjects chosen nor for their subject matter. To guarantee that the matriculants had achieved a certain level of education, a state exam with rules concerning the design, the content and the evaluation of the final exam, was soon drawn up [7].

Unfortunately, this regulation resulted in an enormous overburdening for the HBS and too many subjects. There were also complaints about the outdated subject matter that was taught. Over a long period, ministers and school inspectors tried to produce new order and drafted several reforms but this never resulted in a full-scale reorganisation of the secondary education system.

Nevertheless, the HBS developed into the most successful type of secondary education among those proposed in the law of 1863. Being the counterpart of the Gymnasium with its prevalent Classic languages education the HBS accentuated modern languages, science and mathematics. On average HBS pupils put one third of their time into science and mathematics. As with other school types, classroom teaching was the most widespread method used in the HBS. Which subject matter was dealt with was determined by the requirements set by the final exam. There were two main streams in the exam programme: HBS-A, mainly focussing on languages, and HBS-B, on mathematics and sciences.

As far as mathematics was concerned the final exam for HBS-B consisted of arithmetic and algebra, trigonometry, stereometry and descriptive geometry. A mathematics class followed the pattern 'explanation, demonstration, imitation and practicing endlessly'. Pierre van Hiele, mathematics teacher and didactician, described the attitude at that time with respect to the mathematics that had to be taught as follows [8]:

> Geen leerling betwijfelde de noodzaak van bewijzen in de 'wiskunde'. Er waren er natuurlijk genoeg die daar moeite mee hadden. Als je wiskunde per se nodig had, deed je je uiterste best om er ten minste zoveel van te begrijpen dat je later niet in de moeilijkheden kwam. In het andere geval kon je je werpen op de vraagstukken om zo hoog mogelijk te

> scoren. De leraren speelden daarop in: zij beijverden zich om de stof zo exact mogelijk voor te dragen, maar zij hielpen de leerlingen met de uitwijkmogelijkheid van de vraagstukken. Per slot van rekening nam de bewijsvoering op het eindexamen niet zo'n grote plaats in. (There was no pupil who did not doubt the need of proofs in mathematics. Enough of them, of course, found proving theorems difficult. If you absolutely needed mathematics you did your utmost to understand at least enough to avoid difficulties later. If not, you could launch into solving problems to score as many points as possible. Teachers anticipated this: they tried their hardest to present the subject matter as accurately as possible, but also helped the pupils with the fall-back arrangement of solving problems. After all, proving theorems did not figure largely in the final exams.) [9]

Despite its great success there was also a lot of criticism of the HBS school type. E. Jensema, director of the Rijks-HBS (State-HBS) in Groningen in 1927, distinguished three points of general criticism. The education in the HBS would be too mathematical and too many pupils would leave the school prematurely. In addition, the school would be too dualistic by trying to offer both final and preparatory (higher) education at the same time [10].

2.1.2.2 The Gymnasium

The first Gymnasium was the result of a merger of the long-standing Latin School and the municipal primary school "of the upper class" in The Hague [11]. The Gymnasium had two sections: the first one, a continuation of the Latin School, emphasized teaching Latin and Greek as a preparation for a university study. The emphasis in the second one was on mathematics, physics and modern languages. This section met the society's growing need for education.

The Gymnasium kept its formal and social status after the adoption of the MO-law of 1863. From 1878 Latin schools were also classified as Gymnasia. Since the higher education act of 1876 both the programme and the timetable of the classes had been laid down precisely. Mathematics was one of the compulsory subjects.

The freedom in interpretation of the educational programme such as existed at the HBS was out of the question here. As the practical use in society of studying the Classic languages appeared to diminish, the antique argument of the formative value of the Classic languages was used to preserve the teaching of the Classics. But the competition with the HBS was felt more and more clearly and this led in 1876 to a division of the Gymnasia in a α-department/division (with a language-oriented programme) and a β-department/division (where apart from the Classic languages a great deal of attention was devoted to science and mathematics). The written final exam in mathematics for the β-department consisted of algebra, analytical geometry, stereometry and goniometry.

2.1.2.3 The MMS

The secondary school for girls (Middelbare Meisjes School, MMS) was one of the types of school which could freely develop and hold its ground. It offered an alternative to the HBS which was mainly attended by boys. There were no statutory

regulations to lay down content and form of the education. It was not until 1935 that some rights for further study were laid down for this type of school. Mathematics was not part of the final exam, which offered teachers and authors of the school books a great deal of freedom. Usually mathematics was only taught in the first 3 years of the MMS [12].

2.1.2.4 The Lyceum

The first Lyceum was established in The Hague in 1909 by headmaster R. Casimir (1877–1957) who later became the first professor of pedagogy in the Netherlands. The Lyceum, consisting of a Gymnasium, an HBS and sometimes an MMS too under one roof, originally aimed at postponing the choice for a specific school type by offering education in the first few years in combined classes. After all, when pupils had to choose between Gymnasium and HBS at the age of 12, it directly determined their options for further education. On top of this it was more efficient and less expensive to found three schools simultaneously rather than three separate ones.

The influential didactician and textbook writer Wansink wrote on the importance of the Lycea for the educational reform:

> De vernieuwende tendensen op pedagogisch-didactisch terrein zijn bij de lycea sterker geweest dan bij de meer starre vormen van hogereburgerschool en gymnasium in de V.H.M.O.-periode. Bij het lyceum werd in onze schoolorganisatie voor het eerst rekening gehouden met het kind als medebepalend element. (The innovative tendencies in the pedagogic-didactic field have been stronger in Lycea compared to the more uncompromising school types such as HBS and Gymnasium in the VHMO period. The Lyceum was the first school in the Dutch school organisation in which the child was considered a contributory element.) [13]

In a way the Lyceum can be seen as the predecessor of the modern ‘Scholengemeenschap’ (combined school). It was not until 1968 that Lycea were legally established as an existing school type. Until that time the Lyceum existed merely due to the grace which the law allowed for deviations in the organisation of Gymnasia [14].

2.1.3 The Competition between HBS and Gymnasium

From the moment the MO-law was adopted the view started to take root that the HBS would prepare pupils just as well (if not better!) for specific university studies (in particular medicine, science and mathematics). Increasingly, HBS-graduates continued their education at universities via a detour (for example by attending testimony classes in Greek, Latin and logic, and taking university entrance exams). From 1865 onwards the Education Ministry began granting exemption for these exams in exceptional cases. As from 1878, holders of an HBS diploma could also study medicine by doing exams that paralleled the academic exams.

There was an increasing number of such regulations allowing HBS-graduates to enter universities. This finally induced J. Limburg, member of parliament, to put forward a bill in parliament in 1917. His bill which, among other things, qualified

HBS-graduates for doing exams at the science and mathematics faculty, was passed by a majority vote. This definitively ended the matter-mind dichotomy which Thorbecke advocated. Apart from that it also implied people were no longer convinced that schooling in the Classics was a prerequisite for an academic study and it implicitly emphasized the appreciation for science and mathematics. It was now evident that the original plan for the HBS as a complete and final education for the whole society had failed. An attempt was made to rectify this deficiency by dividing the HBS in two, HBS-A (modern languages and economy) and HBS-B (with the emphasis on science and mathematics).

The assignment of the right to study at a university to HBS-B graduates (and later for some academic studies to HBS-A graduates as well) considerably weakened the monopoly position of the Gymnasium of course. To slightly increase the attraction of the Gymnasium for pupils wanting to study science and mathematics it was decided by Royal Decree to modernize it in 1919. The programme offered more space for science and mathematics. New, modern subjects such as differential calculus and integral calculus now became part of the algebra that was taught. Thus the Gymnasium was still one step ahead of the HBS. However this new difficult Gymnasium-β, combined with the traditional status of the Gymnasium, made it the most demanding school, reserved for a small elite only.

2.2 Discussions on the Mathematics Education at the VHMO

As a result of the industrialization in the Netherlands of the end of the nineteenth century, thanks to its practical value and its social usefulness, science enjoyed increasing prestige. After the First World War, however, a revolution in the appreciation for science and mathematics took place and these subjects consequently lost a lot of their prestige. The mathematician Dijksterhuis called this a spirit of the times hostile to the Mathesis, that influenced many ideas on the reform of the educational system [15].

After studying mathematics in Groningen E.J. Dijksterhuis (1892–1965) was for many years a mathematics teacher at the Rijks-HBS (State-HBS) in Tilburg and during his career he worked for various universities as a science historian (Fig. 2.1). Being an advocate of mathematics education organised on a logical-deductive basis he played an important role in the discussions about the mathematics education between the two World Wars. His demands for the strictness of the mathematics education were very stringent and he was deeply convinced that such a mathematics education would certainly emanate formative value.

The formative value of mathematics was an argument that to some extent kept coming back in the discussions about mathematics education. People were convinced that mathematics would have a positive influence on reasoning power in general and logical reasoning in particular—the so-called ‘stimulation of the mind’ [16]. The underlying idea was that transfer was possible: being able to apply skills

Fig. 2.1 E.J. Dijksterhuis

in a subject field different to the one in which they are acquired. In the literature a distinction is made between horizontal transfer (between tasks of similar order) and vertical transfer (from a task of a lower order to one of a higher order) [17]. It is called a positive transfer when the first task learned facilitates learning the second one. Inherent in the discussion on the formative value is the question whether 'learning to think' can be taught [18].

At the end of the Nineteenth century the consensus was that mathematics education had formative value. But in the United States of America at the beginning of the Twentieth century people started to use the argument of formative value in an opposite sense [19]. Psychologists like E.L. Thorndike published research results that suggested that there was hardly any transfer, if at all. This caused a shift in emphasis on the aim of the American arithmetic and mathematics education towards practical applications of these subjects [20]. In the United States the tendency turned in the 1930s, when new studies apparently showed that mathematics education aimed at 'understanding', in particular, did indeed produce transfer to a certain extent.

In the Netherlands similar tendencies were little noticed in the period between the two World Wars. Teachers still considered the formative value of mathematics an established fact and it was seen as an important goal of mathematics education. Pedagogues also agreed that mathematics education (and Classic languages too) served the purpose of learning to think (logically). The few critical comments, such as those of the Amsterdam University professor G. Mannoury in 1921 and later those of D. van Dantzig, did not have much influence [21]. According to Dijksterhuis mathematics education should "sharpen the intellect of the pupil and give him insight in the aspiration of the human mind that goes beyond the material from which science has arisen" [22].

The educational value of mathematics was not under discussion, but there was a lot of criticism of the overly mathematical nature of the HBS. The mathematician

H.J.E. Beth (1880–1952), director of the Rijks-HBS in Deventer, wrote in 1924 in the *Bijvoegsel van het Nieuw Tijdschrift voor Wiskunde*:

> De uitdrukkingswijze "meer en meer wiskundig karakter der HBS" is reeds zoo ingeburgerd […], dat menigeen, buiten het wiskundig kamp opgesteld (zelfs vele wiskundige collega's zijn het praatje gaan gelooven), zich met verbazing zal afvragen, of het dan niet waar is. Welnu, op deze vraag kunnen we antwoorden met één woord: nonsens! (The expression the "increasingly mathematical character of the HBS" has already been established to such an extent […] that many a person outside the mathematical field (even many colleague mathematicians started to believe the rumour) will wonder whether it is true or not. Well, there is only one answer to this: nonsense!) [23]

According to Beth the mathematical character of the HBS had even been "dubiously decreasing" for a couple of years. Compared to 25 years before there were indeed rather fewer hours reserved for mathematics [24]. The reduced appreciation for mathematics at the HBS mainly resulted from the disappointment of the many drop-outs for whom mathematics would have been a stumbling block. Beth also mentioned much-heard arguments such as "mathematics is of no use to me" and "not everybody is made for mathematics" [25]. However, in addition to these voices a plea was also to be heard in favour of a further modernization of mathematics at the HBS.

In the years between the two World Wars there were three important discussions in the field of mathematics education at the HBS. First, there was the discussion about the organisation of the initial geometry education. In addition there were attempts to modernize mathematics education at the HBS, centred around the curriculum design of the Beth committee and the discussion about the introduction of differential calculus and integral calculus. Finally there was the controversy around the subject of mechanics: where did that belong and the teacher of which discipline should teach it? These three discussions revealed the thoughts and motives of the various advocates and opponents and provided a good picture of the prevailing views regarding how and why mathematics is taught [26].

2.2.1 The Initial Geometry Education and the Foundation of the Journal Euclides

In 1924 mathematician Tatiana Ehrenfest-Afanassjewa (1876–1964) wrote the brochure *Wat kan en moet het Meetkunde-onderwijs aan een niet-wiskundige geven?* (*What can and should geometry education offer a non-mathematician?* [27]) Mrs. Ehrenfest was originally from Russia and completed her study in mathematics and physics in Saint Petersburg (Fig. 2.2). After that she studied in Göttingen with mathematicians Felix Klein and David Hilbert. In addition to her scientific research (in the field of theoretical physics and in other fields) she taught mathematics until 1908. She lived In Leyden for a long time from 1912 with her husband Paul Ehrenfest (1880–1933). Mrs. Ehrenfest was greatly interested in teaching and education and lived up to it by organising monthly mathematical-didactical colloquia for teachers at her house. Discussions about the, in her view fossilized, mathematics education in the Netherlands were brisk [28].

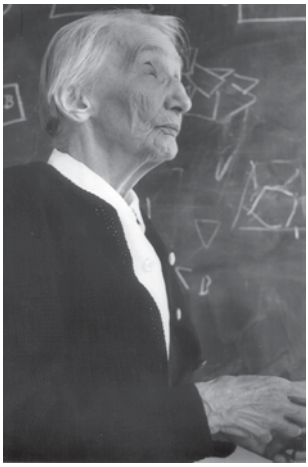

Fig. 2.2 T. Ehrenfest-Afanassjewa

The above-mentioned Dijksterhuis wrote a reaction to Mrs. Ehrenfest's brochure with the title *Moet het Meetkunde-onderwijs gewijzigd worden? (Should the geometry curriculum be changed?)*, which he sent to his publisher Noordhoff. The publisher suggested P. Wijdenes (1872–1972) to include the reaction in *Nieuw Tijdschrift voor de Wiskunde (New Journal for Mathematics)*. Apart from being a mathematics teacher in Amsterdam, Wijdenes was also a prominent author of text books to be used in mathematics education. This eventually led to the foundation of the periodical *Bijvoegsel van het Nieuw Tijdschrift voor Wiskunde gewijd aan Onderwijsbelangen (Supplement to the New Journal of Mathematics devoted to Educational Interests),* of which Wijdenes along with J.H. Schogt (also a mathematics teacher in Amsterdam) would be the editors (Fig. 2.3).

Three years later the *Bijvoegsel* would be renamed *Euclides*. *Tijdschrift voor de Didactiek der Exacte Vakken (Journal for the Didactics of Science and Mathematics)*, and still later the journal would become the bulletin of the associations of teachers, WIMECOS and LIWENAGEL. WIMECOS, the "Vereeniging van Leeraren in de Wiskunde, de Mechanica en de Cosmographie aan Hoogere Burgerscholen met vijfjarigen cursus B, Lycea en Meisjes-Hoogere Burgerscholen met 5-/6- jarigen cursus" (Association of teachers in mathematics, mechanics and cosmography at the 5-year HBS-B, Lycea and the 5 or 6 year HBS-B for girls), was founded in 1925. LIWENAGEL—"Leraren in de Wiskunde en Natuurwetenschappen aan Gymnasia en Lycea" (Teachers in Mathematics and Science at Gymnasia and Lycea)—had been established 4 years before that. All important discussions, modernizations and plans in the field of mathematics education would be brought forward in *Euclides* in the years to come [29].

What had Mrs. Ehrenfest written that provoked such a reaction? In her brochure she raised the question about the initial geometry education. The final aim she had in mind with this kind of education did not differ that much from Dijksterhuis': the lead-up to an Euclidian geometry built on axioms. However, the routes they proposed for getting there immediately made it clear that they differed fundamentally in opinion.

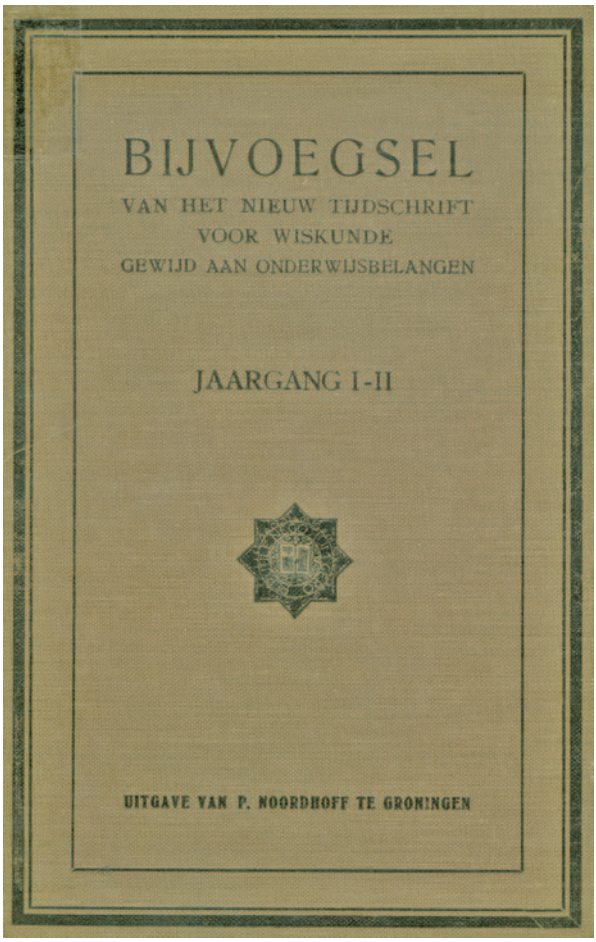

Fig. 2.3 The first two volumes of *Bijvoegsel van het Nieuw Tijdschrift voor Wiskunde gewijd aan Onderwijsbelangen*, later renamed *Euclides. Tijdschrift voor de Didactiek der Exacte Vakken*

Mrs. Ehrenfest argued in favour of preparing the pupils better for the logics of the axiomatically built geometry by first and foremost paying attention to developing their intuition and three-dimensional imagination. She wanted to realise this with an introductory course in which graphics and concrete action would take up a central position. In this course, at any rate in the initial education, the pupils would not have to prove directly the theorems that were the most evident to them.

Later she would develop such an introductory course in a more practical sense in her book *Uebungensammlung zu einer geometrischen Propädeuse (Exercise collection for an Introduction in Geometry)*. In this book she described close on 200 practical exercises to be used in the classroom. According to her the most important result a teacher could achieve if he used these exercises was:

> Man wird imstande sein die Schüler vor solche Probleme zu stellen, wovon der geometrische Inhalt ihnen vollkommen zugänglich sein wird, wo sie aber merken werden, dass für dessen sichere Beantwortung eine kritische Untersuchung nötig ist: sie werden den Impuls bekommen um Sätze zu beweisen! (You will be able to present the pupils with problems

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